MR Semi-active Suspension Accurate Linearization Adaptive LQG Control

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Abstract: In this paper, the MR damping force model of MR damper and 1/4 vehicle semi-active suspension dynamic model were established in terms of the nonlinear characteristic of MR semi-active suspension. Based on the differential geometry theory, the nonlinear model accurate linearization was realized, and the adaptive LQG control strategy was employed to overcome the uncertainty of dynamic system. the performance index weighting coefficients of LQG controller were determined by the improved analytic hierarchy process in terms of the actual requirement of the road. The simulation results show that the proper selection of the weighting coefficient with the improved AHP can be more easily; the body vibration can be reduced effectively by MR semi-active suspension adaptive LQG control, and the ride comfort and handling stability can be improved significantly than passive suspension, and the running safety also can be improved effectively.

Key–Words: MR suspension, Adaptive LQG control, Accurate linearization, Improved AHP

1 Introduction

With the increase demand of the vehicles, the higher requirement on the performance of the vehicle has been put forward. And the suspension system is the key system, which is related to the ride comfort and safety of the vehicle. So it is necessary to conduct a deep study on vehicle suspension system. Compared to the passive suspension and the full active suspension, the semi-active suspension reflects the favorable superiority [1]. Especially the MR semi-active suspension, which has the advantages of simple structure, low cost, flexible control method, fast response and the better security compared to the semi-active suspension by adjusting the opening degree of throttle valve [1-2]. So in this paper, we mainly to conduct a deep study on the MR damper and its application in 1/4 vehicle.

The damper of MR suspension has obvious nonlinear characteristics [3]. For the nonlinear system, the linearization processing is the more effective way, such as the differential geometry method, which can make the nonlinear system through a state transformation and feedback to realize the accuracy linearazition of all or part dynamic characteristics of the system [4]. So the complex nonlinear system problem was transformed into linear system problem. In the linear control field, the LQG control is a widely used control method and with the relatively mature technology. But the LQG control also has the following problems: (1) The determination of index weighting coefficient always depend on trial method or designer's experience, (2) LQG control is unable to satisfy the requirements of time-varying system with the external disturbance exists in the system. Therefore, it is necessary to find a new method that can determine the index weighting coefficient and with the universal [5-7], and study the adaptive LQG control to reduce the influence of uncertain system [8-9]. In the view of the above problem, firstly, we established the damping force model of MR damper, the nonlinear characteristics of the damping force was analyzed, and then 1/4 vehicle MR semi-active suspension dynamic model was established. Secondly the accuracy linearization method was selected to realize linearization of the nonlinear model. Finally the LQG controller of linear system was designed, and on the basis of the LQG controller, the adaptive LQG control strategy was studied. The body acceleration, suspension dynamic travel, tire dynamic displacement, suspension power consumption and control energy as the performance evaluation index of LQG controller, and an improved analytic hierarchy process was used to determine the weighting coefficient of LQG controller.

The remainder of this paper is organized as follows. Section 2 briefly reviews the formulations of MR semi-active suspension. In section 3, the accuracy linearization of MR suspension is described in detail. In section 4, adaptive LQG control of linear model is described in detail. The results of simulation analysis of system are described in section 5. Finally, section 6 concludes this paper.

2 The analysis of MR semi-active suspension

2.1 Damping Force Model

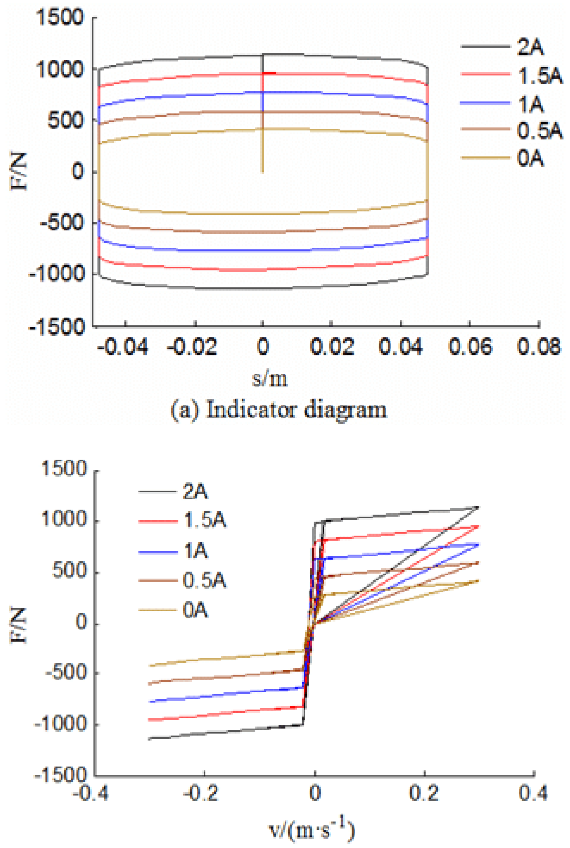
The theory of the MR damper is the strength of the magnetic field be adjusted by adjusting the current, and the rheological effect of magneto rheological fluid will be changed, and then damping force of MR damper will be changed. The motion of MR damper can be mainly divided into three kinds of work modes: flow, shear and extrusion model. And the mixed mode of flow and shear work model is applied to the MR damper of vehicle suspension [10]. In this model, the damping force is composed of a fixed damping force (viscous damping force) and controllable damping force (Kulun damping force) and nitrogen compensation force, which can be defined by the equation (1).

$$\begin{aligned} F &= \left(\frac{12\eta A_p^2 l}{bh^3} + \frac{\eta bl}{h}\right)\nu_y \\ &+ \left[\left(\frac{3lA_p}{h} + bl\right)\tau_y + p_a\frac{\pi d^2}{4}\right]sgn(\nu_y) = C_s\nu_y + \mu \end{aligned} \tag{1}$$

where $A_p = \pi(D^2 - d^2)/4$, D is the inner diameter of damper cylinder, and d is the diameter of piston, $b = \pi D$ is the equivalent width, l is the damping channel length, $\tau_y = k\mu_0 NI(2h)$, k and μ_0 are the parameters associated with magneto rheological fluid, N is the number of turns of the coil, I is the current, h is the gap of damping channel, η is zero field viscosity of MRF, p_a is the gas pressure of inflatable cavity in balance state, sgn is the sign function, ν_y is the velocity of piston.

2.2 Analysis of Damping Force Characteristic

In here, we set external sinusoidal excitation input of the MR damper is $s = Asin2\pi ft$, with the time derivative, we can obtain the input speed is $v = 2\pi fAcos2\pi ft$. The vehicle vertical velocity in the range of $0.15 \sim 0.52$ m/s, so the velocity can be set as $v = 0.3$ m/s, and the frequency can be set as $f = 0.1$ Hz, so the amplitude is $A = 47.7$ mm, and the suspension displacement in this paper is 100 mm, in the suspension travel range. The current input can be selected respectively as 0 A, 0.5 A, 1.0 A, 1.5 A, 2.0 A. So the "displacement, velocity and damping force" of the MR damper can be shown in Fig. 1.



(a) Indicator diagram

(b) Characteristics of velocity and damping force

Figure 1: The "displacement, velocity and damping force" of the MR damper.

From the Fig.1, we can get that when $A = 47.7$ mm and $v = 0.3$ m/s, the adjustable range of damping force is $409.5N \sim 1133N$, which can satisfy the work requirement.

2.3 Nonlinear Dynamic Model of MR Semi-active Suspension

The model of the 1/4 vehicle suspension is shown in Fig. 2. And the dynamic differential equation of system can be established as follow:

$$\begin{cases} m_b\ddot{z}_s = -K_s(z_s - z_w) - C_s(\dot{z}_s - \dot{z}_w) - \mu \\ m_w\ddot{z}_w = K_s(z_s - z_w) + C_s(\dot{z}_s - \dot{z}_w)K_t(z_w \\ -z_g) + \mu \end{cases} \tag{2}$$

where m_b is the sprung mess, m_w is the unsprung mass, C_s is the fixed damping of MR damper, K_s is the stiffness of the suspension, K_t is the stiffness of tire, z_s is the displacement of sprung mess, z_w is

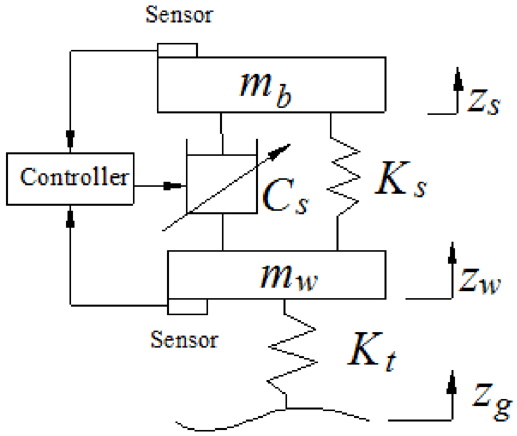


Figure 2: 1/4 Vehicle model of MR semi-active suspension.

the displacement of unsprung mess, and z_g is road input displacement. The state vector can be defined as: $\mathbf{X} = [\dot{z}_s, \dot{z}_w, z_s - z_w, z_w - z_g]^T = [x_1, x_2, x_3, x_4]^T$.

So the general form of MR semi-active suspension nonlinear system can be expressed as equation (3).

$$\dot{\mathbf{X}} = \mathbf{F(X)} + \mathbf{G(X)}\mu + \eta w, \tag{3}$$

where $\eta = [0, 0, 0, -1]^T$, $w = \dot{z}_g$,

$$\mathbf{F(X)} = \begin{cases} -K_s x_3/m_b - C_c(x_1 - x_2)/m_b \\ F_s x_3/m_w + C_c(x_1 - x_2)/m_w - K_t x_4/m_b \\ x_1 - x_2 \\ x_2 \end{cases},$$

$\mathbf{G(X)} = [-1/m_b, 1/m_w, 0, 0]^T$.

3 The accuracy linearization of MR suspension

3.1 The Differential Geometry Theory

The general form of single input and single output nonlinear system can be described by equation (4).

$$\begin{cases} \dot{\mathbf{X}} = \mathbf{F(X)} + \mathbf{G(X)}\mu \\ \mathbf{Y} = \mathbf{h(x)} \end{cases}, \tag{4}$$

where $\mathbf{X} \in \mathbf{R}^n$ is the state variable, $\mathbf{F}$ and $\mathbf{G}$ are the smooth vector field of $\mathbf{R}$, with continuous derivatives of order infinity, $\mathbf{Y} \in \mathbf{R}$ is the output variable, $\mathbf{h(x)}$ is a sufficiently function, $\mu \in \mathbf{R}$ is the input control variable.

For all the x in the neighborhood of $x = x_0$, there is an integer r that can make the equation (5) established:

$$\begin{cases} L_G L_F^k \mathbf{h(X)} = 0, & 0 \le k < r - 1 \\ L_G L_F^{r-1} \mathbf{h(X)} \neq 0, & other \end{cases}. \tag{5}$$

So the integer r is the system relative order. And the state feedback transformation can be described by equation (6).

$$\mu = \frac{1}{L_G L_F^{r-1}\mathbf{h(X)}}[-L_F^r \mathbf{h(X)} + v]. \tag{6}$$

3.2 Accuracy Linearization of System Model

In this subsection, each variable of original system will be analyzed in terms of equations (4-6).

Set $h_1(\mathbf{X}) = x_1$, when $k = 0$, then

$$\begin{cases} L_F^0 h_1(\mathbf{X}) = h_1(\mathbf{X}) = x_1 \\ L_G L_F^0 h_1(\mathbf{X}) = \frac{\partial h_1(\mathbf{X})}{\partial \mathbf{X}} \mathbf{G(X)} = \frac{-1}{m_b} \end{cases}.$$

Hence, the relative order is 1.

$$\begin{cases} L_F^1 h_1(\mathbf{X}) = \frac{\partial h_1(\mathbf{X})}{\partial \mathbf{X}} \mathbf{F(X)} = \frac{-K_s x_3}{m_b} - \frac{C_s(x_1 - x_2)}{m_b} \\ \mu_1 = \frac{-L_F^1 h_1(\mathbf{X}) + v}{L_G L_F^0 h_1(\mathbf{X})} = -K_s x_3 - C_s(x_1 - x_2) - m_b v_1 \end{cases}.$$

Similarity, we can obtain:

When $h_2(\mathbf{X}) = x_2$, the relative order is 1, $\mu_2 = -K_s x_3 - C_s(x_1 - x_2) - K_t x_4 - m_w v_2$.

When $h_3(\mathbf{X}) = x_3$, the relative order is 2, $\mu_3 = -K_s x_3 - C_s(x_1 - x_2) + \frac{m_b K_t x_4}{m_b + m_w} - \frac{m_b m_w}{m_b + m_w} v_3$.

When $h_4(\mathbf{X}) = x_4$, the relative order is 3, $\mu_4 = -K_s x_3 - C_s(x_1 - x_2) - K_t x_3 - m_w v_4$.

Through the above comprehensive analysis, the system semi-active control rate can be described by equation (7).

$$\mu_4 = -K_s x_3 - C_s(x_1 - x_2) - K_t x_3 - v. \tag{7}$$

The equation (7) put in equation (3), and the output variables can be set as:

$$\mathbf{Y} = [\ddot{z}_s, z_s - z_w, z_w - z_g, \dot{z}_s - \dot{z}_w]^T = [y_1, y_2, y_3, y_4]^T$$

So we can obtain the output equation of system state equation:

$$\begin{cases} \dot{\mathbf{X}} = \mathbf{AX} + \mathbf{B}v + \eta w \\ \mathbf{Y} = \mathbf{CX} + \mathbf{D}v \end{cases}, \tag{8}$$

where $\mathbf{B} = [-1/m_b, 1/m_w, 0, 0]^T$, $\mathbf{D} = [-1/m_b, 0, 0, 0]^T$,

$$\mathbf{A} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{-K_t}{m_w} & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

$$\mathbf{C} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 \end{pmatrix}.$$

4 Adaptive LQG control of linear model

4.1 LQG Controller Design

The integral value of weighted square sum of body acceleration, suspension dynamic travel, tire dynamic displacement, suspension power consumption and control energy as the target performance index J of LQG controller. The expression can be described by equation (9).

$$J = \lim_{T\to\infty} \frac{1}{T} \int_0^T [q_1 \dot{x}_1^2 + q_2 x_3^2 + q_3 x_4^2 + q_4 (x_1 - x_2)^2 + \rho v^2] dt \tag{9}$$

Where q_1, q_2, q_3, q_4 and ρ are the weighting coefficients of performance index, respectively. Equation (9) can be transformed into the form of equation (10).

$$J = \lim_{T\to\infty} \frac{1}{T} \int_0^T [\mathbf{Y}^T \mathbf{Q}_0 \mathbf{Y} + \mathbf{v}^T \rho \mathbf{v}] dt. \tag{10}$$

Where $\mathbf{Q}_0 = diag[q_1, q_2, q_3, q_4]$ and $\rho = [\rho]$. The equation $\mathbf{Y} = \mathbf{CX} + \mathbf{Dv}$ put in equation (10), we can obtain the equation (11).

$$J = \lim_{T\to\infty} \frac{1}{T} \int_0^T [\mathbf{X}^T \mathbf{QX} + \mathbf{v}^T \mathbf{Rv} + 2\mathbf{X}^T \mathbf{Nv}] dt. \tag{11}$$

Where $\mathbf{Q} = \mathbf{C}^T \mathbf{Q}_0 \mathbf{C}$ is the weighting matrix of state variables, $\mathbf{R} = \rho + \mathbf{D}^T \mathbf{Q}_0 \mathbf{D}$ is the weighting matrix of control variables, $\mathbf{N} = \mathbf{C}^T \mathbf{Q}_0 \mathbf{D}$ is the weight of cross terms. From the Riccati equation (12), we can obtain $\mathbf{P}$:

$$\mathbf{PA} + \mathbf{A}^T \mathbf{P} - (\mathbf{PB} + \mathbf{N}) \mathbf{R}^{-1} (\mathbf{B}^T \mathbf{P} + \mathbf{N}^T) + \mathbf{Q} = 0. \tag{12}$$

Then $\mathbf{v} = -\mathbf{R}^{-1}(\mathbf{B}^T \mathbf{P} + \mathbf{N}^T)\mathbf{X} = \mathbf{KX}$ is the semi-active optimal control rate, put in equation (7), we can obtain the final nonlinear state feedback.

4.2 Adaptive LQG Control

Because the road excitation, suspension state parameters and measurement noise with time-varying, so the adaptive LQG control can be adopted to reduce the influence of the system by the uncertainty of the model [8]. Through continuous measurement the input and output of system, so that the system model parameter can be identified online. Selected the two order parameter estimation model as the equation (13):

$$A(z^{-1})y(k) = B(z^{-1})v(k) + n(k), \tag{13}$$

where $A(z^{-1}) = 1 + a_1 z^{-1} + a_2 z^{-n}$, $n(k)$ is the measure noise, $B(z^{-1}) = 1 + b_1 z^{-1} + b_2 z^{-n}$. The equation (13) can be transformed into least squares form, as the equation (14):

$$y(k) = \Phi^T(k)\theta + n(k), \tag{14}$$

where $\Phi(k)$ is the beta data vector, θ is the parameter vector of model. The specific form of $\Phi(k)$ and θ can be described by the equation (15):

$$\begin{cases} \Phi(k) = [-y(k-1), -y(k-2), v(k), \\ v(k-1), v(k-2)]^T \\ \theta = [a_1, a_2, b_0, b_1, b_2]^T \end{cases}. \tag{15}$$

The identification method can adopt recursive least square method with the forgetting factor ξ $(0 < \xi < 1)$, which can be described by the equation (16):

$$\begin{cases} \widehat{\theta}(k) = \widehat{\theta}(k-1) + K(k)[y(k) - \Phi^T(k) \\ \theta(k-1)] \\ K(k) = P(k-1)\Phi(k)[\Phi^T(k)P(k-1) \\ \Phi(k) + \xi]^{-1} \\ P(k) = [I - K(k)\Phi^T(k)]P(k-1)/\xi \end{cases}, \tag{16}$$

where $\xi = 0.97$, $\widehat{\theta}(k)$ is the least squares estimation of the unknown parameter vector at a time k.

Parameters identification model transformed into discrete state space form, adaptive LQG control can be realized. Discrete state space form can be described by the equation (17):

$$\begin{cases} X(k+1) = \psi X(k) + \gamma v(k) + \eta w(k) \\ y(k) = HX(k) + M\mu(k) + n(k) \end{cases}, \tag{17}$$

where

$$\psi = \begin{pmatrix} 0 & 1 \\ -a_2 & -a_1 \end{pmatrix},$$

$$\gamma = \begin{pmatrix} 0 \\ 1 \end{pmatrix},$$

$M = (1)$, $H = [b_2 - a_2, b_1 - a_1]$, $w(k)$ is the white noise that the external acted on the system, namely the road excitation speed.

The least squares estimation $\widehat{\psi}(k)$, $\widehat{H}(k)$, $\widehat{M}(k)$, and $\widehat{\gamma}(k)$, can be got by the parameters identification equation (13-16), and then replace the ψ, H, M, γ of equation (17), so the closed loop adaptive LQG control can be conducted.

4.3 The Determination of LQG Controller Weighting Coefficient

Analytic hierarchy process is a method of multi- objective programming and decision making, we can use it to determine the subjective weights of each evaluation index [5,11]. But when the evaluation index is excessive, it is not easy to judge the importance of every index [8]. Because the evaluation index in this paper is more, so we can choose an improved analytic hierarchy process to determine the weighting coefficient of each performance index.

1) Determination steps of weighting coefficient by AHP.

a) The construction of evaluation index comparison matrix H. In here, we set h_{ij} as the importance comparison value of index i and j, the comparison table of the relative importance of each index is shown in table 1.

Table 1: The Comparison Value of Index Importance.

Index i/j	More important	Same important	Not important
h_{ij}	2	1	0

In the complex road, the safety and ride comfort is the most important, so we can obtain the subjective weights comparison matrix H:

$$H = \begin{bmatrix} 1 & 1 & 1/2 & 1 & 2 \\ 1 & 1 & 1/2 & 1 & 2 \\ 2 & 2 & 1 & 2 & 2 \\ 1 & 1 & 1/2 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}.$$

b) The importance order coefficient w_i.

$$w_i = \sum_{j=1}^{n} h_{ij}, (i, j = 1, 2, \cdots, n). \tag{18}$$

c) Constructing judgment matrix P. The element p_{ij} of matrix P can be established as follow:

$$p_{ij} = \begin{cases} \frac{w_i - w_j}{w_{max} - w_{min}}(k_m - 1) + 1, & w_i \geq w_j \\ [\frac{w_i - w_j}{w_{max} - w_{min}}(k_m - 1) + 1]^{-1}, & w_i < w_j \\ i, j = 1, 2, \cdots, n \end{cases}, \tag{19}$$

where $w_{max} = max(w_i)$, $w_{min} = min(w_i)$, $k_m = w_{max}/w_{min}$.

d) Transfer matrix T of judgment matrix P. The element t_{ij} of matrix T can be established as follow:

$$t_{ij} = lgP_{ij}, (i, j = 1, 2, \cdots, n). \tag{20}$$

e) The optimal transfer matrix S of transfer matrix T. The element s_{ij} of matrix S can be established as follow:

$$s_{ij} = \frac{1}{n}\sum_{k=1}^{n}(t_{ik} - t_{jk}), (i, j = 1, 2, \cdots, n). \tag{21}$$

f) The quasi optimal consistent matrix E of judgment matrix P. The element e_{ij} of matrix E is:

$$e_{ij} = 10^{s_{ij}}, (i, j = 1, 2, \cdots, n). \tag{22}$$

g) The weights ranking matrix V of quasi optimal consistent transfer matrix E. The elements V_i of V are the weighting coefficient of each evaluation index.

$$V_i = \sum_{j=1}^{n} e_{ij}, (i, j = 1, 2, \cdots, n). \tag{23}$$

2) The finally determination of weighting coefficient.

There is a great difference between each evaluation index unit and the magnitude order, so when compare its relativity, the same scale quantitative proportion coefficient of the evaluation index should be obtained by the passive suspension [11]. So the performance standard deviation σ_i in a particular condition of passive suspension can be chosen. The same scale quantitative proportion coefficient of body vertical acceleration standard deviation σ_1 can be defined as 1, then the same scale quantitative proportion coefficient β_1 of another performance index can be determined by equation (24).

$$\sigma_1^2 \times 1 = \sigma_i^2 \times \beta_i, (i = 1, 2, \cdots, n). \tag{24}$$

The subjective weighted proportion coefficient of body vertical acceleration can be defined as 1, then the subjective weighting proportion coefficient γ_i and the final weighting proportion coefficient q_i of another performance index can be determined by equation (25).

$$\begin{cases} V_2 = V_i/\gamma_i \\ q_i = \beta_i \times \gamma_i \\ (i = 1, 2, \cdots, n) \end{cases}. \tag{25}$$

5 Simulation analysis of system

Through above analysis, the weighting coefficient of LQG control can be determined, and the weighting coefficient matrix is $\mathbf{Q}_0 = diag[1, 2154, 108000, 20.606]$, $\rho = 1.641 \times 10^{-5}$. The block diagram of the control system is shown in Fig. 3.

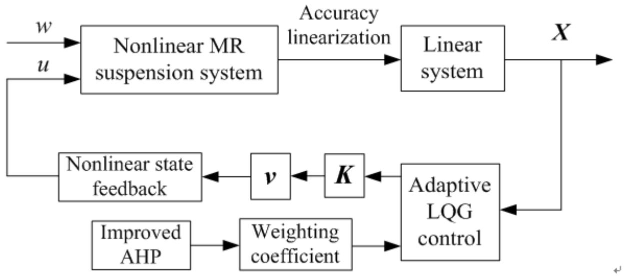


Figure 3: The diagram of control system.

The first-order filtering white noise can be used as the road input:

$$\dot{z}_g(t) = -2\pi f_0 z_g(t) + 2\pi\sqrt{G_0 u_c}w(t), \tag{26}$$

where $f_0 = 0.1$(Hz) is the lower cut-off frequency, $G_0 = 5 \times 10^{-6}(m^3$/cycle) is the road roughness coefficient, $u_c = 20$(m/s) is the vehicle speed, w is the Gauss white noise distribution, the mean value is 0 and the noise intensity is 1.

In this section, we set the variance of measurement noise is $n(k) = 0.136$. Through the simulation analysis in Matlab/Simulink, the performance comparison of MR semi-active suspension and passive suspension can be shown in Fig. 4.

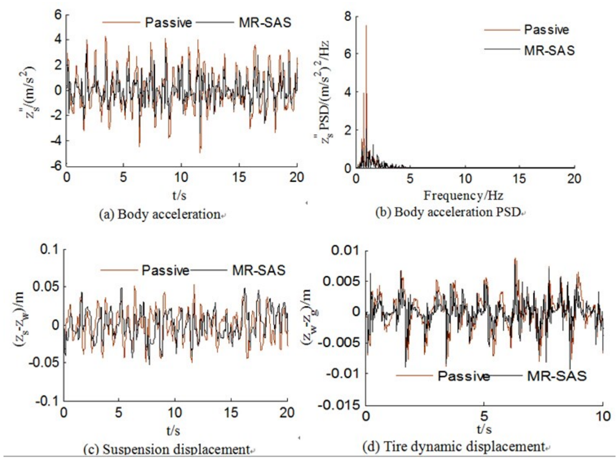


Figure 4: The performance comparison.

From the Fig.4, we can initially find that the each performance of MR semi-active suspension is superior to the passive suspension. Especially the body acceleration PSD of MR semi-active suspension is far less than the passive suspension, so the energy of the semi-active system is far less than the passive suspension, the suspension dynamic performance will be more stable.

In order to show the performance of passive suspension and MR semi-active suspension, table 2 will show each performance standard difference of passive suspension and MR semi-active suspension. Where PC, PS and MRSAS are denote Performance classification, Passive suspension and MR semi-active suspension, respectively.

Table 2: The comparison of each performance standard difference.

PC	PS	MRSAS
$\ddot{z}_s/(m/s^2)$	1.60	0.91
$(z_s - z_w)/cm$	1.96	1.72
$(z_w - z_g)/mm$	2.89	2.18

Form the table 2, we can find that the body acceleration, suspension displacement and tire dynamic displacement of MR semi-active suspension are reduced 43.1%, 12.2% and 24.6%, respectively. The body acceleration got a great slow down, and the ride comfort is improved greatly. The performance of suspension has been improved, and it can meet the requirement of engineering.

6 Conclusion

In this paper, the nonlinear characteristics of MR suspension damping force has been analyzed. The state feedback accuracy linearization nonlinear transformation of vehicle MR semi-active suspension has been realized by accuracy linearization method based on differential geometry theory. Adaptive LQG control theory was applied to the linear model, avoid the uncertainties of the dynamic behavior of the control system. The optimal control rate of linear system that be transformed can be got by LQG control theory, and then the whole nonlinear state feedback of nonlinear systems can also be realized. According to the requirements of ride comfort and safety, the LQG controller performance index weighted coefficient were determined by the improved AHP, realized the weighted coefficient on-line control, which with the more universal and practical. Through the simulation analysis, after the accurate linearization and added the adaptive optimal control rate, the comprehensive performance of nonlinear MR semi-active suspension can be improved effectively. The body vertical acceleration can be reduced greatly, improved the ride comfort, can better meet the security requirement.

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