Color Space Transform for Correlated Images Based on the Recursive Adaptive KLT

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Abstract: - In this work is presented a method for RGB color space transform for a group of correlated images, based on the recursive calculation of the algorithm Adaptive Color KLT (AC-KLT), developed earlier by the authors. However, the use of the algorithm AC-KLT for a group of color images is not sufficiently efficient, because each image should be processed individually. Depending on the image kind (for example, video- or multi-view sequences, medical image sequences, etc.) the pixels of same spatial position can have very high similarity, which in some cases is more than 90%. In such case, there is a high possibility to reduce significantly the information redundancy of the color information for the processed group of images. For this, here is offered the method Recursive AC-KLT, whose basic idea is the AC-KLT to be calculated for the first image in the group only, and for all other images to calculate the values of the difference parameters, needed to restore the color information of the image.

The efficiency of the presented approach depends on the mutual correlation between images in the group. The algorithm efficiency is evaluated, and the obtained results confirm its advantage over the basic AC-KLT algorithm. The new method could be used for processing groups of correlated images in the systems for compression and processing of visual information, computer vision, pattern recognition, digital watermarking, etc.

Key-Words: Karhunen-Loève Transform, Adaptive Color Space Transform, Recursive Adaptive Color KLT, Group of Correlated Color Images

1 Introduction

The processing of sequences of color images became recently an object of many scientific investigations [1-4], aimed at the reduction of the color information redundancy, noise filtration, color-based segmentation and recognition of the objects in the image, tracing of moving objects in video sequences, color analysis of selected objects in groups of images, etc. One of the most efficient transforms of the primary image space RGB in mean square sense is the Karhunen-Loève Transform (KLT) [5-9], also known as the Hotelling Transform (HT) or the Principal Component Analysis (PCA). Its most important advantage in respect of the famous determined color transforms of the kind YCrCb, YUV, YIQ, YCoCg, RCT, HSV (or HSL), CMY, PhotoYCC, Lab, Luv, etc. [2,3,10,11], is the maximum decorrelation of the transformed color components $L_1L_2L_3$. The main disadvantage of the color KLT the higher computational complexity than that of the deterministic transforms. However, together with the development of the contemporary computer technologies, it becomes easily overcome, which defines the color KLT actuality.

In [12] are given some applications of the KLT for recursive image block coding. In this case, the transform is calculated for the first image block, and for the next blocks are calculated the difference parameters only. The efficiency of this approach is strongly dependable on the spatial inter-block correlation, which in some cases (depending on the image contents) is not high enough. As it is known, for many kinds of images obtained from various sources, the correlation between pixels of same spatial position in two neighbor images is much higher than the corresponding inter-block correlation. Depending on the image kind (for example, color video or multi-view sequences, medical sequences, etc.) the similarity between neighbor image couples could be very high (above 90%). In such case, there is a possibility for significant reduction of the information redundancy for the processed group of images.

To enhance the processing efficiency here is offered the method, called Recursive Adaptive Color KLT (RAC-KLT). In accordance with the new approach, on the RGB components of each pixel is applied recursive KLT with transform matrix of size 3×3. The basis of the algorithm is the Adaptive RGB space transform algorithm for a single image, based on the KLT, and called Adaptive Color KLT (AC-KLT) [13]. It has lower computational complexity than the KLT algorithms, which use iterative calculation methods [5,6]. The basic advantage of AC-KLT towards the famous deterministic color transforms is the maximum decorrelation of the transformed three components, in result of which the color energy concentration is maximum in the first one. However, the use of AC-KLT for a group of color images is not efficient enough, because each image should be processed individually, and the computational complexity of AC-KLT is higher than that of the deterministic color transforms. These disadvantages are overcome by RAC-KLT.

The paper is arranged as follows. In Section 2 is given the short presentation of the algorithm AC-KLT for a single color image; in Sections 3 and 4 are presented the method and the corresponding algorithm for RAC-KLT for a group of correlated images; in Section 5 is evaluated the efficiency of RAC-KLT, compared to AC-KLT, in Section 6 are commented the possible applications of the new algorithm, and section 7 contains the Conclusions.

2 Adaptive Color KLT for the Image RGB Components

The use of the Karhunen-Loève Transform for processing of the image primary color components results in their decorrelation, which ensures the enhancement of such operations as: compression, color-based segmentation, etc.

The basic problem is the high computational complexity of KLT. The method Adaptive Color KLT offers precise, and together with this, non-iterative calculation of the covariance matrix eigenvectors and has lower computational complexity. The detailed presentation of the method is given in earlier publication of the authors [13]. The AC-KLT has two basic forms, given below.

2.1 Arithmetic representation of AC-KLT

The direct AC-KLT of the color components R,G,B for an image of S pixels and of size m×n=S, is:

$$\vec{L}_s=[\Phi](\vec{C}_s-\vec{m}) \text{ or } \vec{L}_s=[\Phi]\vec{X}_s \text{, for s=1,2,..,S.} \quad (1)$$

Here $\vec{C}_s=[R_s,G_s,B_s]^T$ is the color vector which represents the pixel s in the image of S pixels; $\vec{L}_s=[L_{1s},L_{2s},L_{3s}]^T$ - the transformed color vector of the pixel s; $\vec{m}=[E(R_s),E(G_s),E(B_s)]^T$ - the mean color vector, where $\bar{x}=E(x_s)=(1/S)\sum_{s=1}^{S}x_s$ is the mean operator (for x_s in respect to s);

$\vec{X}_s=\vec{C}_s-\vec{m}=[x_{1s},x_{2s},x_{3s}]^T$ - the modified color vector s, for which $E(\vec{X}_s)=0$; $[\Phi]^T=[\vec{\Phi}_1,\vec{\Phi}_2,\vec{\Phi}_3]$ - the transposed matrix of size 3×3 used for the adaptive color KLT; $\vec{\Phi}_k=[\Phi_{1k},\Phi_{2k},\Phi_{3k}]^T$ for k=1,2,3 - the eigen vectors of the color covariance matrix $[K_c]$, defined on the basis of the modified vectors $\vec{X}_s=[x_{1s},x_{2s},x_{3s}]^T$:

$$[K_c]=E(\vec{X}_s\vec{X}_s^T)=\frac{1}{S-1}\sum_{s=1}^{S}\vec{X}_s\vec{X}_s^T. \quad (2)$$

The elements k_{ij} of this matrix are symmetric in respect to its main diagonal, and it is represented as:

$$[K_c]=\begin{bmatrix}k_{11} & k_{12} & k_{13}\\ k_{21} & k_{22} & k_{23}\\ k_{31} & k_{32} & k_{33}\end{bmatrix}=\begin{bmatrix}k_1 & k_4 & k_5\\ k_4 & k_2 & k_6\\ k_5 & k_6 & k_3\end{bmatrix}. \quad (3)$$

In the relation above are used the substitutions: $k_1=k_{11}$, $k_2=k_{22}$, $k_3=k_{33}$, $k_4=k_{12}=k_{21}$, $k_5=k_{13}=k_{31}$, $k_6=k_{23}=k_{32}$. The elements $k_1 \div k_6$ of the covariance matrix are calculated from the relations:

$$k_1=E(R_s^2)-(\bar{R})^2,\ k_2=E(G_s^2)-(\bar{G})^2,\ k_3=E(B_s^2)-(\bar{B})^2; \quad (4)$$

$$k_4=E(R_sG_s)-(\bar{R})(\bar{G}),\ k_5=E(R_sB_s)-(\bar{R})(\bar{B}),$$
$$k_6=E(G_sB_s)-(\bar{G})(\bar{B}). \quad (5)$$

The eigenvectors $\vec{\Phi}_k$ of the matrix $[K_C]$ are the solution of the set of equations:

$$[K_C]\vec{\Phi}_k=\lambda_k\vec{\Phi}_k \text{ and } |\vec{\Phi}_k|^2=\sum_{i=1}^{3}\Phi_{ki}^2=1 \text{ for k=1, 2, 3.} \quad (6)$$

The eigenvalues $\lambda_1,\lambda_2,\lambda_3$ of the matrix $[K_C]$ are the solution of its characteristic equation:

$$\det|k_{ij}-\lambda\delta_{ij}|=\lambda^3+a\lambda^2+b\lambda+c=0, \quad (7)$$

where: $\delta_{ij}=\begin{cases}1, & \text{for } i=j,\\ 0, & \text{for } i\neq j,\end{cases}$

$$a=-(k_1+k_2+k_3),\ b=k_1k_2+k_1k_3+k_2k_3-(k_4^2+k_5^2+k_6^2),$$
$$c=k_1k_6^2+k_2k_5^2+k_3k_4^2-(k_1k_2k_3+2k_4k_5k_6). \qquad (8)$$

Since the matrix $[K_C]$ is symmetrical, its eigenvalues are always real numbers. They can be defined using the Cardano relations for the "casus irreducibilis" ("trigonometric solution"):

$$\lambda_1=2\sqrt{|p|/3}\cos(\varphi/3)-(a/3),$$
$$\lambda_{2,3}=-2\sqrt{|p|/3}\cos[(\varphi\mp\pi)/3]-(a/3). \qquad (9)$$

For these eigenvalues are satisfied the relations: $\lambda_1\geq\lambda_2\geq\lambda_3\geq 0$ and $\lambda_1+\lambda_2+\lambda_3=k_1+k_1+k_1$.

In Eqs. (9) are used the following quantities:

$$q=2(a/3)^3-(ab)/3+c,\ p=-(a^2/3)+b<0,$$
$$\varphi=\arccos[-q/2\big/\sqrt{(|p|/3)^3}]. \qquad (10)$$

The solution of the systems of equations (6) defines the elements Φ_{ik} of the matrix $[\Phi]$, when i,k=1,2,3, and the rows of the matrix comprise the eigen vectors $\vec{\Phi}_k$. The corresponding elements are:

$$\Phi_{i1}=A_i/P_i,\ \Phi_{i2}=B_i/P_i,\ \Phi_{i3}=D_i/P_i \text{ for } i=1,2,3, \quad (11)$$

$$A_i=(k_3-\lambda_i)[k_5(k_2-\lambda_i)-k_4k_6],$$
$$B_i=(k_3-\lambda_i)[k_6(k_1-\lambda_i)-k_4k_5], \qquad (12)$$
$$D_i=k_6[2k_4k_5-k_6(k_1-\lambda_i)]-k_5^2(k_2-\lambda_i),$$

$$P_i=\sqrt{A_i^2+B_i^2+D_i^2}. \qquad (13)$$

Then from Eq. (11) it follows, that the direct AC-KLT could be presented as:

$$\begin{bmatrix}L_{1s}\\L_{2s}\\L_{3s}\end{bmatrix}=\begin{bmatrix}A_1/P_1 & B_1/P_1 & D_1/P_1\\A_2/P_2 & B_2/P_2 & D_2/P_2\\A_3/P_3 & B_3/P_3 & D_3/P_3\end{bmatrix}\times\left(\begin{bmatrix}R_s\\G_s\\B_s\end{bmatrix}-\begin{bmatrix}E(R_s)\\E(G_s)\\E(B_s)\end{bmatrix}\right). \quad (14)$$

The inverse AC-KLT is defined by the relations:

$$\vec{C}_s=[\Phi]^T\vec{L}_s+\vec{m}, \text{ or } \vec{X}_s=[\Phi]^T\vec{L}_s, \qquad (15)$$

From Eqs. (14) and (15) it follows that the inverse AC-KLT could be represented as:

$$\begin{bmatrix}R_s\\G_s\\B_s\end{bmatrix}=\begin{bmatrix}A_1/P_1 & A_2/P_2 & A_3/P_3\\B_1/P_1 & B_2/P_2 & B_3/P_3\\D_1/P_1 & D_2/P_2 & D_3/P_3\end{bmatrix}\times\begin{bmatrix}L_{1s}\\L_{2s}\\L_{3s}\end{bmatrix}+\begin{bmatrix}E(R_s)\\E(G_s)\\E(B_s)\end{bmatrix}. \quad (16)$$

As a result of the direct AC-KLT, the energy of the first component (L_{1s}) of the transformed vector $\vec{L}_s$ is maximum, the second (L_{2s}) is much smaller, and the last (L_{3s}) is close to zero.

2.2. Trigonometric representation of AC-KLT

The matrix $[\Phi]$ could be represented through the Euler rotation angles (α,β,γ), as follows:

$$[\Phi]=\begin{bmatrix}\Phi_{11} & \Phi_{12} & \Phi_{13}\\\Phi_{21} & \Phi_{22} & \Phi_{23}\\\Phi_{31} & \Phi_{32} & \Phi_{33}\end{bmatrix}=$$
$$=[\Phi_1(\alpha)][\Phi_2(\beta)][\Phi_3(\gamma)]=[\Phi(\alpha,\beta,\gamma)], \qquad (17)$$

where the angles α,β,γ define the position of the transform coordinate axes L_1, L_2, L_3 in respect to the original color RGB space;

$$[\Phi_1(\alpha)]=\begin{bmatrix}\cos\alpha & -\sin\alpha & 0\\\sin\alpha & \cos\alpha & 0\\0 & 0 & 1\end{bmatrix};$$
$$[\Phi_2(\beta)]=\begin{bmatrix}\cos\beta & 0 & \sin\beta\\0 & 1 & 0\\-\sin\beta & 0 & \cos\beta\end{bmatrix}; \qquad (18)$$
$$[\Phi_3(\gamma)]=\begin{bmatrix}\cos\gamma & -\sin\gamma & 0\\\sin\gamma & \cos\gamma & 0\\0 & 0 & 1\end{bmatrix}.$$

From Eqs. (17) and (18) it follows, that the elements of the matrix $[\Phi]$ are defined by the relations:

$$\Phi_{11}=\cos\alpha\cos\beta\cos\gamma-\sin\alpha\sin\gamma;$$
$$\Phi_{12}=-\cos\alpha\cos\beta\sin\gamma-\sin\alpha\cos\gamma; \qquad (19)$$
$$\Phi_{13}=\cos\alpha\sin\beta;$$
$$\Phi_{21}=\sin\alpha\cos\beta\cos\gamma+\cos\alpha\sin\gamma;$$
$$\Phi_{22}=-\sin\alpha\cos\beta\sin\gamma+\cos\alpha\cos\gamma; \qquad (20)$$
$$\Phi_{23}=\sin\alpha\sin\beta;$$
$$\Phi_{31}=-\sin\beta\cos\gamma;\ \Phi_{32}=\sin\beta\sin\gamma;\ \Phi_{33}=\cos\beta. \qquad (21)$$

The matrix of the inverse AC-KLT in this case is defined as:

$$[\Phi]^{-1}=[\Phi]^T=[\Phi_1(-\gamma)][\Phi_2(-\beta)][\Phi_3(-\alpha)]. \qquad (22)$$

Hence, to define $[\Phi]^{-1}$ are needed the Euler angles α, β and γ only. These angles are calculated using the matrix elements:

$$\alpha=\arcsin(\Phi_{23}\big/\sqrt{1-\Phi_{33}^2})=\arcsin(D_2P_3\big/P_2\sqrt{A_3^2+B_3^2}); \quad (23)$$

$$\beta=\arccos(\Phi_{33})=\arccos(D_3/P_3); \qquad (24)$$

$$\gamma=\arccos(\Phi_{32}\big/\sqrt{1-\Phi_{33}^2})=\arccos(B_3\big/\sqrt{A_3^2+B_3^2}). \quad (25)$$

The elements of the matrix $[\Phi]^{-1}$ are restored by using the angles α, β, γ. As a result, to the decoder are transferred the values of angles α, β, γ only, instead of all 9 elements of the matrix $[\Phi]^{-1}$, i.e. the number of needed coefficients is 3 times smaller.

3 Recursive Adaptive Color KLT for a Group of Color Images

The method, called Recursive Adaptive Color KLT (RAC-KLT), is aimed at the processing of a sequence of correlated images, separated into groups with fixed, or adaptively changing length. In the first case, the fixed length is defined on the basis of the averaged range of the mutual correlation calculated for the images in the group. In the second case, the length is set in accordance with the value of the mutual correlation between the couples of neighbor images in the group, which should be higher than a pre-selected threshold. The method RAC-KLT is applied for the images from each group, got after its separation into sub-groups with fixed, or adaptive changing length (number of images in the group).

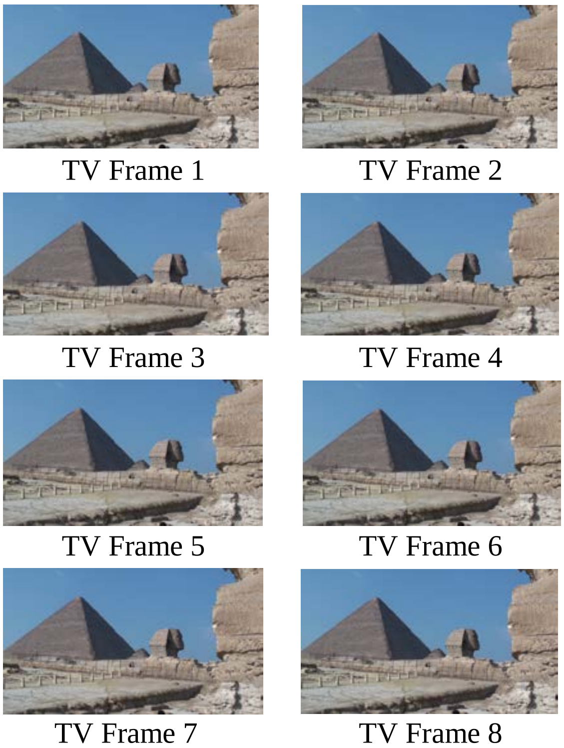

Fig. 1. A sequence of 8 TV frames from video camera with 25 fps

Examples for groups of correlated images of different kind (sequence of TV frames) with fixed length (N=8) are shown on Fig. 1.

The RAC-KLT processing for one image in the group is based on the following operations:

- Recursive calculation of the color covariance matrix $[K_{c,t}]$ of the processed image with sequential number t, based on $[K_{c,t-1}]$, calculated for the preceding image, with number (t-1):

$$[K_{c,t}] = [K_{c,t-1}] + [\Delta K_{c,t}], \tag{26}$$

where

$$[\Delta K_{c,t}] = \begin{bmatrix} (k_{1,t}-k_{1,t-1}) & (k_{4,t}-k_{4,t-1}) & (k_{5,t}-k_{5,t-1}) \\ (k_{4,t}-k_{4,t-1}) & (k_{2,t}-k_{2,t-1}) & (k_{6,t}-k_{6,t-1}) \\ (k_{5,t}-k_{5,t-1}) & (k_{6,t}-k_{6,t-1}) & (k_{3,t}-k_{3,t-1}) \end{bmatrix}. \tag{27}$$

It is supposed here that in the related equations all quantities with index (t-1) are already calculated for the previous image with number (t-1) and are stored in the dynamic buffer memory.

- Recursive calculation of the eigen vectors $\vec{\Phi}_{k,t}$

$$\vec{\Phi}_{k,t} = \vec{\Phi}_{k,t-1} + \Delta\vec{\Phi}_{k,t}, \text{ for } k = 1,2,3, \tag{28}$$

where each k^{th} difference eigen vector $\Delta\vec{\Phi}_{k,t} = \vec{\Phi}_{k,t} - \vec{\Phi}_{k,t-1}$ is the solution of the system of linear equations for k=1,2,3:

$$[\Delta K_{c,t}]\Delta\vec{\Phi}_{k,t} = \Delta\lambda_{k,t}\Delta\vec{\Phi}_{k,t}, \ \left|\Delta\vec{\Phi}_k\right|^2 = \sum_{i=1}^{3}\Delta\Phi_{ki}^2 = 1. \tag{29}$$

The difference eigen values $\Delta\lambda_{k,t}$ of the matrix $[\Delta K_{c,t}]$ are defined by relations, similar to Eqs. (9), (10), and the matrix $[\Phi_t]$ - by relations, similar to Eqs. (19)-(21). Each of the participating quantities should be assigned the index t.

- Recursive calculation of the angles $\alpha_t, \beta_t, \gamma_t$ for the processed image t:

$$\alpha_t = \alpha_{t-1} + \Delta\alpha_t, \ \beta_t = \beta_{t-1} + \Delta\beta_t, \ \gamma_t = \gamma_{t-1} + \Delta\gamma_t, \tag{30}$$

where the difference angles are defined similarly to Eqs. (23) - (25) as follows:

$$\begin{aligned} &\Delta\alpha_t = \arcsin\left(\Delta D_{2,t}\Delta P_{3,t} \Big/ \Delta P_{2,t}\sqrt{\Delta A_{3,t}^2 + \Delta B_{3,t}^2}\right), \\ &\Delta\beta_t = \arccos\left(\Delta D_{3,t}/\Delta P_{3,t}\right), \\ &\Delta\gamma_t = \arccos\left(\Delta B_{3,t} \Big/ \sqrt{\Delta A_{3,t}^2 + \Delta B_{3,t}^2}\right). \end{aligned} \tag{31}$$

- Calculation of the direct RAC-KLT for the vectors $\vec{X}_{s,t} = \vec{C}_{s,t} - \vec{m}_t$ in accordance with the relations:

$$\vec{L}_s = [\Phi(\alpha_t, \beta_t, \gamma_t)]\vec{X}_{s,t}, \text{ for } s=1,2,..,S. \tag{32}$$

The matrix $[\Phi(\alpha_t, \beta_t, \gamma_t)]$ is defined by the relations from Table 1, where in the last line the mutual correlation between the corresponding two sequential color images with sequential numbers (t) and (t-1) is close to 100 %. This permits the calculations needed for AC-KLT to be reduced many times, because the matrix $[\Phi_t]$ should not be calculated (instead, the already calculated matrix $[\Phi_{t-1}]$ is used).

Table 1

$[\Phi_t]=[\Phi(\alpha_t,\beta_t,\gamma_t)]$	Condition
$[\Phi_1(\alpha_t)][\Phi_2(\beta_t)][\Phi_3(\gamma_t)]$	$\Delta\alpha_t\neq 0,\Delta\beta_t\neq 0,\Delta\gamma_t\neq 0$
$[\Phi_1(\alpha_{t-1})][\Phi_2(\beta_t)][\Phi_3(\gamma_t)]$	$\Delta\alpha_t\approx 0,\Delta\beta_t\neq 0,\Delta\gamma_t\neq 0$
$[\Phi_1(\alpha_t)][\Phi_2(\beta_{t-1})][\Phi_3(\gamma_t)]$	$\Delta\alpha_t\neq 0,\Delta\beta_t\approx 0,\Delta\gamma_t\neq 0$
$[\Phi_1(\alpha_t)][\Phi_2(\beta_t)][\Phi_3(\gamma_{t-1})]$	$\Delta\alpha_t\neq 0,\Delta\beta_t\neq 0,\Delta\gamma_t\approx 0$
$[\Phi_1(\alpha_{t-1})][\Phi_2(\beta_{t-1})][\Phi_3(\gamma_t)]$	$\Delta\alpha_t\approx 0,\Delta\beta_t\approx 0,\Delta\gamma_t\neq 0$
$[\Phi_1(\alpha_{t-1})][\Phi_2(\beta_t)][\Phi_3(\gamma_{t-1})]$	$\Delta\alpha_t\approx 0,\Delta\beta_t\neq 0,\Delta\gamma_t\approx 0$
$[\Phi_1(\alpha_t)][\Phi_2(\beta_{t-1})][\Phi_3(\gamma_{t-1})]$	$\Delta\alpha_t\neq 0,\Delta\beta_t\approx 0,\Delta\gamma_t\approx 0$
$[\Phi_1(\alpha_{t-1})][\Phi_2(\beta_{t-1})][\Phi_3(\gamma_{t-1})]$	$\Delta\alpha_t\approx 0,\Delta\beta_t\approx 0,\Delta\gamma_t\approx 0$

- Calculation of the inverse RAC-KLT (IRAC-KLT) by analogy with Eq. (15):

$$\vec{X}_{s,t}=[\Phi_t]^{-1}\vec{L}_{s,t}=[\Phi_t]^T\vec{L}_{s,t} \text{ or } \vec{C}_{s,t}=[\Phi_t]^T\vec{L}_{s,t}+\vec{m}_t. \quad (33)$$

Here the inverse transform matrix $[\Phi_t]^T$ and the vectors $\vec{L}_{s,t},\vec{C}_{s,t},\vec{m}_t$ are calculated recursively:

$$[\Phi_t]^T=[\Phi_{t-1}]^T+[\Delta\Phi_t]^T, \quad \vec{L}_{s,t}=\vec{L}_{s,t-1}+\Delta\vec{L}_{s,t}, \quad (34)$$

$$\vec{C}_{s,t}=\vec{C}_{s,t-1}+\Delta\vec{C}_{s,t}, \ \vec{m}_t=\vec{m}_{t-1}+\Delta\vec{m}_t. \quad (35)$$

The difference matrix $[\Delta\Phi_t]^T$ for the inverse transform is calculated in correspondence with Eq. (22), but using the inverse difference angles, $(-\Delta\gamma_t),(-\Delta\beta_t),(-\Delta\alpha_t)$.

4. Algorithm RAC-KLT

On the basis of the above presented method was developed the corresponding algorithm (RAC-KLT) for the processing of a group of N correlated images. The consecutive number of the image is given as t = 0,1,..,N-1.

4.1. Algorithm for direct RAC-KLT

For the first image in the group (with number t=0) is executed the algorithm initialization, which comprises the following steps:

Step 1. Calculation of the elements of the initial color covariance matrix $[K_{c,0}]$:

$$k_{1,0}=E(R^2_{s,0})-(\bar{R}_0)^2,\ k_{2,0}=E(G^2_{s,0})-(\bar{G}_0)^2,\ k_{3,0}=E(B^2_{s,0})-(\bar{B}_0)^2,$$

$$k_{4,0}=E(R_{s,0}G_{s,0})-(\bar{R}_0)(\bar{G}_0),$$
$$k_{5,0}=E(R_{s,0}B_{s,0})-(\bar{R}_0)(\bar{B}_0), \quad (36)$$
$$k_{6,0}=E(G_{s,0}B_{s,0})-(\bar{G}_0)(\bar{B}_0).$$

Step 2. Calculation of the parameters, defined by the elements of the matrix $[K_{c,0}]$:

$$a_0=-(k_{1,0}+k_{2,0}+k_{3,0}), \quad (37)$$

$$b_0=k_{1,0}k_{2,0}+k_{1,0}k_{3,0}+k_{2,0}k_{3,0}-(k^2_{4,0}+k^2_{5,0}+k^2_{6,0}), \quad (38)$$

$$c_0=k_{1,0}k^2_{6,0}+k_{2,0}k^2_{5,0}+k_{3,0}k^2_{4,0}-(k_{1,0}k_{2,0}k_{3,0}+2k_{4,0}k_{5,0}k_{6,0}).$$

$$q_0=2(a_0/3)^3-(a_0b_0)/3+c_0,\ p_0=-(a_0^3/3)+b_0,$$

$$\varphi_0=\arccos[-(q_0/2)\Big/\sqrt{(|p_0|/3)^3}\,], \quad (39)$$

$$\lambda_1=2\sqrt{|p_0|/3}\cos(\varphi_0/3)-(a_0/3),$$
$$\lambda_{2,3}=-2\sqrt{|p_0|/3}\cos[(\varphi_0\mp\pi)/3]-(a_0/3), \quad (40)$$

$$A_{3,0}=(k_{3,0}-\lambda_{3,0})[k_{5,0}(k_{2,0}-\lambda_{3,0})-k_{4,0}k_{6,0}],$$
$$A_{3,0}=(k_{3,0}-\lambda_{3,0})[k_{5,0}(k_{2,0}-\lambda_{3,0})-k_{4,0}k_{6,0}], \quad (41)$$

$$D_{2,0}=k_{6,0}[2k_{4,0}k_{5,0}-k_{6,0}(k_{1,0}-\lambda_{2,0})]-k^2_{5,0}(k_{2,0}-\lambda_{2,0}),$$

$$P_{2,0}=\sqrt{A^2_{2,0}+B^2_{2,0}+D^2_{2,0}},\ P_{3,0}=\sqrt{A^2_{3,0}+B^2_{3,0}+D^2_{3,0}}. \quad (42)$$

Step 3. Calculation of the trigonometric functions sin(.) и cos(.) for the angles α_0, β_0, γ_0:

$$\sin\alpha_0=D_{2,0}P_{3,0}\Big/P_{2,0}\sqrt{A^2_{3,0}+B^2_{3,0}},\ \cos\alpha_0=\sqrt{1-(\sin\alpha_0)^2}, \quad (43)$$

$$\cos\beta_0=D_{3,0}/P_{3,0},\ \sin\beta_0=\sqrt{1-(\cos\beta_0)^2}, \quad (44)$$

$$\cos\gamma_0=B_{3,0}\Big/\sqrt{A^2_{3,0}+B^2_{3,0}},\ \sin\gamma_0=\sqrt{1-(\cos\gamma_0)^2}, \quad (45)$$

Step 4. Calculation of the initial AC-KLT matrix $[\Phi_0]$ as a product of the corresponding rotation matrices:

$$[\Phi_0]=[\Phi_1(\alpha_0)][\Phi_1(\beta_0)][\Phi_1(\gamma_0)]=[\Phi(\alpha_0,\beta_0,\gamma_0)]. \quad (46)$$

Step 5. Calculation of the initial mean color vector

$$\vec{m}_0=(1/S)\sum_{s=1}^{S}\vec{C}_{s,0}=[E(R_{s,0}),E(G_{s,0}),E(B_{s,0})]^T. \quad (47)$$

Step 6. Calculation of the transformed color vectors for the image with number t=0:

$$\vec{L}_{s,0}=[\Phi_0]\vec{X}_{s,0}=[\Phi_0](\vec{C}_{s,0}-\vec{m}_0) \text{ for } s=1,2,.,S. \quad (48)$$

For each of the remaining images in the group (t=t+1 for t≤N-1) are executed steps, similar to these for the image with number t=0:

Step 7. Calculation of the rotation angles $\alpha_t,\beta_t,\gamma_t$ for the image t=t+1 using relations, similar to Eqs. (43)-(45), where the index 0 is replaced by t.

Step 8. Calculation of the difference rotation angles for the image t:

$$\Delta\alpha_t=\alpha_t-\alpha_{t-1},\ \ \Delta\beta_t=\beta_t-\beta_{t-1},\ \ \Delta\gamma_t=\gamma_t-\gamma_{t-1}. \quad (49)$$

Step 9. Calculation of the approximated angles $\hat{\alpha}_t,\hat{\beta}_t,\hat{\gamma}_t$ for the image t:

$$\hat{\alpha}_t=\begin{cases}\hat{\alpha}_{t-1}, & \text{if} \quad |\Delta\alpha_t|<\delta;\\ \hat{\alpha}_{t-1}+\Delta\alpha_t & \text{- in other cases,}\end{cases}$$
$$\hat{\beta}_t=\begin{cases}\hat{\beta}_{t-1}, & \text{if} \quad |\Delta\beta_t|<\delta;\\ \hat{\beta}_{t-1}+\Delta\beta_t & \text{– in other cases,}\end{cases} \quad (50)$$
$$\hat{\gamma}_t=\begin{cases}\hat{\gamma}_{t-1}, & \text{if} \quad |\Delta\gamma_t|<\delta;\\ \hat{\gamma}_{t-1}+\Delta\gamma_t & \text{– in other cases.}\end{cases}$$

In the relations above, $\hat{x}$ means the approximation of x, and δ is a preset threshold (a small number, for example, in the range 0,01-0,05), which defines the accuracy of the restored color vectors after the inverse RAC-KLT, and the data which represents the difference angles.

Step 10. The values of functions $\sin\hat{\alpha}_t$, $\cos\hat{\alpha}_t$, $\sin\hat{\beta}_t$, $\cos\hat{\beta}_t$, $\sin\hat{\gamma}_t$, $\cos\hat{\gamma}_t$ are set from the pre-calculated tables. After that, the matrix $[\Phi_t(\hat{\alpha}_t,\hat{\beta}_t,\hat{\gamma}_t)]$ is calculated as the product of the three matrices $[\Phi_1(\hat{\alpha}_t)]$, $[\Phi_2(\hat{\beta}_t)]$, $[\Phi_3(\hat{\gamma}_t)]$ in correspondence with Eqs. (43-45), where the index 0 is replaced by t.

Step 11. Calculation of the mean color vector $\vec{m}_t$ for the image t, in correspondence with Eq. (47) where the index 0 is replaced by t, and the vectors $\vec{X}_{s,t}=\vec{C}_{s,t}-\vec{m}_t$;

Step 12. Calculation of the restored vectors $\hat{\vec{L}}_{s,t}=[\Phi(\hat{\alpha}_t,\hat{\beta}_t,\hat{\gamma}_t)]\vec{X}_{s,t}$ on the basis of Table 1 and Eq. (50), for s=1,2,..,S, as given in Table 2.

Table 2

$[\Phi(\hat{\alpha}_t,\hat{\beta}_t,\hat{\gamma}_t)]$	Condition
$[\Phi_1(\hat{\alpha}_t)][\Phi_2(\hat{\beta}_t)][\Phi_3(\hat{\gamma}_t)]$	$\lvert\Delta\hat{\alpha}_t\rvert\geq\delta,\lvert\Delta\hat{\beta}_t\rvert\geq\delta,\lvert\Delta\hat{\gamma}_t\rvert\geq\delta$
$[\Phi_1(\hat{\alpha}_{t-1})][\Phi_2(\hat{\beta}_t)][\Phi_3(\hat{\gamma}_t)]$	$\lvert\Delta\hat{\alpha}_t\rvert<\delta,\lvert\Delta\hat{\beta}_t\rvert\geq\delta,\lvert\Delta\hat{\gamma}_t\rvert\geq\delta$
$[\Phi_1(\hat{\alpha}_t)][\Phi_2(\hat{\beta}_{t-1})][\Phi_3(\hat{\gamma}_t)]$	$\lvert\Delta\hat{\alpha}_t\rvert\geq\delta,\lvert\Delta\hat{\beta}_t\rvert<\delta,\lvert\Delta\hat{\gamma}_t\rvert\geq\delta$
$[\Phi_1(\hat{\alpha}_t)][\Phi_2(\hat{\beta}_t)][\Phi_3(\hat{\gamma}_{t-1})]$	$\lvert\Delta\hat{\alpha}_t\rvert\geq\delta,\lvert\Delta\hat{\beta}_t\rvert\geq\delta,\lvert\Delta\hat{\gamma}_t\rvert<\delta$
$[\Phi_1(\hat{\alpha}_{t-1})][\Phi_2(\hat{\beta}_{t-1})][\Phi_3(\hat{\gamma}_t)]$	$\lvert\Delta\hat{\alpha}_t\rvert<\delta,\lvert\Delta\hat{\beta}_t\rvert<\delta,\lvert\Delta\hat{\gamma}_t\rvert\geq\delta$
$[\Phi_1(\hat{\alpha}_{t-1})][\Phi_2(\hat{\beta}_t)][\Phi_3(\hat{\gamma}_{t-1})]$	$\lvert\Delta\hat{\alpha}_t\rvert<\delta,\lvert\Delta\hat{\beta}_t\rvert\geq\delta,\lvert\Delta\hat{\gamma}_t\rvert<\delta$
$[\Phi_1(\hat{\alpha}_t)][\Phi_2(\hat{\beta}_{t-1})][\Phi_3(\hat{\gamma}_{t-1})]$	$\lvert\Delta\hat{\alpha}_t\rvert\geq\delta,\lvert\Delta\hat{\beta}_t\rvert<\delta,\lvert\Delta\hat{\gamma}_t\rvert<\delta$
$[\Phi_1(\hat{\alpha}_{t-1})][\Phi_2(\hat{\beta}_{t-1})][\Phi_3(\hat{\gamma}_{t-1})]$	$\lvert\Delta\hat{\alpha}_t\rvert<\delta,\lvert\Delta\hat{\beta}_t\rvert<\delta,\lvert\Delta\hat{\gamma}_t\rvert<\delta$

Step 13. Calculation of the modified angles $\Delta\alpha_t^*,\Delta\beta_t^*,\Delta\gamma_t^*$ satisfying the rules:

$$\Delta\alpha_t^*=\begin{cases}0, & \text{if} \quad |\Delta\alpha_t|<\delta;\\ \Delta\alpha_t & \text{- in other cases,}\end{cases}$$
$$\Delta\beta_t^*=\begin{cases}0, & \text{if} \quad |\Delta\beta_t|<\delta;\\ \Delta\beta_t & \text{- in other cases,}\end{cases} \quad (51)$$
$$\Delta\gamma_t^*=\begin{cases}0, & \text{if} \quad |\Delta\gamma_t|<\delta;\\ \Delta\gamma_t & \text{- in other cases.}\end{cases}$$

Step 14. Calculation of the difference vectors:

$$\Delta\hat{\vec{L}}_{s,t}=\hat{\vec{L}}_{s,t}-\hat{\vec{L}}_{s,t-1},\ \ \Delta\vec{m}_t=\vec{m}_t-\vec{m}_{t-1}. \quad (52)$$

When t reaches the value t=N, the execution of the direct RAC-KLT for the group of images is finished. As a result, the values of the vectors' components of $\vec{L}_{s,0}$, $\vec{m}_0$, $\Delta\hat{\vec{L}}_{s,t}$, $\Delta\vec{m}_t$ and of the angles $\alpha_0,\beta_0,\gamma_0$, $\Delta\alpha_t^*,\Delta\beta_t^*,\Delta\gamma_t^*$ for t=0,1,2,..,N-1 and s=1,2,..,S, are got. After that they could be efficiently coded using various methods, selected in accordance with the application of the algorithm RAC-KLT.

4.2. Algorithm of the inverse RAC-KLT

The input parameters, needed for the inverse RAC-KLT are:

- the initial angles $\alpha_0,\beta_0,\gamma_0$ and the components of the vectors $\vec{L}_{s,0}$ and $\vec{m}_0$ for the image with group number t=0, when s=1,2,..,S;
- the difference angles $\Delta\alpha_t^*,\Delta\beta_t^*,\Delta\gamma_t^*$ and the components of the vectors $\Delta\vec{L}_{s,t}$ and $\Delta\vec{m}_t$ for each image with number t=1,2,..,N-1, for s=1,2,..,S.

For the first image (with number t=0) the inverse RAC-KLT comprises the following steps:

Step 1. Calculation of the matrix $[\Phi_0]$, using the values of $\alpha_0,\beta_0,\gamma_0$ in correspondence with (46).

Step 2. Calculation of the initial restored colors, got after the inverse RAC-KLT:

$$\hat{\vec{C}}_{s,0}=[\Phi_0]^T\hat{\vec{L}}_{s,0}+\hat{\vec{m}}_0 \ \text{ for s=1,2,..,S.} \quad (53)$$

For each of the next images with number t=t+1 when t ≤ N-1, the inverse RAC-KLT comprises the steps:

Step 3. Calculation of the difference matrix $[\Delta\Phi_t]$ based on the values of $\Delta\alpha_t^*, \Delta\beta_t^*, \Delta\gamma_t^*$ in correspondence with relations from Table 3.

Table 3

$[\Delta\Phi_t]$	Condition
$[\Phi_1(\Delta\alpha_{t-1})][\Phi_2(\Delta\beta_t)][\Phi_3(\Delta\gamma_t)]$	$\Delta\alpha_t^*=0, \Delta\beta_t^*\neq 0, \Delta\gamma_t^*\neq 0$
$[\Phi_1(\Delta\alpha_t)][\Phi_2(\Delta\beta_{t-1})][\Phi_3(\Delta\gamma_t)]$	$\Delta\alpha_t^*\neq 0, \Delta\beta_t^*=0, \Delta\gamma_t^*\neq 0$
$[\Phi_1(\Delta\alpha_t)][\Phi_2(\Delta\beta_t)][\Phi_3(\Delta\gamma_{t-1})]$	$\Delta\alpha_t^*\neq 0, \Delta\beta_t^*\neq 0, \Delta\gamma_t^*=0$
$[\Phi_1(\Delta\alpha_{t-1})][\Phi_2(\Delta\beta_{t-1})][\Phi_3(\Delta\gamma_t)]$	$\Delta\alpha_t^*=0, \Delta\beta_t^*=0, \Delta\gamma_t^*\neq 0$
$[\Phi_1(\Delta\alpha_{t-1})][\Phi_2(\Delta\beta_t)][\Phi_3(\Delta\gamma_{t-1})]$	$\Delta\alpha_t^*=0, \Delta\beta_t^*\neq 0, \Delta\gamma_t^*=0$
$[\Phi_1(\Delta\alpha_t)][\Phi_2(\Delta\beta_{t-1})][\Phi_3(\Delta\gamma_{t-1})]$	$\Delta\alpha_t^*\neq 0, \Delta\beta_t^*=0, \Delta\gamma_t^*=0$
$[\Phi_1(\Delta\alpha_t)][\Phi_2(\Delta\beta_t)][\Phi_3(\Delta\gamma_t)]$	$\Delta\alpha_t^*\neq 0, \Delta\beta_t^*\neq 0, \Delta\gamma_t^*\neq 0$
$[\Phi_1(\Delta\alpha_{t-1})][\Phi_2(\Delta\beta_{t-1})][\Phi_3(\Delta\gamma_{t-1})]$	$\Delta\alpha_t^*=0, \Delta\beta_t^*=0, \Delta\gamma_t^*=0$

Step 4. Calculation of the matrix $[\Phi_t]^T$ for the inverse RAC-KLT of the image t:

$$[\Phi_t]^T=[\Phi_{t-1}]^T+[\Delta\Phi_t]^T. \tag{54}$$

Step 5. Calculation of vectors $\vec{\hat{L}}_{s,t}$ and $\vec{m}_t$ for the image t:

$$\vec{\hat{L}}_{s,t}=\vec{\hat{L}}_{s,t-1}+\Delta\vec{\hat{L}}_{s,t},\ \ \vec{m}_t=\vec{m}_{t-1}+\Delta\vec{m}_t. \tag{55}$$

Step 6. Restoration of color vectors $\vec{C}_{s,t}$ and $\vec{m}_t$:

$$\vec{C}_{s,t}=[\Phi_t]^T\vec{L}_{s,t}+\vec{m}_t \quad \text{for s=1,2,..,S.} \tag{56}$$

The execution of RAC-KLT stops when t=N is satisfied.

The presented algorithm RAC-KLT is asymmetric, because the number of calculations needed for the inverse transform is much lower than that for the direct.

5 Evaluation of RAC-KLT efficiency

The efficiency of the algorithm RAC-KLT for a group of correlated color images is evaluated here by comparison with that of the AC-KLT. For the evaluation are used the number of the parameters needed for the inverse color transform (which are added to the coded image information), and the computational complexity of the algorithm.

5.1. Parameters of the inverse color transform

These parameters are got after the execution of the direct transforms AC-KLT and RAC-KLT.

To execute the inverse color transform for each pixel *s* in the processed image should be calculated the following parameters:

- for AC-KLT (for one image) - the parameters $\vec{L}_s, \vec{m}, \alpha, \beta, \gamma$, which are represented by (4S+1)N numbers in total (S is the number of pixels in the image, N - the number of images in the group);

- for RAC-KLT of the initial image t=0 - the parameters $\vec{L}_{s,0}, \vec{m}_0, \alpha_0, \beta_0, \gamma_0$. For each of the next images with number t=1,2,..,N-1 - the parameters $\Delta\vec{\hat{L}}_{s,t}, \Delta\vec{m}_t, \Delta\hat{\alpha}_t, \Delta\hat{\beta}_t, \Delta\hat{\gamma}_t$. These parameters are represented by (S+4)+(S+4)(N-1)=(S +4)N numbers for the whole group.

Hence, there is no difference between both algorithms in respect of the needed parameters number. However, due to high mutual correlation between neighbor images, significant part of the RAC-KLT parameters for the processed group are close or equal to zero. For AC-KLT such dependence does not exist and this is the main difference between both algorithms.

5.2. Computational complexity of algorithms AC-KLT and RAC-KLT

The computational complexity is evaluated on the basis of the mathematical operations (additions and multiplications), needed for their execution. For this are used the generalized data, obtained after analysis of the operations needed for the direct and inverse color transforms for the compared algorithms. The comparison is done for a group of N images, of S pixels each:

- direct AC-KLT: (37S+124)N≈37SN; (57)

- direct RAC-KLT: 37S+124+[37S+129](N-1)≈ ≈37SN; (58)

- inverse AC-KLT: (15S+23)N≈15SN; (59)

- inverse RAC-KLT: (15S+47)N-24≈15SN. (60)

Hence, the computational complexity of the direct and inverse RAC-KLT is approximately same as that of AC-KLT.

The main advantage of RAC-KLT towards AC-KLT is the ability for higher reduction of the data needed for the representation of the color parameters of the processed image group. Taking into account the high correlation between couples of sequential images, the values of the difference angles $\Delta\alpha_t, \Delta\beta_t, \Delta\gamma_t$ and of the components of the difference vectors $\Delta\vec{L}_{s,t}$ for significant part of the pixels in each image are close, or equal to zero. As a consequence, the volume of data representing the color components for each image in the processed RAC-KLT group could be reduced, by using various coding methods. However, the algorithm RAC-KLT needs larger memory to keep the color information for the preceding image in the group. The problems, related to the larger memory volume are easily surmountable by new contemporary computer technologies.

The selected value of δ influences the accuracy of the restored color vectors of each image in the processed group after the first one, and also - the volume of the data which represents the image color parameters. For smaller values of δ the accuracy of the restored color vectors is higher, but the color data is increased. Therefore, the value of δ should be selected by compromise, depending on the RAC-KLT application.

Experimental results for the comparison of AC-KLT with other algorithms for deterministic color space transform are given in [15]. These results confirm the higher efficiency of AC-KLT compared for example, with the well-known RGB - YC_RC_B. Similar conclusions could be made for the algorithm RAC-KLT. Its experimental investigation will be executed for various application areas, so that to fix best values of the used parameters.

6. Application areas for RAC-KLT

On the basis of the RAC-KLT efficiency evaluation given above, the following application areas are defined:

- coding of video sequences obtained from TV cameras with fixed spatial position (for example - surveillance systems);
- coding of groups of images got at the same moment from a group of TV cameras, aimed at same object but from difference spatial angle (multi-view);
- search of closest contents in images from large databases, got from TV cameras with fixed position;
- objects segmentation on the basis of their color in correlated group of images, etc.

RAC-KLT could be also used in digital TV systems, but in this case the difference color vectors for each pixel and the difference rotation angles for the consecutive frames from the processed group should be calculated using corresponding methods for movement compensation. For real-time applications of RAC-KLT should be developed its modification, based on the algorithm reversible integer KLT [7].

7. Conclusions

A basic method for recursive color space transform of a group of correlated images is developed, based on the AC-KLT. It could be used as a basis for the development of:

- New algorithms for parallel processing of the images in the group, based on the RAC-KLT, which will enhance the calculation speed of the direct and inverse transforms;
- RAC-KLT algorithm with higher efficiency, based on the adaptive division of the processed group of images into groups of various length, where the correlation between the neighbor image couples is higher that a preset threshold. This approach will ensure higher reduction of the color components number for each group;
- New algorithms, aimed at various application areas.

Maximum efficiency for each algorithm could be achieved when the basic parameters of RAC-KLT are adapted depending on the application requirements in respect of accuracy, reversibility, execution time, implementation, methods for color parameters coding, etc.

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