

Similarity analysis for a heated ferrofluid flow in magnetic field

GABRIELLA BOGNÁR, KRISZTIÁN HRICZÓ

Faculty of Mechanical Engineering and Informatics

University of Miskolc

H-3515 Miskolc-Egyetemváros

HUNGARY

v.bognar.gabriella@uni-miskolc.hu, mathk@uni-miskolc.hu

Abstract: - The aim of this paper is to introduce new results on the magneto-thermomechanical interaction between heated viscous incompressible ferrofluid and a cold wall in the presence of a spatially varying magnetic field. Similarity transformation is applied to convert the governing nonlinear boundary layer equations into coupled nonlinear ordinary differential equations. This system is numerically solved using higher derivative method. The effects of governing parameters corresponding to various physical conditions are investigated. Numerical results are represented for the distributions of velocity and temperature, for the dimensionless wall skin friction and for heat transfer coefficients. Our results show excellent agreement with previous studies and obtained two solutions in some cases.

Key-Words: ferrofluid, magnetic field, boundary layer, similarity transformation (6 - 10 words)

1 Introduction

During the last several decades, liquids are intensive investigated by various researchers due to them number of application in industry. One of them is the nanofluid, which is a homogenous combination of base fluid and nanoparticles. These suspensions are made of various metals or non-metals e.g., aluminium (Al), copper (Cu), Silver (Ag), and graphite or carbon nanotubes respectively, and the base fluid, which includes water, oil or ethylene glycol. Ferrofluid is a special type of nanofluids, where nanoparticles can be magnetized in the suspension.

Nanofluids can be used in many areas in our daily lives and technological processes. Such type of applications includes heat exchanger, vehicle cooling, nuclear reactor, cooling of electronic devices. The magneto nanofluids are also very much helpful in magnetic drug targeting in cancer diseases, hyperthermia, wound treatments, removal of blockage in the arteries, magnetic resonance imaging (MRI) etc. (see [8]).

When magnetizable materials are subjected to an external magnetizing field \mathbf{H} , the magnetic dipoles or line currents in the material will align and create a magnetization \mathbf{M} .

Problem of magnetohydrodynamic (MHD) flow near infinite plate has been studied intensively by a number of investigators (see, e.g., [1], [2], [4], [5],

[6], [9], [10], [15]). The hydrodynamic flow of MHD fluids was studied when the applied transverse magnetic field is assumed to be uniform.

Neuringer [14] has investigated numerically the dynamic response of ferrofluids to the application of non-uniform magnetic fields with studying the effect of magnetic field on two cases, the two-dimensional stagnation point flow of a heated ferrofluid against a cold wall and the two-dimensional parallel flow of a heated ferrofluid along a wall with linearly decreasing surface temperature.

The aim of this paper is investigating the static behaviour of ferrofluids in magnetic fields with similarity analysis. This technique is applied on the governing equations to transform partial differential equations to nonlinear ordinary differential equations. A numerical solution is obtained. Wall shear stress, heat transfer, velocity and temperature boundary layer profiles are obtained and compared with the results obtained in [14]. The behaviour of the velocity and thermal distribution is studied. In some cases, the existence of two different solutions will be presented. It will be graphically illustrated the effects of the parameters involved in the boundary value problem.

2 Problem Formulation

Consider a steady two-dimensional flow of an incompressible, viscous and electrically

nonconducting ferromagnetic fluid over a flat surface in the horizontal direction seen in Fig. 1. The two magnetic dipoles are equidistant a from the leading edge. The field is due to two-line currents perpendicular to and directed out of the flow plane.

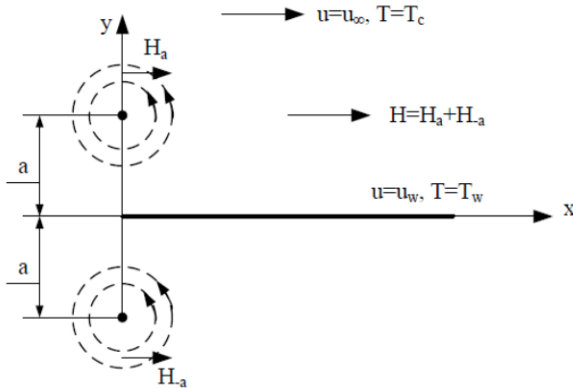


Fig. 1. Parallel flow along a flat surface in magnetic field

The existence of spatially varying fields is required in ferrohydrodynamic interactions [13].

The following assumptions are needed:

- (i) the direction of magnetization of a fluid element is always in the direction of the local magnetic field,
- (ii) the fluid is electrically non-conducting and
- (iii) the displacement current is negligible.

Introducing the magnetic scalar potential ϕ whose negative gradient equals the applied magnetic field, i.e. $\mathbf{H} = -\nabla\phi$, the scalar potential can be given by

$$\phi(x, y) = -\frac{I_0}{2\pi} \left(\tan^{-1} \frac{y+a}{x} + \tan^{-1} \frac{y-a}{x} \right),$$

where I_0 denotes the dipole moment per unit length and a is the distance of the line current from the leading edge.

In the boundary layer for regions close to the wall when distances from the leading edge large compared to the distances of the line sources from the plate, i.e. $x \gg a$, then one gets

$$[\nabla H]_x = -\frac{I_0}{\pi} \frac{1}{x^2}, \quad (1)$$

where H is the magnetic field.

The boundary layer equations for a two-dimensional and incompressible flow are based on expressing the conservation of mass, continuity, momentum and energy.

The analysis is based on the following four assumptions [14]:

(i) the applied field is of sufficient strength to saturate the ferrofluid everywhere inside the boundary layer,

(ii) within the temperature extremes experienced by the fluid, the variation of magnetization with temperature can be approximated by a linear equation of state, the dependence of M on the temperature T is described by $M = K(T_c - T)$, where K is the pyromagnetic coefficient and T_c denotes the Curie temperature as proposed in [3], [14],

(iii) the induced field resulting from the induced magnetization compared to the applied field is neglected; hence, the uncoupling of the ferrohydrodynamic equations from the electromagnetic equations and

(iv) in the temperature range to be considered, the thermal heat capacity c , the thermal conductivity k , and the coefficient of viscosity ν are independent of temperature.

The governing equations are described as follows

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (2)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{I_0 \mu_0 k}{\pi \rho} (T_c - T) \frac{1}{x^2} + \nu \frac{\partial^2 u}{\partial y^2}, \quad (3)$$

$$c \left[u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right] = k \frac{\partial^2 T}{\partial y^2}, \quad (4)$$

where u and v are the parallel and normal velocity components to the plate, the x and y axes are taken parallel and perpendicular to the plate, respectively, ν is the kinematic viscosity and ρ denotes the density of the ambient fluid, which will be assumed constant. The system (2)-(4) of nonlinear partial differential equations is considered under the boundary conditions at the surface ($y = 0$)

$$u(x, 0) = 0, \quad v(x, 0) = 0, \quad T(x, 0) = T_w \quad (5)$$

with $T_w = T_c - Ax^{m+1}$ and

$$u(x, y) \rightarrow u_\infty, \quad T(x, y) \rightarrow T_\infty \quad (6)$$

as y leaves the boundary layer ($y \rightarrow \infty$) with $T_\infty = T_c$, and u_∞ is the exterior streaming speed which is assumed throughout the paper to be $u_\infty = U_\infty x^m$ ($U_\infty = \text{const.}$). Parameter m is relating to the power law exponent. The parameter $m = 0$ refers to a linear temperature profile and constant exterior streaming speed. In case of $m = 1$, the temperature profile is quadratic, and the streaming speed is linear. The value of $m = -1$ corresponds to no temperature variation on the surface.

Introducing the stream function ψ , defined by $u = \partial\psi / \partial y$ and $v = -\partial\psi / \partial x$, so equation (2) is automatically satisfied, and equations (3) – (4) can be formulated as

$$\frac{\partial\psi}{\partial y} \frac{\partial^2\psi}{\partial yx} - \frac{\partial\psi}{\partial x} \frac{\partial^2\psi}{\partial y^2} = \nu \frac{\partial^3\psi}{\partial y^3} - \frac{I_0\mu_0 K}{\pi\rho} (T_c - T), \quad (7)$$

$$c \left[\frac{\partial\psi}{\partial y} \frac{\partial T}{\partial x} - \frac{\partial\psi}{\partial x} \frac{\partial T}{\partial y} \right] = k \frac{\partial^2 T}{\partial y^2}. \quad (8)$$

Boundary conditions (5) and (6) are transformed to

$$\psi'_y(x, 0) = 0, \quad \psi'_x(x, 0) = 0, \quad T(x, 0) = T_c - Ax^{m+1}, \quad (9)$$

$$\psi'_y(x, y) \rightarrow U_\infty x^m, \quad T(x, y) = T_c \text{ as } y \rightarrow \infty. \quad (10)$$

Now, we have two single unknown functions and two partial differential equations. The system of (8)–(10) allows us to look for similarity solutions of a class of solutions ψ and T in the form (see [7])

$$\left. \begin{aligned} \psi(x, y) &= C_1 x^b f(\eta) \\ T &= T_c + Ax^{m+1} \theta(\eta) \\ \eta &= C_2 x^d y \end{aligned} \right\} \quad (11)$$

where b and d satisfy the scaling relation $b + d = m$ and for coefficients C_1 and C_2 the relation $C_1 / C_2 = \nu$ must be fulfilled. The real numbers b, d are such that $b - d = 1$ and $C_1 C_2 = U_\infty$, i.e.

$$b = \frac{m+1}{2}, \quad d = \frac{m-1}{2}$$

$$C_1 = \sqrt{\nu U_\infty}, \quad C_2 = \sqrt{\frac{U_\infty}{\nu}}.$$

By considering (11), equations (7) and (8) and conditions (9) and (10) lead to the following system of coupled ordinary differential equations

$$f''' - mf'^2 + \frac{m+1}{2} ff' - \beta\theta = 0, \quad (12)$$

$$\theta'' + (m+1)Pr \left(\frac{1}{2} f\theta' - \theta f' \right) = 0 \quad (13)$$

subjected to the boundary conditions

$$f(0) = 0, f'(0) = 0, \theta(0) = 1 \quad (14)$$

$$f'(\eta) = 1, \theta(\eta) = 0 \text{ as } \eta \rightarrow \infty \quad (15)$$

where $Pr = c\nu / k$ is the Prandtl number $\beta = I_0\mu_0 KA / \pi\rho U_\infty^2$.

The components of the non-dimensional velocity $\vec{v} = (u, v, 0)$ can be expressed by

$$u = U_\infty x^m f'(\eta),$$

$$v = -\sqrt{\nu U_\infty} x^{\frac{m-1}{2}} \left(\frac{m+1}{2} f(\eta) + \frac{m-1}{2} f'(\eta)\eta \right).$$

The shear stress and the heat transfer at the wall are derived by the drag coefficient $f''(0)$ and the $\theta'(0)$.

According to our knowledge, the coupled boundary-layer equations for the case when $m = 0$ were first examined by Neuringer [14]. If $m = 0$ and $\beta = 0$, equation (12) is equivalent to the well-known Blasius equation

$$f'''' + \frac{1}{2} ff' = 0 \quad (16)$$

which appears when analysing a laminar boundary-layer problem for Newtonian fluids [2], [10].

In the mathematical investigation of a model describing the dynamics of heat transfer in an incompressible magnetic fluid under the action of an applied magnetic field, the fluid is supposed nonelectrically conducting and the calculated solutions are valid only for distances greater than a .

3 Numerical Solution

There are several proceedings for the numerical solution of boundary value problems of coupled strongly nonlinear differential equations as (12)–(13).

One of them is the higher derivative method (HDM), which is implemented in Maple by Chen et al. [11]. This code plays a very important role in the numerical analysis of boundary value problems, because it can be achievable an increase in accuracy while ensuring stability by using higher derivatives [12].

A discretization scheme using higher derivative method (HDM) suggested by Chen et al. [11] is applied for the solution of the boundary value problem (12)–(15). The setting of digits in our case is digits:=15. The boundary value problem is considered as a first order system, where $y1(x) = f(\eta)$, $y2(x) = f'(\eta)$, $y3(x) = f''(\eta)$ and $y4(x) = \theta(\eta)$, $y5(x) = \theta'(\eta)$. The left and right boundary conditions are defined by bc1 and bc2. It is necessary to give the range (bc1 to bc2) of the boundary value problem (Range:= [0.0, η_{max}]). We

have three parameters, m , β and Pr (e.g., pars:= m=0.0, β =0.0, Pr=10.0).

The next step is to define the initial derivative in order and the number of the nodes in nele (nder:= 3; nele := 5;). Next settings of the absolute and relative tolerance for the local error are (atol:= 1e-6; rtol:= atol /100;). The HDMadapt procedure is applied to determine the approximate numeric solution. The simulation gives the figure of all solution functions (from y1 to y5).

4 Results and Conclusion

We studied a heated ferrofluid flow in magnetic field over a flat surface with boundary conditions.

The coupled, nonlinear partial differential equations of MHD flow was transformed into a system of coupled and nonlinear ordinary differential equations by similarity analysis.

After then, we solved the boundary value problem in Maple with HDM method.

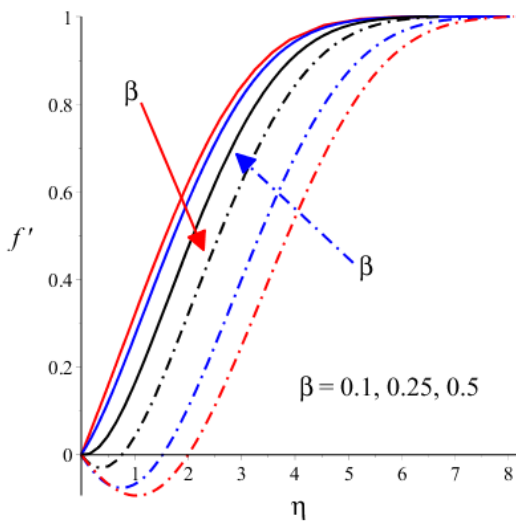


Figure 2. The upper and lower velocity distribution (Pr=10, m=0)

We obtained two different solutions, call them upper and lower solutions for velocity distribution (see Fig. 2), where the upper solution is in a good agreement with those published by Neuringer [14]. Figure 3. represents the lower velocity profiles obtained by numerical simulations. The solutions of the non-dimensional temperature can be seen on Fig 4., where we denoted the temperature profiles corresponding to lower and upper velocity solutions by dashed and continuous lines, respectively. Similarly, in this case, the solutions by continuous lines are in a good agreement with published by Neuringer [14].

Figures 2-5. show the effect of parameter β for the velocity and thermal distributions. If the parameter value β increases, then the boundary layer thickness increases for both the velocity and thermal distribution solutions with continuous lines. An opposite effect can be seen in case of solutions denoted by dashed lines.

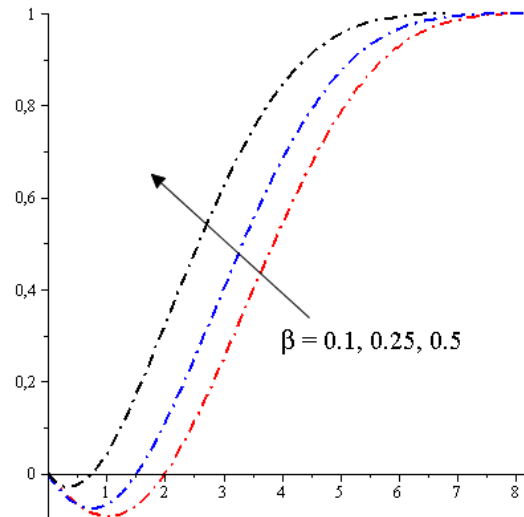


Figure 3. Lower velocity distribution (Pr=10, m=0)

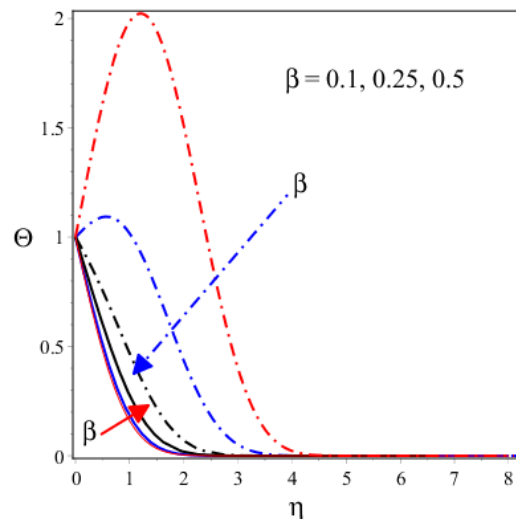


Figure 4. The dual solutions for the thermal distribution (Pr=10, m=0)

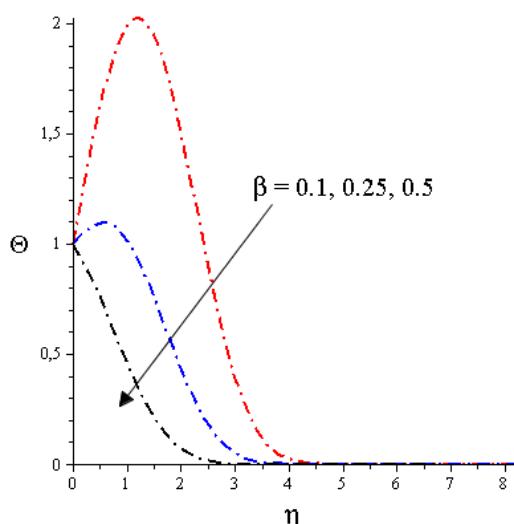



Figure 5. Upper thermal distribution
(Pr=10, m=0)

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