Modelling of Thermo Magnetic Field Effects on Stability Analysis of a Single Walled Carbon Nanotube Rested on Polymer Matrix

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Abstract: - This paper presents the study of non local thermo elastic waves in a single walled carbon nanotube resting on polymer matrix via in plane magnetic effect. The analytical formulation is developed based on Eringen's non-local elasticity model. The governing equations that contains partial differential equations for single walled carbon nanotube is derived by considering thermal field and longitudinal Lorenz magnetic force. The ultrasonic wave propagation analysis is carried out by spectral analysis method. The polymer elastic matrix is considered as a function of temperature change. The computed non dimensional wave frequency, phase velocity and group velocity are investigated and are presented in the form of dispersion curves. Table values are presented for different physical values and are compared with exiting literature

Key-Words: - Euler-beam theory; Nanotube; Nonlocal elasticity; Polymer support; NEMS

1 Introduction

Mechanical behavior of magneto thermo elastic(MTE) material will enhance the potential of nanostructure for amplification and for many applications in engineering. The interaction between Carbon Nanotubes (CNTs) and polymer matrix have gained great attention due to their vital mechanical properties. The CNT resting on a polymeric matrix will create a remarkable volume fraction in the interfacial layer between CNT and the bulk polymer matrix. Interface region plays a vital role to measure the overall elastic moduli of polymeric Nano composites so that the mechanical analysis of Nano composite structures will give the accurate result which may not possible in the absence of interface layer. Thus, modelling actual response of the interfacial region is the one of the most important issues in this type of study and also the impudence of Nano composites material as the inelastic systems. The investigations of this field have vividly depicted that the extent of impact of CNTs greatly depends on the condition and response of interfacial layer between CNTs and the polymer matrix.

[1] proposed the chemistry of fullerenes which leads to the discovery of the synthesis of carbon nanotubes and its properties. The mechanical properties of carbon nanotubes (CNTs) have been proposed from the dates of CNTs discovered by [2] Recently, continuum elastic beam models have been widely used to study vibration [3],[4] and Sound wave propagation [5]-[6] in CNTs. The nonlocal beam models have been further applied to the investigations of static and vibration properties of single and multi-walled carbon nanotubes [7]-[10]. Several studies [11], [12] and [13] shows the fact that atomistic interaction in axial direction is significant for short CNTs. [14] remarked that a higher small length scale effect can be found for shorter CNTs. The mechanical properties of CNTs with temperature change are of great interest in Nano Electro Mechanical System (NEMS). [15] investigated the thermal expansion of helical CNTs arrays. [16] studied the thermal effects on interfacial stress transfer characteristics of single walled and multi walled carbon nanotubes / polymer composite systems via thermal loading by means of thermo elastic theory and conventional fibre pilot models. [17] conducted an analysis of buckling behaviour of SWCNTs subjected to axial compression under a thermal environment. [18] investigated the temperature dependent elastic properties of SWCNTs by molecular dynamics simulation. They found the effect of temperature on the buckling of carbon nanotubes, the bending, torsion, and radial compression buckling of a double-walled carbon nanotube and multi-walled nanotubes. [19] , [20] analyzed the thermal effect on torsional and axially compressed buckling of MWCNTs. Zhang *et al*.[21] studied the thermal effect on transverse vibrations of DWCNTs using classical continuum model.

The mechanical behaviour of single and multi-walled carbon nanotubes bedded in polymer matrix has attaracted much by the recent days of researchers. [22] investigated the thermal effect on wave propagation in double walled carbon nanotubes embedded in a polymer matrix using nonlocal elasticity. They identified that the non local effect becomes larger at higher values of vibration mode. The nonlinear vibration of embedded carbon nanotubes was analysed by [23]. The Influence of thermal and longitudinal magnetic field vibration of a fluid conveying double walled carbon nanotubes embedded in an elastic medium was studied by [24]. [25] studied the different wave modes coupled in Longitudinal or transverse magnetic field. More recently, [26] have studied the vibrations of nonlocal Flugge shell model for SWCNTs under the longitudinal magnetic field based on wave propagation approach. Their analysis shows that the vibration frequencies of SWCNTs drop dramatically in the presence of the magnetic field for various circumferential wave numbers.

Based on the above literature, the non local thermo elastic waves in a single walled carbon nanotube resting on polymer matrix with the longitudinal magnetic effect is studied using Eringen's non local elasticity theory. The governing equations that contains partial differential equations for single walled carbon nanotube is derived by considering thermal and Lorentz magnetic force along with the nonlocal parameters. The computed non dimensional wave frequency, phase velocity and group velocity with respect to the polymer matrix support and thermal constant are presented in the form of dispersion curves.

2 Mathematical Formulations

2.1 Eringen nonlocal theory of elasticity

This theory assumes that stress state at a reference point X in the body is regarded to be dependent not only on the strain state at X but also on the strain states at the all other points of the body. The general form of the constitutive equations in the non-local form of elasticity contains an integral over the entire region of interest. The integral contains a non-local kernel function, which describes the relative influences of the strains at various locations on the stress at a given location. The constitutive equations of linear, homogeneous, isotropic, non-local elastic solid with zero body forces are given by Eringen [32] as follows

$$\sigma_{ij} + \rho(f_j - \ddot{u}_j) = 0 \tag{1}$$

$$\sigma_{ij}(X) = \int_v \xi(|X - X'|), \tau)\sigma_{ij}{}^c(X')dv(X') \tag{2}$$

$$\sigma_{ij}{}^c = C_{ijkl}\varepsilon_{kl} \tag{3}$$

$$e_{ij}(X') = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right) \tag{4}$$

Eq. (1) is the equilibrium equation, where $\sigma_{ij,i}, \rho, f_j, u_j$ are the stress tensor, mass density, body force density and displacement vector at a reference point X in the body, respectively, at the time t, Eq. (3) is the classical constitutive relation where $\sigma^c{}_{ij}(X')$ is the classical stress tensor at any point x' in the body, which is shown by $\sigma^c{}_{ij}(X')$ which is related to the linear strain tensor $e_{ij}(X')$ at the same point. $\sigma^c{}_{ij}(X')$ is the classical stress tensor at any point X' in the body, which is related to the linear strain tensor $e_{ij}(X')$ at the same point. Equations (4) are the classical strain displacement relationship. The kernel function $\xi(|X - X'|, \tau)$ is the attenuation function which incorporated the nonlocal effect in the constitutive equations. The volume integral in (2) is over the region v occupied the body. It is clear that, the only difference between (1)-(4) and the corresponding equations of classical elasticity in (2) replaces the Hooke's law in (3) by (2). (2) consists the parameters which correspond to the non-local modulus has dimensions of $(lengh)^{-3}$ and so it depends on a characteristic length (lattice parameter, size of grain, granular distance, etc.) and "$l.$" is an external characteristic length of the system (wavelength, crack length, size or dimensions of sample, etc.) Therefore the non-local modulus can be written in the following form;

$$\lambda = \frac{Ev}{(1+v)(1-2v)}, \quad \eta = \frac{E}{(1+v)} \tag{5a}$$

$$\xi = \xi(|X - X'|, \tau), \tau = \frac{e_o a}{l} \tag{5b}$$

Where E and v are the elastic modulus and poisson's ratio respectively. The difference between the classical and nonlocal elastic theory lies un the materials constitutive relation (2) where e_0 a is a constant corresponding to the material's and has to be determined for each materials independently and "$|X - X'|$" is the Euclidian distance. Then, the integro-partial differential (2) of non-local elasticity can be simplifies to partial differential equation as follows

$$(1-\tau^2 l^2 \nabla^2)\sigma_{ij}(X) = \sigma_{ij}{}^{c}(X) = C_{ijkl}e_{kl}(X) \qquad (6)$$

where C_{ijkl} is the elastic modulus tensor of classical isotropic elasticity and e_{ij} is the strain tensor. Where ∇^2 denotes the second-order spatial gradient applied on the stress tensor $\sigma_{ij,i}$ and $\tau = e_o a / l.$ Eringen proposed $e_o = 0.39$ by the matching of the dispersion curves via non-local theory for place wave and Born-Karman model of lattice dynamics at the end of the Brillouin zone $(ka = \pi)$, where a is the distance between atoms is and k is the wave number in the phonon analysis. On the order hand Eringen proposed $e_o = 0.31$ in his study for Rayleigh surface wave via non-local continuum mechanics and lattice dynamics.

2.2 Formulation of SWCNT with nonlocal relations

The partial differential equation which governs of free vibration of the nanotube under the influence of thermal can be expressed as [27-28]

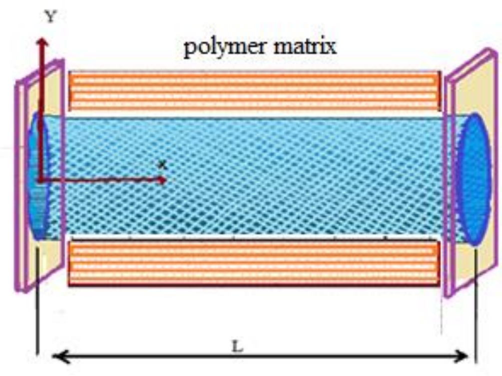

Fig. 1. Geometry of the problem.

$$\frac{\partial Q}{\partial X} + N_T \frac{\partial^2 y}{\partial X^2} + q_{(x)} + f(x) = \rho A \frac{\partial^2 Y}{\partial t^2} \qquad (7)$$

where $f(X)$ is the interaction pressure per unit axial length between the nanotube and the surrounding elastic medium. A is the cross section of CNT. The resultant shear force Q on the cross section of the nanotube is defined in the following equilibrium equation

$$Q = \frac{\partial M}{\partial X} \qquad (8)$$

N_t denotes the temperature dependent axial force with thermal expansion coefficient α . This constant force is defined as [26]

$$N_T = -EA\alpha_x T \qquad (9)$$

where A is the cross section of nanotube and T is the temperature change . The longitudinal magnetic flux due to Lorentz force exerted on the tube in z direction is represented by the term $q(x)$ and is read from [27]

$$q(x) = \eta A H_x{}^2 . \frac{\partial^2 Y}{\partial X^2}, \qquad (10)$$

where H_x is the magnetic field strength and η is the magnetic permeability. For the Euler –beam theory the resultant bending moment M in (8) can be taken as follows

$$M = \int_A z\,\sigma_{XX}\, dA, \qquad (11)$$

where σ_{XX} is the nonlocal axial stress defined by nonlocal continuum theory. The constitutive (6) of a homogeneous isotropic elastic solid in non local form for one-dimensional nanotube is taken as

$$\sigma_{XX} - (e_0 a)^2 \frac{\partial^2 \sigma_{XX}}{\partial X^2} = E\varepsilon_{XX} \qquad (12)$$

where E is the Young's modulus of the tube , ε_{XX} is the axial strain, $(e_0 a)$ is a nonlocal parameter which represents the impact of nonlocal scale effect on the structure. a is an internal characteristic length. The nonlocal relations in (12) can be written with temperature environment as follows

$$\sigma_{XX} - (e_0 a)^2 \frac{\partial^2 \sigma_{XX}}{\partial X^2} = E\varepsilon_{XX} - E\alpha T \qquad (13)$$

In the context of Bernoulli –Euler model, the axial strain ε_{xx} for small deflection is defined as [28]

$$\varepsilon_{xx} = -z \frac{\partial^2 Y}{\partial X^2} \qquad (14)$$

where Y is the transverse co-ordinate in the positive direction of deflection. By using (13) and (14), in (11), the bending moment M can be expressed in terms of generalized displacement as;

$$M - (e_0 a)^2 \left[\frac{\partial^2 M}{\partial X^2}\right] = EI \frac{\partial^2 Y}{\partial X^2} \qquad (15)$$

where $I=\int_A z^2 dA$ is the moment of inertia. By substituting (7) and (8) into (15), the nonlocal bending moment M and shear force Q can be expressed as follows

$$M-(e_0a)^2\left[(\rho A)\frac{\partial^2 Y}{\partial t^2}+q_{(x)}-f(x)+N_t\right]=EI\frac{\partial^2 Y}{\partial X^2} \quad (16)$$

$$Q-(e_0a)^2\left[\rho A\frac{\partial^3 Y}{\partial X^2\partial t^2}+\frac{\partial^2 q_{(x)}}{\partial X^2}-\frac{\partial f(x)}{\partial X}+N_t\right]=EI\frac{\partial^3 Y}{\partial X^3} \quad (17)$$

For the transverse vibration, the equation of motion (7) can be expressed under distributed pressure and thermal interaction with surrounding polymer elastic medium as

$$f(x)=EI\frac{\partial^4 Y}{\partial X^4}-EA\alpha T\frac{\partial^2 Y}{\partial x^2}+(\rho A)\frac{\partial^2 Y}{\partial t^2}+q(x)\frac{\partial^2 Y}{\partial X^2} -\left((e_0a)^2\left(EA\alpha T\frac{\partial^4 Y}{\partial X^4}+q(x)\frac{\partial^4 Y}{\partial X^4}-\frac{\partial^2 f(x)}{\partial X^2}\right)\right) \quad (18)$$

The pressure per unit length acting on the surface of the tube due to the surrounding elastic medium can be described by a Winkler type model [22]

$$f(X)=-k\ \ Y \quad (19)$$

where the negative sign indicated that the pressure f is opposite to the deflection of the tube, and k is the spring constant of the surrounding polymer matrix. It is noted that the spring constant k is proportional to the young's modulus of the surrounding elastic medium E_m [30] and is given by the following relations

$$\varepsilon^X{}_N=\frac{\partial u_N{}^X}{\partial r}=\frac{1}{E_n}\left[\sigma^X{}_N-v[\sigma^Y{}_N\right]+\alpha_N\Delta T_N$$

$$\varepsilon^Y{}_m=\frac{\partial u_m{}^Y}{\partial r}=\frac{1}{E_m}\left[\sigma^Y{}_m-v[\sigma^X{}_m\right]+\alpha_m\Delta T_m \quad (20)$$

where, E_m, E_n, α are respectively, express young modulus and thermal expansion coefficients of CNTs and polymer matrix, under temperature change environments, which is defined as

$$E_n=E^0(1-0.0005T) \quad \alpha=\alpha^0(1+0.0002T) \quad \text{and}$$

$$E_m=E_m{}^0(1-0.0003T) \quad (21)$$

where E^0 and α^0 represents the modulus and thermal expansion of CNTs under room temperature environment. From the above relations, we can write the spring constant of the polymer matrix as follows

$$k=k^0(1-0.0003T) \quad (22)$$

where E_m^0 and k^0 are the Young modulus and spring constant of polymer matrix under a room temperature environment , respectively. Introduction of (20) into (19) yields the following non local Euler-Bernoulli relation

$$EI\frac{\partial^4 Y}{\partial X^4}+N_t\frac{\partial^4 Y}{\partial t^4}+\eta AH_x{}^2\frac{\partial^2 Y}{\partial X^2}+\rho A\frac{\partial^2 Y}{\partial t^2} -(e_0a)^2\left(+k\frac{\partial^2 Y}{\partial X^2}-N_t\frac{\partial^4 Y}{\partial t^4}+\eta AH_x{}^2\frac{\partial^4 Y}{\partial x^4}+\rho A\frac{\partial^2 Y}{\partial X^2}\right)+kY=0 \quad (23)$$

3 Ultrasonic Wave Dispersion

In order to analysis the elastic wave charectertic of SWCNT, a harmonic wave solution for the displacement $Y(X,t)$ is taken from [26] in the complex form as follows

$$Y(X,t)=\sum_{n=1}^{N}\bar{Y}(x)e^{-j(kn-\omega_n t)} \quad (24)$$

where $\bar{Y}$ is the amplitude of the wave motion , $j=\sqrt{-1}$, k_n is the wave number, ω_n is the circular frequency of the nth sampling point and N is the Nyquist frequency. The sampling rate and the number of sampling points should be sufficiently large to have relatively good resolution of both high and low frequencies respectively. Substitution of (24) into (23), we get the following coupled equations

$$\sum_{n=1}^{N}\left[\left(EI\frac{\partial^4\bar{Y}}{\partial X^4}+N_t\frac{\partial^2\bar{Y}}{\partial X^2}+\rho A\frac{\partial^2\bar{Y}}{\partial t^2}+\left(\eta AH_x{}^2\frac{\partial^2\bar{Y}}{\partial X^2}\right) -(e_0a)^2\left(N_t\frac{\partial^2\bar{Y}}{\partial X^2}+\rho A\frac{\partial^2\bar{Y}}{\partial X^2}-\left(\eta AH_x{}^2\frac{\partial^2\bar{Y}}{\partial X^2}\right)\right)\right)+(K)\right]e^{i\omega_n t}=0 \quad (25)$$

The above equation must full filled for each values for small n and can be written in the following with single variable X. (25) can be reduced as

$$\left[EI\frac{\partial^4\bar{Y}}{\partial X^4}+EA\alpha T\frac{\partial^4\bar{Y}}{\partial X^4}+\eta AH_x{}^2\frac{\partial^2\bar{Y}}{\partial X^2} -\frac{\partial^2}{\partial t^2}\left[\rho A-\left(\rho A-k+EA\alpha T+\eta AH_x{}^2\right)(e_0a)^2\frac{\partial^2\bar{Y}}{\partial X^2}\right]-k\bar{Y}\right]=0 \quad (26)$$

The following non dimensional parameters are used for the convenience of the problem

$$\frac{X}{l}=x; \quad \frac{\bar{Y}}{l}=\bar{y}; \quad e_i=\frac{AL^4}{EI};$$
$$\eta=\frac{1}{\left(1+\bar{H}_x e_i\right)}; \quad \bar{N}_t=\frac{N_t l^2}{EI}; \quad \bar{H}_x=\frac{\eta H_x{}^2}{E}; \qquad (27)$$

Using (27)in (26), we obtain the following relation with dimensionless form

$$\left(\left(1+\bar{H}_x e_i\right)\frac{\partial^4 \bar{y}}{\partial x^4}+\left(\begin{bmatrix} N_t+\bar{H}_x e_i-\bar{k} \\ +(1-m)\beta^2\tau^2 \end{bmatrix}\frac{\partial^2 \bar{y}}{\partial x^2} \\ -2\beta v\sqrt{m}\rho A\frac{\partial \bar{y}}{\partial x}-\beta^2\rho A\right)\right)=0 \qquad (28)$$

For the spectrum relation the following term is considered

$$\bar{y}(x,t)=ye^{-ik_n x} \qquad (29)$$

Substituting (29) into (28), the following equation is obtained for non-trivial solution of the wave amplitude y

$$\left(1+\bar{H}_x e_i\right)k_n^4+\begin{bmatrix} N_t+\bar{H}_x e_i \\ -\bar{k}+\tau^2 \end{bmatrix}k_n^2-\left(2\rho A\right)k_n-\rho A=0 \qquad (30)$$

which represents the characteristic equation for a continuum structure (ECS) coupled with surrounding medium of an SWCNT. From (30), we can derive the wave numbers in the following form

$$k_n=\pm\sqrt{-\frac{1}{2}\frac{\lambda_1(\eta)}{\lambda_2(\eta)}\pm\sqrt{\frac{1}{2}\left(\frac{\lambda_1{}^2(\eta)}{\lambda_2(\eta)}\right)(\eta)-\frac{4\lambda_2\lambda_0(\eta)}{2\lambda_2(\eta)}}}, \qquad (31)$$

where $\lambda_2=\overline{N}_t+\bar{H}_x e_i-\bar{k}+\tau^2$, $\lambda_1=-2\rho A$, $\lambda_0=-\rho A$

$$\eta=1\Big/\left(1+\bar{H}_x e_i\right)$$

It is clear that the above wave numbers relation are a function of the nonlocal scaling parameter, wave frequency, longitudinal magnetic field strength, stiffness of elastic medium and other material parameters of the nano tube. Among the four wave numbers two are real and the other two are imaginary. The real and imaginary parts represent the propagating and damped modes, respectively. From (31), if $\bar{k}=0$, the spatially damped mode turns to be propagating nature. The resonant frequency of n the order of the SWCNT with thermal effect can be obtained via nonlocal model by

$$\omega_n=\frac{1}{2}\left(\overline{N}_t\pm\sqrt{\overline{N}_t{}^2-4\bar{k}}\right), \qquad (32)$$

In which

$$\overline{N}_t=[1+\bar{H}_x e_i]\alpha^2-\left[\bar{H}_x e_i-\bar{k}+\tau^2\right]-\frac{(2\rho A)}{\alpha}-\frac{[\rho A]}{\alpha^2}$$
$$\bar{k}=\left[1+\bar{H}_x e_i\right]\alpha^2-\left[\bar{H}_x e_i+N_t+\tau^2\right]-\frac{(2\rho A)}{\alpha}-\frac{[\rho A]}{\alpha^2}$$

The vibration modes of wave speed or phase speed is calculated from

$$C_p=\mathrm{Re}\left(\frac{\omega_n}{k_n}\right) \qquad (33)$$

The phase speed is defined with respect to real k_n , since the real part represents the propagative component of the wave. As a result, the speeds change with frequencies, which makes the wave highly dispersive. The dispersion curve between wave velocity and wave frequency will explain the entire description of the wave propagation in nanostructures. The wave number is mainly function of the nonlocal scaling parameter $(e_0 a)$ and the wave circular frequency. The corresponding group velocity $C_g=\mathrm{Re}\left(\frac{\partial\omega_n}{\partial k_n}\right)$ are derived from the relation given in Eq.(33).

4 Boundary Conditions

Here, an analytical solution of the governing equations for vibration of a polymer elastic Nano beam having simply-supported (S) and clamped (C) boundary conditions is presented which they are given as:

4.1 Simply Supported SWCNT

The boundary conditions for the simply supported problem are

$$Y(x)|_{X=0}=0\ , \qquad M(X)=Y''|_{X=0}=0,$$

$$Y(x)|_{X=L}=0, \qquad M(X)=Y''|_{X=L}=0,$$

4.2 Clamped - Clamped SWCNT

Assume the case where both the ends of the beam are clamped and are subjected to axial compressive load . The boundary conditions for this case are given as

$$Y(x)|_{X=0}=0\ , \qquad \frac{\partial Y(X)}{\partial X}|_{X=0}=0\ ,$$
$$Y(x)|_{X=L}=0\ , \qquad \frac{\partial Y(X)}{\partial X}|_{X=L}=0\ ,$$

5 Numerical Results And Discussion

In this paper thermo magneto elastic wave in a single walled carbon nanotube resting on polymer matrix is discussed using nonlocal Euler beam theory. From [31], we considered the Young modulus E=1 Tpa, thickness to be 0.35 nm and mass density as 2.3 g/cm3. To analyze the influence of velocity on the vibration of SWCNT, while the mass density of CNTs is $2300\ kg/m^3$ with the bending rigidity EI of $1.1122\times10^{-25} N\,m^9$. According to the calculation, The thermal expansion coefficient in room temperature is taken as $\alpha^0=-1.5\times10^{-6}C^{-1}$ [22].

The dispersion curves are drawn in Fig. 2 and 3 for the variation of wave frequency versus the wave number of the elastic SWCNT for the varying non local parameter with respect to thermal parameter $N_t=0.2,0.5$ and spring constant of polymer matrix $k=0,0.2$, respectively. From Fig. 2 and 3, it is observed that the wave frequency is increasing with respect to its wave number for the different values of non-local parameters. Figure.3 reveals dispersion trend in the wave propagation due to the surrounded polymer matrix support. It can be noted that the increasing values of thermal parameter N_t also influence the values of wave frequency in Fig. 2 and 3. A comparative illustration is made between the group velocity and wave number of the SWCNT for the thermal and spring constant values is respectively shown in the Figs.4 and 5. From the Figs.4 and 5, it is clear that, at the lower range of wave number the group velocity attain maximum value in both cases of $k=0$ and $k=0.2$.

Fig. 6 and 7 investigates the dispersion curves for the phase velocity of thermo elastic SWCNT with respect to thermal parameter $N_t=0.2,0.5$ and spring constant of polymer matrix $k=0,0.2$ and for the varying non local parameter. From the Fig. 6 and 7, it is observed that the phase velocity reaches higher values at lower wave number for the increasing values of thermal, polymer matrix value and nonlocal parameters. The crossings over trend of the dispersion curves explains that the energy exchange among the vibrational modes when the polymer matrix support is absent. The 3D curves in Fig. 8 and 9, clarify the relative variation of wave frequency against the constant values of temperature and non local constant in the presence of magnetic field strength. These curves explain the dependence of wave frequency on the non local scale values, magnetic field strength and temperature. Tables 1 show the different geometrical parameters from the literature. Table 2 exhibit the numerical results of the natural frequencies for different stiffness parameter and non local scale values. From these tables it is observed that the frequencies are increasing when the non local scale values increases .The result also show that as the foundation stiffness increases the effect of non local scale values diminishes. The same natural frequency is obtained for both local and nonlocal boundaries in Table.3.These results show that for both local and nonlocal boundary conditions the amplitude of the natural frequency at different stiffness and nonlocal values are same. The natural frequency of simply supported and clamped CNT is calculated at different thermal parameter and nonlocal values in Table.4.From these results it is observed that as the thermal parameter grows the frequency also increases but the small scale effects reduces the values of frequency in both boundary conditions. Table. 5 presents the comparative study between the numerical results of the natural frequency of simply supported CNT at different half wave and non local parameter. Results predicts the reasonable agreement with the literature.

Table 1 Parameters of the materials

Properties	**Value**	**Units**
EI	1.1122×10^{-25}	$N\,m^9$
α^0	-1.5×10^{-6}	C^{-1}
ρ	2.3	g/cm^3
e_0	0.31	nm

Table 2 Natural frequency(THz) of a simply supported -clamped CNT for different non local parameters.

	$K^0=10^{17}\ N/m^3$			$K^0=10^{19}\ N/m^3$		
n	$e_0a=.5$	$e_0a=1.0$	$e_0a=1.5$	$e_0a=.5$	$e_0a=1.0$	$e_0a=1.5$
1	0.3813	0.7706	0.7916	0.5574	0.0886	0.0887
2	0.5719	0.8238	0.9141	0.5959	0.1776	0.2986
3	0.7626	0.8738	1.0220	0.6321	0.2663	0.3649

4	0.9211	0.9532	1.1195	0.6663	0.3551	0.4214
5	0.9660	1.1439	1.2092	0.6988	0.4439	0.4511
6	1.0090	1.3345	1.2927	0.7299	0.5327	0.5361
7	1.0502	1.5252	1.3711	0.7597	0.6215	0.6321
8	1.0898	1.7188	1.4453	0.8243	0.7102	0.7199
9	1.1281	1.9064	1.6479	0.9184	0.7990	0.8160
10	1.1651	2.0971	2.0945	1.0104	1.0643	1.0654

Table 3 Natural frequency (THz) of a simply supported –clamped CNT in both local and nonlocal boundary

	$K^0=10^{17}\ N/m^3$		$K^0=10^{18}\ N/m^3$		$K^0=10^{19}\ N/m^3$	
τ	*L.BC.*	*NL.BC*	*L.BC.*	*NL.BC*	*L.BC.*	*NL.BC*
0	0.0106	0.0103	0.0113	0.0110	0.0123	0.0120
.5	0.0148	0.0145	0.0173	0.0171	0.0129	0.0126
1	0.0198	0.0196	0.0214	0.0212	0.0237	0.0236
1.5	0.0216	0.0214	0.0450	0.0448	0.0274	0.0273

Table 5 Comparison of the obtained natural frequencies of a simply supported CNT with [37] at different circumferential half waves and nonlocal parameters

n	$\tau=0(nm)$		$\tau=0.1(nm)$		$\tau=0.2(nm)$	
	Ref.[33]	**Author**	**Ref.[33]**	**Author**	**Ref.[33]**	**Author**
2	1.4792	1.4781	1.5765	1.5722	1.8377	1.8312
3	0.5331	0.5215	0.5977	0.5944	0.7800	0.7723
4	0.2728	0.2707	0.3390	0.3368	0.4861	0.4849
5	0.1687	0.1665	0.2294	0.2270	0.3538	0.3526
6	0.1150	0.1149	0.1715	0.1704	0.2792	0.2774
7	0.0835	0.0832	0.1364	0.1347	0.2313	0.2313
8	0.0635	0.0629	0.1132	0.1109	0.1978	0.1902
9	0.0499	0.4445	0.0967	0.0951	0.1731	0.1704
10	0.0403	0.0366	0.0845	0.0823	0.1540	0.1512

Table 4 Natural frequency(THz) of a simply supported – clamped CNT in both local and nonlocal boundary

	$\alpha^0=-1.5\times10^{-6}\,C^{-1}$		$\alpha^0=-1.5\times10^{-6}\,C^{-1}$		$\alpha^0=-1.5\times10^{-6}\,C^{-1}$	
τ	*L.BC.*	*NL.BC*	*L.BC.*	*NL.BC*	*L.BC.*	*NL.BC*
0	0.0180	0.0178	0.0176	0.0173	0.0256	0.0259
0.5	0.0145	0.0142	0.0169	0.0167	0.0231	0.0229
1	0.0134	0.0131	0.0150	0.0148	0.0212	0.0214
1.5	0.0039	0.0027	0.0116	0.0113	0.0207	0.0205

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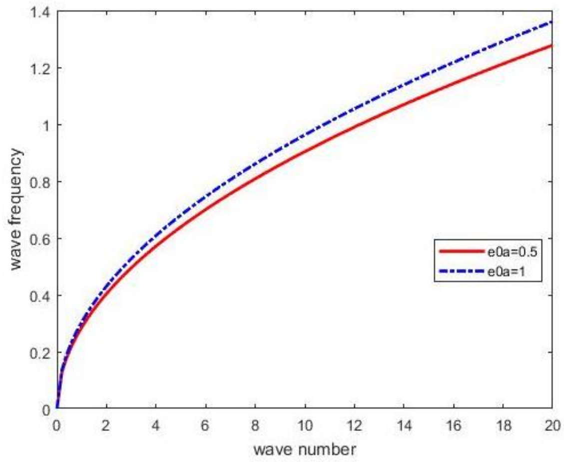

Fig. 2 Distribution of wave frequency versus wave number with $N_t=0.2$, $k=0$.

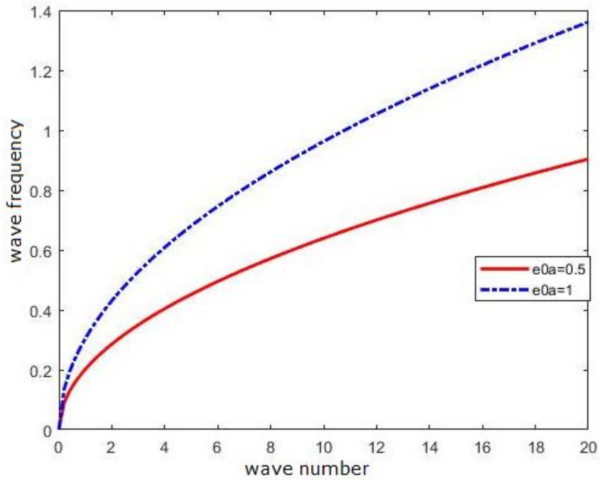

Fig. 3 Distribution of wave frequency versus wave number with $N_t=0.5$, $k=0.2$

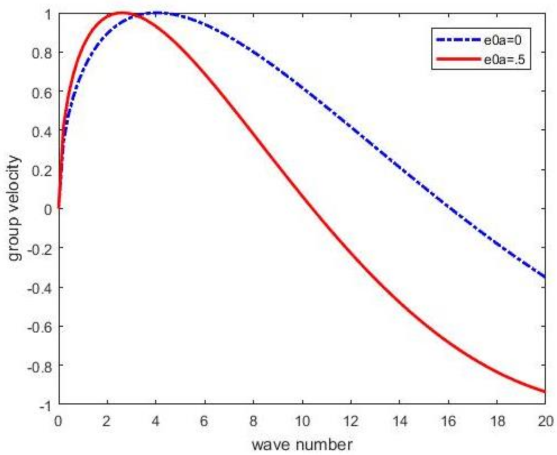

Fig.4 Distribution of group velocity versus wave number with $N_t=0.5, k=0$

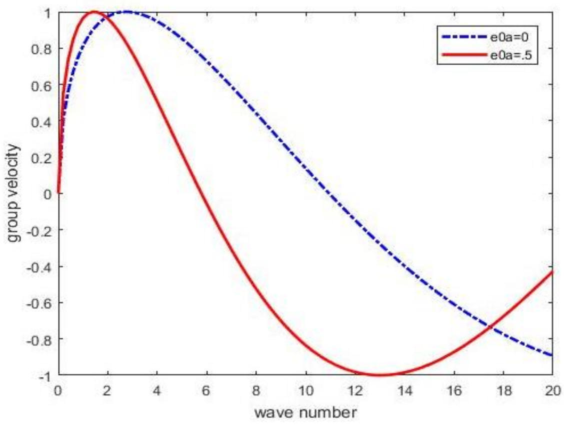

Fig. 5 Distribution of group velocity versus wave number with $N_t = 0.5, k = 0.2$

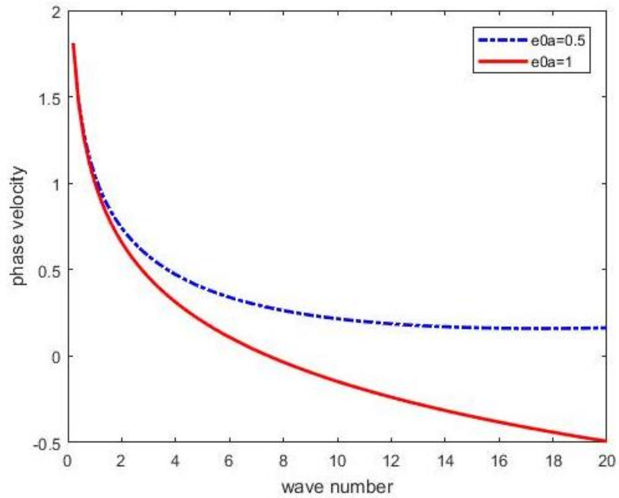

Fig. 6 Distribution of phase velocity versus wave number with $N_t = 0.2, k = 0.2$

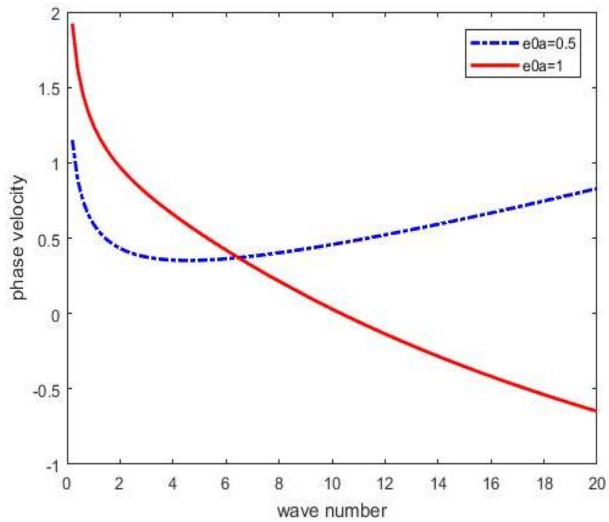

Fig. 7 Distribution of phase velocity versus wave number with $N_t = 0.5, k = 0$

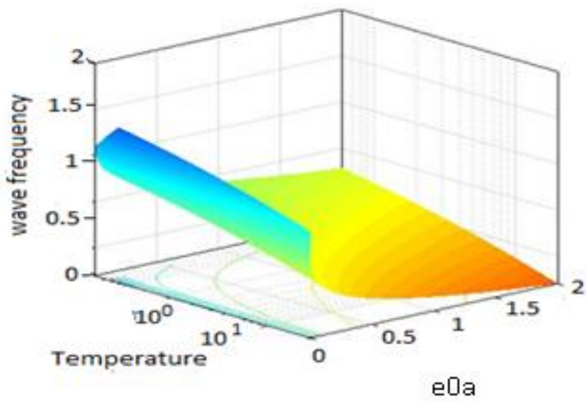

Fig. 8 3 D Distribution of wave frequency with non local constant and temperature via $H_x = 0.5$

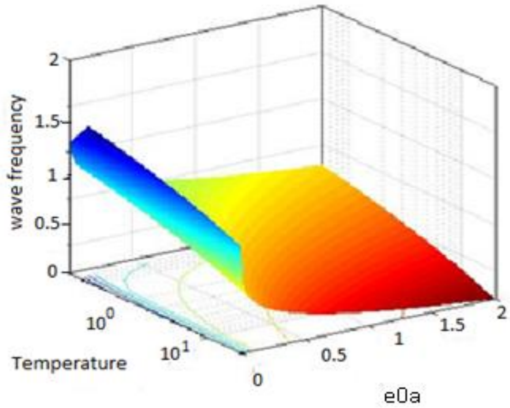

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Fig. 9 3 D Distribution of wave frequency via temperature and non local values with $H_x = 1.0$

6 Conclusions

This paper presents the study of non local thermo elastic waves in a single walled carbon nanotube embedded on polymer matrix with the longitudinal magnetic effect. The analytical formulation is developed based on Eringen's non-local elasticity theory. The governing equations that contains partial differential equations for single walled carbon nanotube is derived by considering thermal and Lorenz magnetic force. The computed non dimensional wave frequency, phase velocity and group velocity are presented in the form of dispersion curves.

- It is found that the non local scaling constant enhances the wave frequency and reduces the phase and group velocities in the presence temperature fields.

- Further it is observed that the increase in the spring constant value of polymer matrix influences the variation of physical variables under the temperature change environment.
- The result also show that as the foundation stiffness increases the effect of non local scale values diminishes via room temperature environment
- It is noticed that the wave frequency decay in the presence of temperature via magnetic and non local values.
- It is observed that the natural frequency is arrived below 1% in both local and non local boundary conditions in the presence of stiffness and temperature coefficients.
- The results presented in this study can provide mechanism for the study and design of the nano devices like component of nano oscillators, micro wave absorbing, nano-electron technology and nano–electro– magneto–mechanical systems (NEMMS) that make use of the wave propagation properties of single-walled carbon nanotubes embedded on polymer matrix.

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