s: source's signal.

ɳ: The noise term has a mean of 0 and a variance of $\alpha^2 I$.

The algorithms in this paper are based on the received data's autocorrelation matrix [10][11]. A Root-MUSIC algorithm which is a derived version (alternative version) of MUSIC is the algorithm in question.

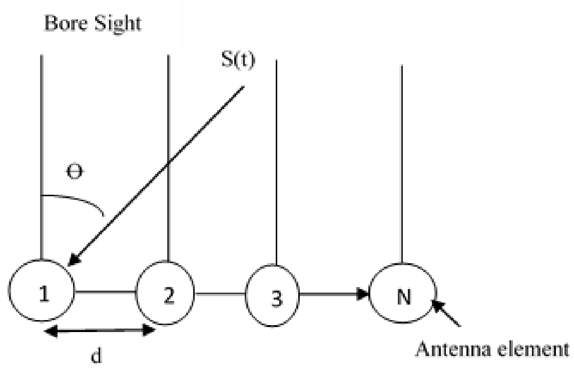

Fig.1 The Direction of Arrival (DoA) and the Uniform Linear Array (ULA).

2.1 Subspace-based techniques approach

The spectral decomposition of the covariance matrix is used to evaluate the space of the covariance matrix in subspace-based approaches for DOA estimation. Techniques based on subspaces are based on the fact that the covariance matrix space can be divided into two subspaces. These are signal and noise subspaces [1][12]. The eigenvectors of the covariance matrix that correspond to the largest eigenvalues span the signal subspace. The eigenvectors with smaller eigenvalues, on the other hand, traverse the noise subspace.

2.2 Root-MUSIC algorithm

It is an updated version of the MUSIC algorithm that was proposed by Barabell [13]. The root of the polynomial in the MUSIC spectrum is utilized in the "*Angle of Arrival (AOA)*" estimation. The main difference between the MUSIC and Root-MUSIC, is, MUSIC always provides the results as visual plots, but the Root-MUSIC provides the results as numbers. The algorithm starts with estimating the covariance matrix of the input data. The advantage of this method lies in calculating the DoA directly by searching for nulls of a polynomial, thus replacing the search for the maximum in MUSIC. This approach is limited to the uniformly spaced networks of linear antennas. It also reduces the calculation time by using certain properties of the received signals to increase the angular resolution. The Root-MUSIC algorithm is based on the formation of a degree 2(M-1) polynomial and the extraction of roots [14],[15]. The estimation of signal arrival directions corresponds to the search for max. pseudo-spectrum F(Θ) values of MUSIC:

$$F_{MUSIC}(\Theta) = \frac{1}{b^H(\Theta)\ U_n\, U_n^H\ b(\Theta)} \qquad (2)$$

Where U_n is the matrix of eigenvectors that span the noise subspace and $E = U_n . U_n^H$ is the projection matrix and $b^H(\Theta)\ U_n U_n^H\ b(\Theta)$ is the projection of the vector b(Θ) on the noise subspace.

The projection of the steering vector on the noise subspace according to (2) can be expressed by the following relation for a linear antenna array that is uniformly separated:

$$F_{MUSIC}^{-1}(\Theta) = g_{R-MUSIC}(\Theta) = b^H(\Theta)\ U_n U_n^H\ b(\Theta)) \qquad (3)$$

Let $E = U_n . U_n^H$, equation (3) becomes:

$$F_{MUSIC}^{-1}(\Theta) = g_{R-MUSIC}(\Theta) = b^H(\Theta). E. b(\Theta) \qquad (4)$$

Applying both the analytical representation and the expression of the steering vector

$b_n(\Theta) = e^{\frac{i2\pi d(n-1)\sin\Theta}{\Psi}}$ of the n^{th} element of the linear array (n= 1, 2, …., N), where Ψ is the wavelength, we can write [16]:

$$F_{MUSIC}^{-1}(\Theta) = g_{R-MUSIC}(\Theta) = \sum_{n=1}^{N} . \sum_{p=1}^{N} e^{\frac{-i2\pi(n-1)d\sin\Theta}{\Psi}}\ E_{np}\ e^{\frac{i2\pi(p-1)d\ \sin\Theta}{\Psi}} \qquad (5)$$

Where E_{np} are the elements of the n^{th} row and the p^{th} column of E. By combining both sums in (4), the following expression is obtained [16]:

$$F_{MUSIC}^{-1}(\Theta) = g_{R-MUSIC}(\Theta) = \sum_{L=-N+1}^{N-1} E_L\, e^{\frac{-2\pi Ld\ \sin\Theta}{\Psi}} \qquad (6)$$

Where $E_L = \sum_{n-p=L} E_{np}$

Equation (6) can be converted into Root-MUSIC polynomial which is a function of z defined by [16]:

$$R(z) = \sum_{L=-N+1}^{N-1} E_L z^L \qquad (7)$$

Where $z = e^{\frac{-i2\pi d \sin Ѳ}{ϒ}}$

The functions of z, which is, therefore, the problem of calculating the 2(M-1) roots of the polynomial, with useful zeros on the unit circles. These complex root phases are consistent with the electrical phase shifts that are desired. From the following equation, the angles of signal arrival can be deduced:

$$Ѳ_n = \sin^{-1}\left(\frac{ϒ}{2\pi d}\arg(z_n)\right) \qquad (8)$$

Where z_n are the n closest roots to the unit circle.

3. Practical Results

For the Direction of Arrival (DOA) estimate test with one source and two sources, the ultrasonic transducers are used the received signal is obtained by scanning an ultrasonic transducer and taking a sample at d interval. For DoA estimations, the FFT and Root-MUSIC methods are used.

A comparison between conventional and high-resolution methods is then made for different numbers of samples. The system parameters are N (number of samples), d (the distance between two samples), F (frequency), ϒ (wavelength), Ѳ (Angle of the source).

We can determine the percentage error in all methods by using the following equation:

$$\%\, error = \frac{Ѳ_{practical} - Ѳ_{actual}}{Ѳ_{actual}} * 100 \qquad (9)$$

3.1 Single source

Two experiments for single-source are achieved. One for a negative angle and the other for a positive angle.

3.1.1 Single-source with a negative angle

The experiment uses a single source with a negative angle. The actual angle is equal to Ѳ = -10°. The following parameters are used: N=17, d=0.2 cm, F=40 KHz, ϒ=0.8 cm. Fig.2 refers to the result of the FFT algorithm method. The practical (apparent) angle is equal to -6°. The percentage error is very high and equal to 40%.

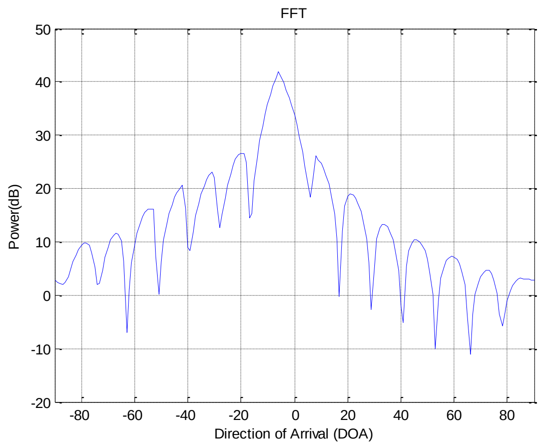

Fig.2 DOA estimation for a single-source (Ѳ = -10°) using the FFT method (N=17).

The other method that is used is the Root-MUSIC algorithm method for the same number of samples, N=17. The practical (apparent) angle is -10.4836°. Hence the percentage error is 4.836%. It means that the percentage error of the Root-MUSIC algorithm method is very less than that of the FFT method for this reason the Root-MUSIC algorithm method is better than the FFT algorithm method.

Fig.3 shows the percentage errors for a single source (Ѳ = -10°) of both FFT and Root-MUSIC algorithm methods for different numbers of samples. It is noticed that the percentage error for the FFT algorithm method is much greater than that of the Root-MUSIC algorithm method. The error for the FFT method exceeds 30% and reaches up to 40% while for the Root-MUSIC method is less than 20% and decreases down to less than 10%.

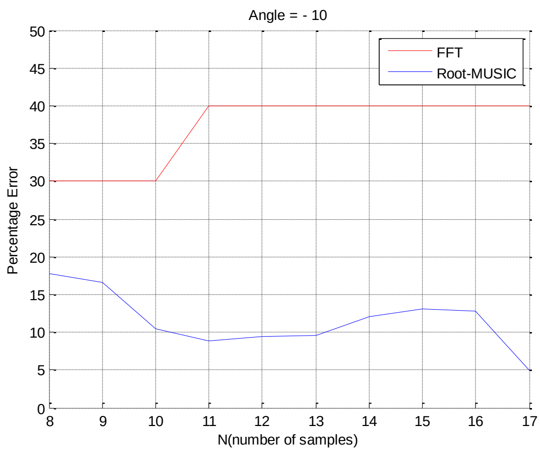

Fig.3 Percentage error versus N for DOA Estimation for the single source (Ѳ = -10°).

3.1.2 Single-source with a positive angle

The experiment uses a single source with a positive angle. The actual angle is positive and equal to $\Theta=6^0$. The following parameters are used: N=17, d=0.2 cm, F=40 KHz, Ɏ=0.8 cm.

The values of d=0.2cm and Ɏ=0.8cm is chosen according to the criteria d≤ Ɏ/2. The value of frequency equal to 40KHz is chosen since the ultrasonic waves have less attenuation than that of higher frequencies.

Fig.4 shows the result of the FFT algorithm method. The practical (apparent) angle is equal to 3^0. The percentage error is high and equal to 50%. The side lobe is high, i.e. less than 10 dB from the peak that is corresponding to the practical (apparent) angle. This is one of the disadvantages of the FFT algorithm method.

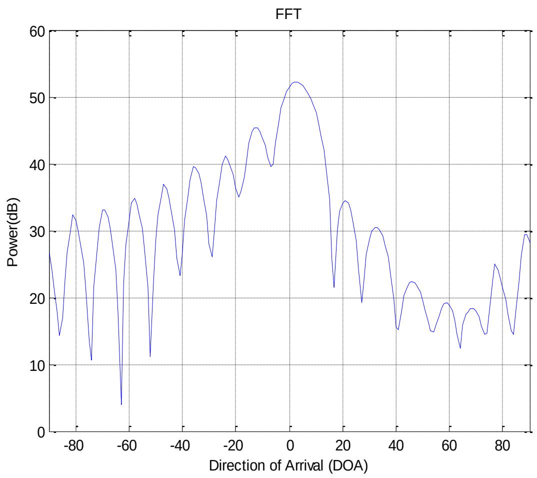

Fig.4 DOA estimation for single-source ($\Theta=6^0$) using the FFT method (N=17).

The other method that is used is the Root-MUSIC algorithm method for the same number of samples, N=17. The practical (apparent) angle is 6.1675^0. Hence the percentage error is 2.791%. It means that the percentage error of the Root-MUSIC algorithm method is very less than that of the FFT algorithm method for this reason the Root-MUSIC algorithm method is better than the FFT algorithm method. Moreover, the sidelobe that appears with the Root-MUSIC algorithm method is higher than 10dB.

Fig.5 shows the percentage error for a single source ($\Theta=6^o$) of both FFT and Root-MUSIC algorithm methods for different numbers of samples. It is noticed that the percentage error for the FFT algorithm method is much greater than the Root-MUSIC algorithm method. The error for the FFT method exceeds 50% and reaches up to 100% while for Root-MUSIC is less than 20% and decreases down to less than 10%.

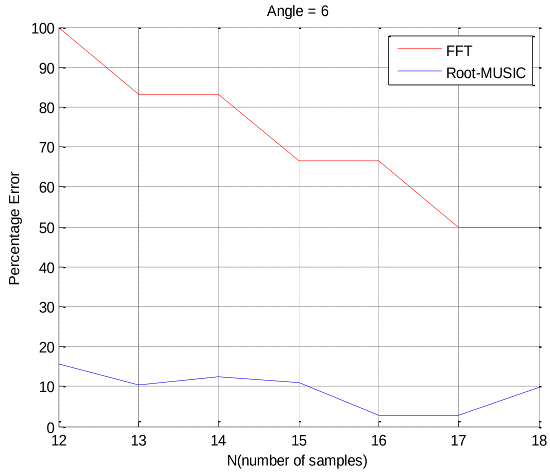

Fig.5 Percentage error versus N for DOA Estimation for the single source ($\Theta=6^o$).

3.1.3 Two sources

In other experiments, two sources are used. The first angle is negative and the second angle is positive. The first actual angle is equal to $\Theta_1 = -29^0$ and the second actual angle is equal to $\Theta_2=26^0$. The following parameters are used: N=17, d=0.2cm, F=40 kHz, Ɏ=0.8cm.

Fig.6 refers to the result of the FFT algorithm method. The first practical (apparent) angle is equal to -24^0. The percentage error is equal to 17.241%. The second practical (apparent) angle is equal to 19^0, hence the percentage error is 26.923%. The side lobe is high, less than 10dB from the peak that is corresponding to the highest practical (apparent) angle. This is one of the disadvantages of the FFT algorithm method.

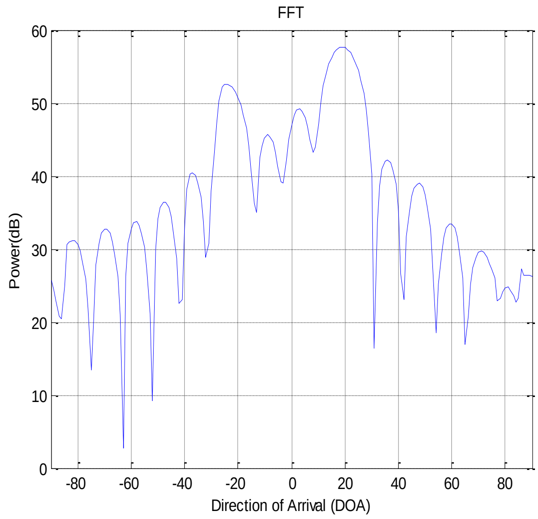

Fig.6 DOA estimation for double (two) sources ($\Theta_1 = -29^0$, $\Theta_2=26^0$) using the FFT method (N=17).

The other method that is used is the Root-MUSIC algorithm method for the same number of samples N=17. The first practical (apparent) angle is equal to -30.2004^0. The percentage error is 4.139%, while the second practical (apparent) angle is equal to 25.2695^0 with a percentage error equal to 2.809%. This means that the percentage error of the Root-MUSIC is very less than that of the FFT method for this reason the Root-MUSIC algorithm method is better than the FFT algorithm method. Moreover, the side lobe that appears with the Root-MUSIC method is higher than 10dB.

These advantages are because the Root-MUSIC method is based on the eigendecomposition of a covariance matrix of data.

Fig.7 shows the percentage error for the first angle ($\Theta_1 = -29^0$) for two sources of both FFT and Root-MUSIC algorithm methods for different numbers of samples. It is noticed that the percentage error for the FFT method is much greater than the Root-MUSIC method. The error for the FFT method exceeds 10% and reaches up to 27.586% while for Root-MUSIC is less than 10% and decreases down to less than 5%.

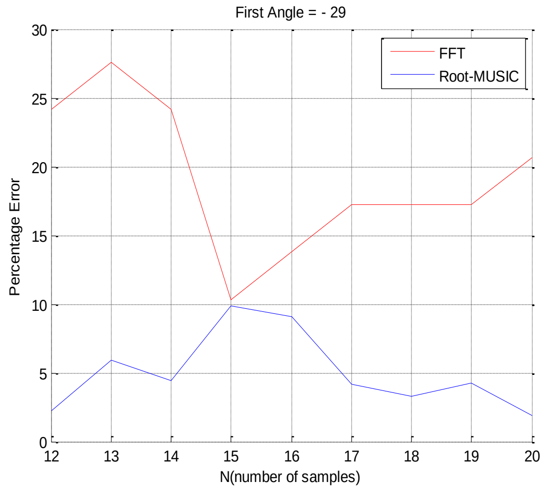

Fig.7 Percentage error versus N for double (two) sources ($\Theta_1 = -29^0$, $\Theta_2=26^0$) for the first actual angle ($\Theta_1 = -29^0$) for DOA Estimation.

Fig.8 shows the percentage error for the second angle ($\Theta_2 = 26^0$) for two sources of both FFT and Root-MUSIC methods for different numbers of samples. The percentage error for the FFT method is much greater than the Root-MUSIC method. The error for the FFT method exceeds 23% and reaches up to 34.615% while for Root-MUSIC is less than 7.5% and down to about 0%.

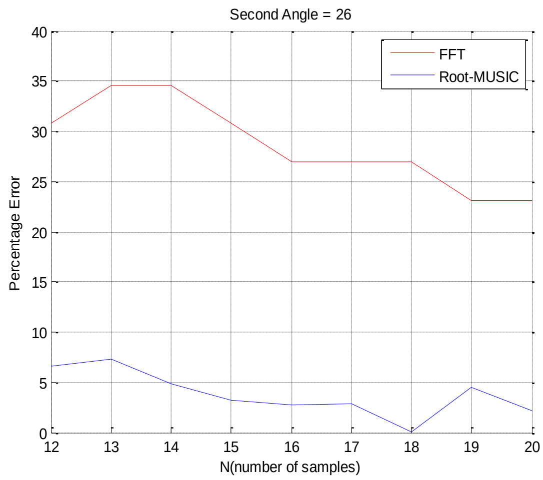

Fig.8 Percentage error versus N for double (two) sources ($\Theta_1 = -29^0$, $\Theta_2=26^0$) for the second actual angle ($\Theta_2 = 26^0$) for DOA Estimation.

4. Discussion

Many parameters are taken into account for comparing between the methods used in this paper, such as the power of sidelobe, and percentage error in the resulting angles.

5. Conclusion

From the practical results along with an investigation of the performance of both the conventional method (FFT) and super-resolution method (Root-MUSIC), we can conclude many points: The percentage errors of the FFT method is much higher, in most cases than of Root-MUSIC for both types of experiments, the two single-source experiments, and the single two sources experiment. In the FFT method the error is increased with fewer values of the number of samples N and begins decreasing, but still high, as N increases. For the Root-MUSIC method, it is noticed that the percentage error in most cases less than 15% for the two experiments of single-source, go less than 10% for the experiment of two sources.

Also, it is noticed that high side lobes have appeared for the FFT method in most experiments and it was less than 10 dB from the peak that is corresponding to the highest practical (apparent) angle. While in the Root-MUSIC method, the side lobes are of small values, higher than 10 dB from the peak that is corresponding to the highest practical (apparent) angle.

Also, it is noticed, from the results (curves) of using the FFT method, that the peak(s) corresponding to appearent (measured) angles is (are) not sharp enough.

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