Improvement of SwoR-rKA Precoding Method for Extremely Large-Scale MIMO With the Same Calculational Complexity

TATSUKI FUKUDA
Organization for Liberal Arts and Education
Shimonoseki City University
JAPAN

Abstract: - Extremely massive MIMO (Multiple-Input Multiple-Output) is a crucial technology in wireless communication systems. By deploying a large number of antennas, extremely massive MIMO enables spatial diversity, but it also introduces significant computational complexity due to the large number of antennas involved in various processing tasks. One such task is precoding, where numerous improvements have been proposed in the literature. However, most existing methods assume spatially stationary channels and do not adequately account for the spatial non-stationarity that arises when the number of antennas increases. The randomized Kaczmarz algorithm (rKA) method and the sampling without replacement rKA (SwoR-rKA) method, that is an enhancement method of rKA, are proposed for the spatial non-stationarity downlink. The bit error ratio of rKA and SwoR-rKA are good at lower signal-to-noise ratio but the performances level off due to the residuals at higher signal-to-noise ratio, so the performances of bit error ratio are limited with the residuals. This paper focuses on reducing the residuals with the same or lower computational complexity. In the rKA method, the factors of precoding matrix are updated iteratively and a column to be updated is selected by a probability. The rKA method select each column at randomly and the SwoR-rKA tends to select a column corresponding to the channel in better condition. It, however, is not effective to update the column corresponding to good conditions because the columns corresponding to the good channels can be estimated to some extent with a small number of iterations. Our idea to improve the SwoR-rKA is that the policy to select a column to be updated is set that the columns corresponding to channels in bad condition tend to be selected. With this policy, all the columns corresponding to each channel are considered to reach a certain degree of estimation criteria. The result of computer simulation shows that proposed method defeats the original SwoR-rKA in view of bit error ratio performance with the same complexity, particularly at high signal-to-noise ratios of about 16 dB or more. The simulation results provide evidence of the effectiveness of the proposed approaches in mitigating the impact of spatial non-stationarity, leading to improved bit error ratio performance.

Key-Words: - MIMO, regularized zero forcing, non-stationarity, precoding, randomized Kaczmarz algorithm

1 Introduction

Wireless communication technology has become an indispensable technology in modern society. The Internet is utilized in various scenarios, such as video streaming services, web conferences, and remote learning. In the past, wired connections were primarily used due to speed limitations. However, in recent years, wireless communication has provided sufficient speed, enabling the reception of numerous services through wireless means. Furthermore, the proliferation of the Internet of Things (IoT) [1-3] has resulted in an exponential increase in wireless communication devices. To enable multiple wireless communication devices to simultaneously achieve high-speed and high-reliability communication, the technology known as massive MIMO (Multiple-Input Multiple-Output) wireless communication system [4] is actively researched. Conventional MIMO wireless communication systems utilized a few antennas for transmission and reception, whereas extremely massive MIMO wireless communication enhances spectral efficiency by deploying hundreds of antennas at base stations.

An increase in the number of antennas brings not only advantages but also disadvantages. One of them is the computational complexity. As the number of antennas increases, the benefits obtained in Massive MIMO systems are substantial. However, it is also important to address the increased computational complexity associated with the processing required for transmission and reception. Signal detection [5-6], decoding [7-8], and precoding [9-10] are among the typical tasks that are affected by this complexity.

Additionally, the increased number of antennas introduces certain differences compared to conventional MIMO systems. For instance, the non-stationarity of the spatial environment becomes

significant and cannot be ignored. Currently, many existing methods discuss this issue while disregarding the non-stationarity [11]. As an example, the tensor zero-forcing (ZF) method is a prominent linear signal estimation technique [12], but it is not practical from a spectral efficiency perspective. Of course, improved algorithms such as regularized ZF (RZF) methods [13] have been proposed, leading to enhanced spectral efficiency. However, lots of approaches primarily focus on signal detection in the uplink scenario [14-16].

On the other hand, regarding signal detection in the downlink scenario, methods based on the randomized Kaczmarz algorithm (rKA) method [17] and the sampling without replacement rKA (SwoR-rKA) method [18] have been proposed. They can solve the issue of RZF that the computational complexity to obtain the inverse matrix is too large for extremely massive MIMO. With the rKA method, however, the bit error ratio (BER) levels off due to residuals even though the noise becomes low, so it is not useful in lower-noise environment. The SwoR-rKA method has less residuals than the rKA.

In this paper, the precoding process in the downlink for extremely massive MIMO is focused on and there are two goals. The first is to further reduce the residuals of SwoR-rKA, and that makes the BER performances better. The second is to reduce the computational complexity of SwoR-rKA to make it practical for the extremely massive MIMO systems with many antennas. In order to achieve the goals, we update the probability to select the column of precoding matrix in rKA and confirm the performances of BER with the computer simulation.

Organization: The rest of this paper is organized as follows. In Section 2, the model considered in this paper is shown. The conventional precoding methods are shown in Section 3. The idea of improvement of SwoR-rKA is also described in the section. In Section 4, the BER performances of proposed method and the other methods are shown as the result of computer simulations. In the section, we show that our method can reduce the residuals, and that the reduction of residuals is more pronounced when the number of iterations in the algorithm is low. The conclusions are drawn in Section 5.

2 Model

Let us define the model for the Massive MIMO wireless communication system under investigation in this study. Firstly, the base station is equipped with M antennas, and it serves K user devices. Each user device has a single antenna. The M antennas at the base station can be divided into S subarrays, where the s-th subarray has M_s antennas and each subarray accommodates a distributed set of user devices. The number of user devices connected to subarray j is denoted as K_j, that is, (1) holds.

$$K = \sum_j K_j. \tag{1}$$

In this case, let's define $y_{j,k}$ as the received signal by the k-th user device $U_{j,k}$ connected to the j-th subarray. $\boldsymbol{x}_s$ represents the signal transmitted from the s-th subarray, and $\boldsymbol{h}_{j,k}^s$ represents the channel vector between the s-th subarray and $U_{j,k}$. The channel matrix between s-th subarray in the base station and K_j user devices in the j-th subarray is denoted as $\boldsymbol{H}_j^s = [\boldsymbol{h}_{j,1}^s, \boldsymbol{h}_{j,2}^s, \dots, \boldsymbol{h}_{j,k(j)}^s]$. The relation between received signal and transmitted signal is represented in (2).

$$y_{j,k} = \sum_s (\boldsymbol{h}_{j,k}^s)^{\mathrm{H}} \boldsymbol{x}_s + n_j^k \tag{2}$$

Here, $(\cdot)^{\mathrm{H}}$ represents the complex conjugate transpose, n_j^k denotes the additive circularly symmetric complex Gaussian noise at $U_{j,k}$ with zero mean and a covariance of σ^2.

In this study, the base station is assumed have imperfect channel information. Specifically, the estimated value of the communication channel vector $\widetilde{\boldsymbol{h}}_{j,k}^s$ is obtained by (3).

$$\widetilde{\boldsymbol{h}}_{j,k}^s = \sqrt{(1-\tau^2)}\, \boldsymbol{h}_{j,k}^s + \tau \boldsymbol{n}_{j,k}^s \tag{3}$$

Here, $\boldsymbol{h}_{j,k}^s$ and $\boldsymbol{n}_{j,k}^s$ denotes the true value of the channel vector and the independent error vector. The estimated channel matrix between s-th subarray in the base station and K_j user devices in the j-th subarray is denoted as $\widehat{\boldsymbol{H}}_j^s = [\widetilde{\boldsymbol{h}}_{j,1}^s, \widetilde{\boldsymbol{h}}_{j,2}^s, \dots, \widetilde{\boldsymbol{h}}_{j,K_j}^s]$.

By the way, in this study, the Massive MIMO system is assumed to exhibit non-stationarity. Specifically, the communication channel vector between $U_{j,k}$ and M_s antennas in the s-th subarray is defined as follows:

$$\boldsymbol{h}_{j,k}^s = \sqrt{M_s}\, (\boldsymbol{\Phi}_{j,k}^s)^{\frac{1}{2}} \boldsymbol{z}_{j,k}^s \in \mathbb{C}^{M_s \times 1}, \tag{4}$$

$$\boldsymbol{\Phi}_{j,k}^s = (\boldsymbol{D}_{j,k}^s)^{\frac{1}{2}} \boldsymbol{R}_{j,k}^s (\boldsymbol{D}_{j,k}^s)^{\frac{1}{2}} \in \mathbb{C}^{M_s \times M_s}, \tag{5}$$

where $z_{j,k}^s \in \mathbb{C}^{M_s \times 1}$ follows a Gaussian distribution with mean 0 and covariance $\frac{1}{M_s} \mathbf{I}_{M_s}$, $\mathbf{I}_{M_s}$ denotes the identity matrix of order M_s, $\boldsymbol{R}_{j,k}^s \in \mathbb{C}^{M_s \times M_s}$ denotes the spatial correlation matrix between $U_{j,k}$ and s-th subarray in the base station, and $\boldsymbol{D}_{j,k}^s \in \mathbb{C}^{M_s \times M_s}$ denotes a diagonal matrix and has $D_{j,k}^s$ non-zero diagonal elements between $U_{j,k}$ and s-th subarray in the base station.

The channel normalization scheme in this study is that the energy of the user devices served by different numbers of antennas may be the same.

3 Precoding

Precoding techniques are signal processing methods employed at the transmitter to modulate the transmitted signal with coefficients pre-determined based on the state of the communication channel. This technique aims to enhance the signal quality at the receiver's end.

3.1 Overview

The data symbol transmitted from the j-th subarray to $U_{j,k}$ is denoted as $s_{j,k}$ which follows a Gaussian distribution with mean 0 and covariance $p_{j,k}$. The vector of data symbols transmitted from the j-th subarray is represented as $\boldsymbol{s}_j = \left[s_{j,1}, s_{j,2}, \dots, s_{j,K_j}\right]^T$. In other words, $\boldsymbol{s}_j$ encompasses all the data symbols transmitted to $U_{j,k}(1 \le k \le K_j)$. With $\boldsymbol{s}_j$, the transmitted signal $\boldsymbol{x}_j$ in (2) can be transformed into (6).

$$\boldsymbol{x}_j = \sum_{k=1}^{K_j} \boldsymbol{g}_{j,k} s_{s,k} = \boldsymbol{G}_j \boldsymbol{s}_j, \qquad (6)$$

where $\boldsymbol{G}_j = \left[\boldsymbol{g}_{j,1}, \boldsymbol{g}_{j,2}, \dots, \boldsymbol{g}_{j,K_j}\right]$ is the precoding matrix for K_j user devices in the j-th subarray, and $\boldsymbol{g}_{j,k}$ represents the precoding vector for $U_{j,k}$. $\boldsymbol{g}_{j,k}$ satisfies the power constraint, the expectation value of $\left\|\boldsymbol{g}_{j,k}\right\|^2$ is 1. Now, (2) is transformed into (7).

$$y_{j,k} = \left(\boldsymbol{h}_{j,k}^j\right)^H \boldsymbol{g}_{j,k} s_{j,k} + \sum_{\substack{i=1 \\ i \ne k}}^{K_j} \left(\boldsymbol{h}_{j,k}^j\right)^H \boldsymbol{g}_{j,i} s_{s,i}$$

$$+ \sum_{\substack{s=1 \\ s \ne j}}^{S} \sum_{i=1}^{K_j} \left(\boldsymbol{h}_{j,k}^s\right)^H \boldsymbol{g}_{s,i} s_{s,i} + n_{j,k}, \qquad (7)$$

where the first term denotes a desired signal, the second term denotes intra-subarray interference, and

the third term denotes inter-subarray interference. One of the linear precoding schemes is RZF [13] whose precoding matrix $\boldsymbol{G}_j^{RZF}$ is denoted by (8).

$$\boldsymbol{F}_j = \boldsymbol{H}_j^j \left(\left(\boldsymbol{H}_j^j\right)^H \boldsymbol{H}_j^j + \xi \boldsymbol{I}_{K_j}\right)^{-1},$$

$$\boldsymbol{G}_j^{RZF} = \sqrt{P/\mathrm{Tr}\left(\boldsymbol{F}_j^H \boldsymbol{F}_j\right)} \cdot \boldsymbol{F}_j, \qquad (8)$$

where $\xi = \frac{\sigma^2}{P}$, P denotes signal power, and $\mathrm{Tr}(\cdot)$ denotes the trace operator. The complexity of (8) increased with the size of antennas and subarrays, so it is difficult for extremely massive MIMO systems to calculate it.

3.2 rKA method

The rKA method refers to the randomized Kaczmarz algorithm [17]. It leverages randomization to iteratively update the precoding coefficients based on the received signal and the estimated channel information. This method helps mitigate interference and improve system performance.

In order to obtain $\boldsymbol{G}_j$, (9) is required to be solved for $\boldsymbol{z}_j$ [18].

$$\left(\boldsymbol{A}_j\right)^H \boldsymbol{z}_j = \boldsymbol{s}_j. \qquad (9)$$

where $\boldsymbol{A}_j = \left[\boldsymbol{H}_j^j; \sqrt{\xi} \boldsymbol{I}_{K_j}\right] \in \mathbb{C}^{(M_j + K_j) \times K_j}$ and $\boldsymbol{z}_j = \boldsymbol{A}_j \boldsymbol{w}_j$. K_j rKA program are run where $\boldsymbol{s}_j$ for k-th program is $\boldsymbol{e}_j$ which denotes k-th canonical basis [11]. The process of rKA is shown in the following steps 0 to 2.

step 0: Initialize the state vectors $\boldsymbol{m}_j^0 \in \mathbb{C}^{M_j}$ and $\boldsymbol{n}_j^0 \in \mathbb{C}^{K_j}$ with zero vector. Define user canonical basis $\mathbf{e}_k$, where $[\mathbf{e}_k]_k = 1$ and $[\mathbf{e}_k]_j = 0 \ (j \ne k)$. Initialize the output matrix $\mathbf{W}_j \in \mathbb{C}^{K_j \times K_j}$ with zeros.

step 1: Repeat step 1a to 1e for τ times. After that, go to step 2.

step 1a: Select the r_t-th row of $\left(\boldsymbol{H}_j^j\right)^H$ and denotes it $\boldsymbol{h}_{jr_t}^j$, where the probability of selecting r_t is given by $P_{r_t}^j = 1/K_j$.

step 1b: Calculate the residual $\eta_t = \frac{[\mathbf{e}_k]_{r_t} - \langle \mathbf{h}_{jr_t}^j, \mathbf{m}_j^t \rangle - \xi n_{jr_t}^t}{\left\| \mathbf{h}_{jr_t}^j \right\|^2 + \xi}$, where $[\mathbf{e}_k]_{r_t}$ denotes the r_t-th factor of $\mathbf{e}_k$, ξ denotes the inverse of the signal to noise ratio (SNR), $\langle \mathbf{h}_{jr_t}^j, \mathbf{m}_j^t \rangle$ denotes the inner product of $\mathbf{h}_{jr_t}^j$ and $\mathbf{m}_j^t$, and $\|\cdot\|$ denotes the l_2-norm.

step 1c: Update $\mathbf{m}_j^{t+1} = \mathbf{m}_j^t + \eta_t \mathbf{h}_{jr_t}^j$.

step 1e: Copy $\mathbf{n}_j^t$ to $\mathbf{n}_j^{t+1}$.

step 1d: Update $\left[\mathbf{n}_j^{t+1} \right]_{r_t} = \left[\mathbf{n}_j^{t+1} \right]_{r_t} + \eta_t$.

step 2: Update the k-th column of $\mathbf{W}_j$ with $\mathbf{n}_j^\tau$.

Finally, the precoding matrix $\mathbf{G}_j^{RZF}$ is approximated as (10).

$$\mathbf{G}_j^{RZF} = \sqrt{P / \mathrm{Tr}\left(\mathbf{F}_j^H \mathbf{F}_j \right)} \mathbf{H}_j^j \mathbf{W}_j, \qquad (10)$$

Note that obtaining the precoding matrix with rKA method results in a residual term, so it does not yield the desired value.

3.3 SwoR-rKA method
The SwoR-rKA method [18] is a technique that improves the convergence speed and enhances the spectral efficiency with the selection probability shown in (11) for step 1a of rKA.

$$P_{r_t}^j = \frac{\left(\left\| \mathbf{h}_{jr_t}^j \right\|^2 + \xi \right)}{\left\| \mathbf{H}_j^j \right\|_F^2 + K_j \xi} \qquad (11)$$

where $\|\cdot\|_F$ denotes Frobenius norm.

Unlike the rKA method, SwoR-rKA allows for the use of different $\mathbf{h}_{jr_t}^j$ and $\mathbf{n}_{jr_t}^j$ values each time the selection is made. This flexibility enables the proactive selection of users who have better channel conditions.

By adjusting the probabilities of selection based on the channel quality, SwoR-rKA focuses on prioritizing users with favorable communication channels. This approach aims to improve system performance by allocating more resources to users with better channel conditions, leading to enhanced SE and faster convergence speed compared to traditional methods like rKA. Furthermore, the

SwoR-rKA can reduce the BER effectively when the channel estimation is not perfect at the transmitter.

3.4 Proposed method
In SwoR-rKA, the selection of users is actively performed based on the communication channel information to enhance the accuracy through iterative methods. However, this approach tends to select users with better channel conditions more frequently, leading to variations in the accuracy of signal estimation. As a result, there is a possibility of deteriorating the overall bit error rate.

When the channel conditions are favorable, there is no need to allocate a significant computational load to the estimation process. Conversely, when the channel conditions are poor, a larger computational load is required. Therefore, in this paper, (12) is proposed to determine the user selection probability.

$$P_{r_t}^j = 1 - \frac{\left(\left\| \mathbf{h}_{jr_t}^j \right\|^2 + \xi \right)}{\left\| \mathbf{H}_j^j \right\|_F^2 + K_j \xi} \qquad (12)$$

(12) means that the columns corresponding to the bad channel are tends to be selected.

The number of complex multiplications of our method is the same as one of SwoR-rKA because ours is different from SwoR-rKA in just probability to select the column of precoding matrix to be updated.

4 Simulation
To evaluate the performance of the proposed method, some simulations were conducted. The aims of simulations are to confirm the BER performances with the various normalized transmit powers and to confirm those with various number of iterations in rKA.

The simulation program is made by Python, and other environment of software is shown in Table 1.

TABLE 1. SIMULATION ENVIRONMENT

Software/Library	Version
Python	3.10.6
NumPy	1.24.3
SciPy	1.10.1
Matplotlib	3.7.1

Through the simulations shown in this section, some parameters of simulations and precoding methods were fixed to specific values. They are presented in Table 2.

TABLE 2. SIMULATION PARAMETERS

Parameter	Value	Explanation
K_j	16	The number of users in a subarray.
M	256	The number of antennas at base station.
S	16	The number of subarrays.
M_j	16	The number of antennas in a subarray.
T	10	The length of timeslot that channel states remain unchanged.
modulation	64QAM	A kind of digital modulation method.
Trials	10^5	The number of simulation traials with fixed channel states.
τ	0.3	The quality of CSI at base station.
σ^2	1 dBm	The power of noise.

4.1 BER vs. normalized transmit power

In order to confirm the change in BER due to normalized transmit power (NTP), the simulations with each of the original rKA, the SwoR-rKA, and ours were conducted, respectively. Furthermore, the simulations were conducted in case that the base stations have perfect channel information and that ones have imperfect channel information.

The Fig. 1 shows the BER performances of each precoding methods in case of the base stations with imperfect channel information. In the simulation, the number of iterations in rKA algorithm is fixed to 100.

As you see, the BER performance of the original rKA method levels off due to residuals. Fig. 1 also shows that our method defeats SwoR-rKA and rKA methods at the relatively high NTP of about 16 dB or higher. At the lower NTP, BER performance of ours is slightly better than others, but the different is so small.
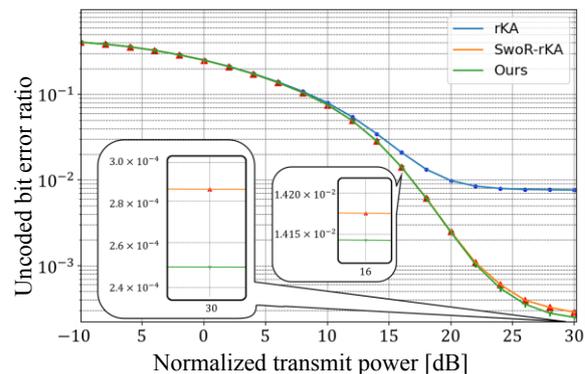

Fig. 1 BER performances of original rKA, SwoR-rKA, and ours against the normalized transmit power over *imperfect* non-stationary channels.
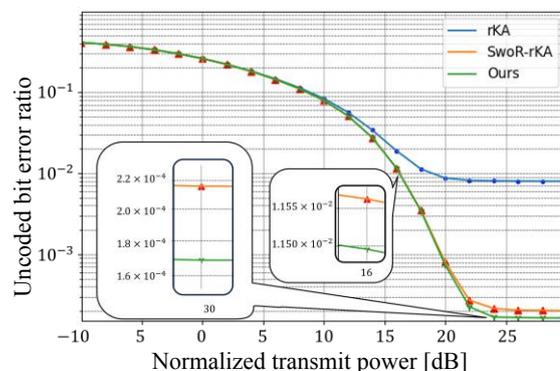

Fig. 2 BER performances of original rKA, SwoR-rKA, and ours against the normalized transmit power over *perfect* non-stationary channels.

Fig. 2 shows the BER performances in case of the base stations with perfect channel information. The number of iterations is 100 for each method.

Compared to Fig. 1, Fig. 2 shows that ours and SwoR-rKA are converged to the residuals at lower NTP while the performance of original rKA is almost the same as those in Fig. 1. Similar to the results in Fig. 1, Fig.2 shows that the BER performance of ours is slightly better than others at the lower NTP and that ours defeats others at the high NTP. The difference between ours and SwoR-rKA got bigger from about 16 dB of NTP.

4.2 BER vs. the number of iterations

Next, the simulations to confirm the BER performances with changes in the number of iterations in rKA algorithm were conducted. The number of iterations in the simulations are 20, 40, 60, 80, 100, 120, 140, 160, 180, and 200, and the results are shown in Fig. 3 to Fig. 12, respectively. The base

stations in the simulations have the imperfect channel information.
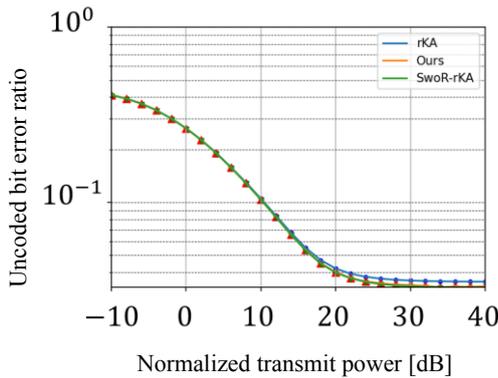

Fig. 3   BER performances of original rKA, SwoR-rKA, and ours against the normalized transmit power over *imperfect* non-stationary channels. The number of iterations in each method is 20.
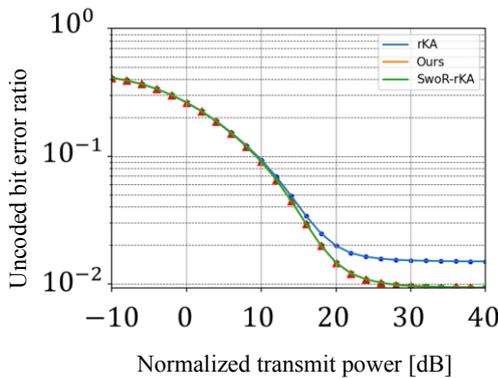

Fig. 4   BER performances of original rKA, SwoR-rKA, and ours against the normalized transmit power over *imperfect* non-stationary channels. The number of iterations in each method is 40.
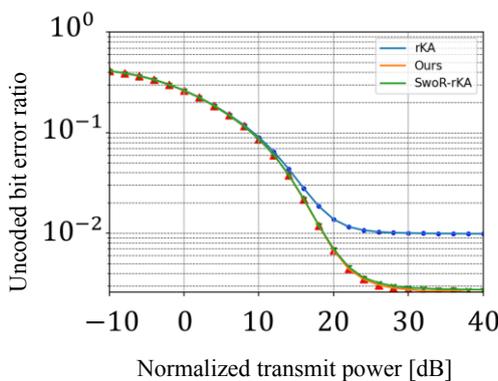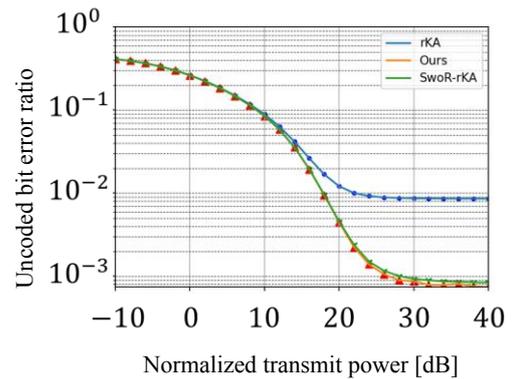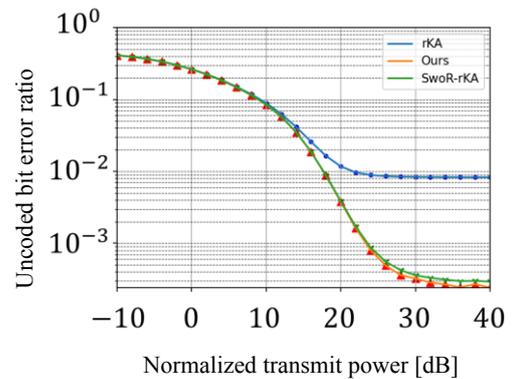

Fig. 5   BER performances of original rKA, SwoR-rKA, and ours against the normalized transmit power over *imperfect* non-stationary channels. The number of iterations in each method is 60.

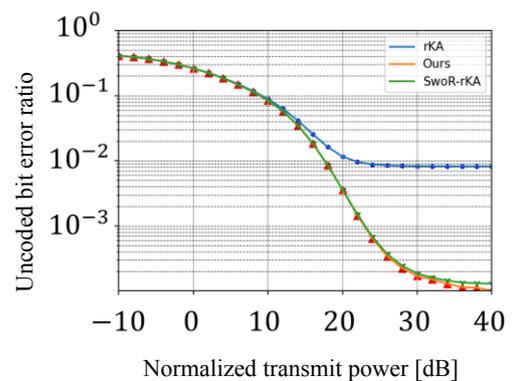Fig. 6   BER performances of original rKA, SwoR-rKA, and ours against the normalized transmit power over *imperfect* non-stationary channels. The number of iterations in each method is 80.

Fig. 7   BER performances of original rKA, SwoR-rKA, and ours against the normalized transmit power over *imperfect* non-stationary channels. The number of iterations in each method is 100.
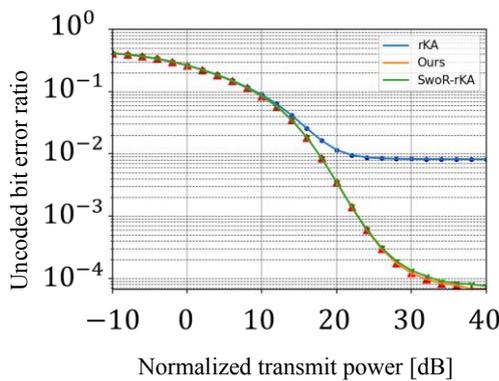

Fig. 8   BER performances of original rKA, SwoR-rKA, and ours against the normalized transmit power over *imperfect* non-stationary channels. The number of iterations in each method is 120.

Fig. 9 BER performances of original rKA, SwoR-rKA, and ours against the normalized transmit power over *imperfect* non-stationary channels. The number of iterations in each method is 140.
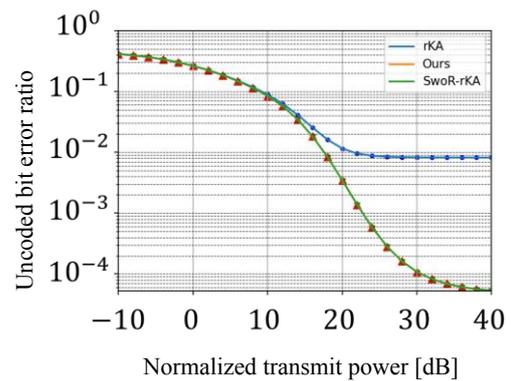

Fig. 12 BER performances of original rKA, SwoR-rKA, and ours against the normalized transmit power over *imperfect* non-stationary channels. The number of iterations in each method is 200.
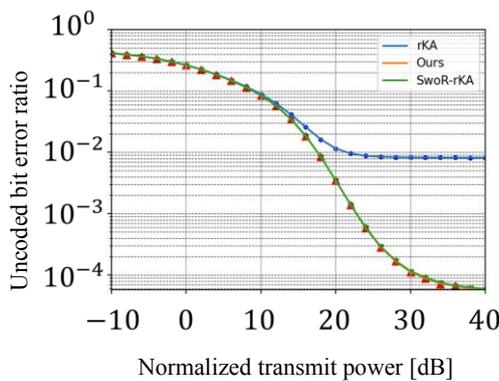

Fig. 10 BER performances of original rKA, SwoR-rKA, and ours against the normalized transmit power over *imperfect* non-stationary channels. The number of iterations in each method is 160.
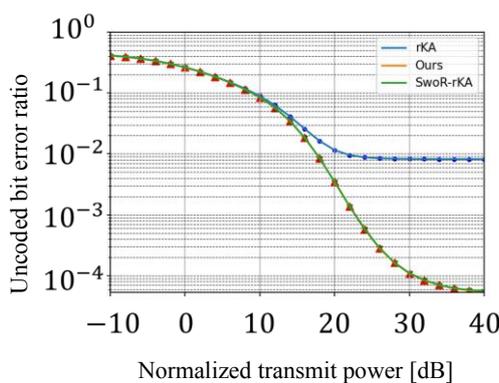
From the figures, SwoR-rKA and ours are improved in BER performance with the increase of the number of iterations in rKA algorithm while those of the original rKA are improved slightly due to the residuals. SwoR-rKA and ours, however, also has residuals, so the performances of BER is considered to plateau as the number of iterations becomes even higher. Compared to SwoR-rKA, ours shows better performance clearly at high NTP in case of the low number of iterations especially from 80 to 120. The BER performances of SwoR-rKA and ours does not change well in case of 140 or more iterations.

That means that the BER performance of ours is the best in them with the same number of iterations at relatively high NTP environment and that the number of iterations should be 120 or smaller.

4 Conclusion

The extremely massive MIMO wireless communication systems require the precoding method with low complexity and non-stationarity channel considered. The SwoR-rKA method is a kind of them. The BER performance is better than original rKA method. The SwoR-rKA differs from rKA in the probability of choosing a column of the channel matrix. The policy of SwoR-rKA is to select columns corresponding to channel in the better condition. However, since it is inefficient to allocate a long processing time to columns corresponding to channels in good condition. The proposed method in this paper, the probability is set to select a channel in poor condition more frequently.

The results of some simulations shows that our method was found to be more effective than the

Fig. 11 BER performances of original rKA, SwoR-rKA, and ours against the normalized transmit power over *imperfect* non-stationary channels. The number of iterations in each method is 180.

SwoR-rKA method and original rKA method at high SNR for both of cases that the base station has imperfect and perfect channel information. The BER performance of ours shows better than others especially at 16dB or more in NTP. With low number of iterations in rKA, our method also defeats others in BER performance, especially at high NTP. Compared to conventional rKA methods, therefore, our method can improve the BER performance without the increase of computational complexity.

However, our method can be seen to have the residuals as SwoR-rKA has because the BER performance seems level off at 30dB or higher NTP.

For the future work, the improvement method to realize linear complexity is required for high NTP environment because the number of antennas in MIMO wireless communication systems will be expected more and more.

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