

# Enhancement of MIMO QoS Using RNS Coding over Various Channels

M. I. YOUSSEF, A. E. EMAM, M. ABD ELGHANY  
Faculty of Engineering, Department of Electrical engineering  
Al-Azhar University  
Nasr City, Cairo  
EGYPT  
E-mail: mohamedgheth@yahoo.com

**Abstract:** - The performance of wireless communication systems is affected mainly by the environment of its associated channel, which is characterized by a dynamic and unpredictable behaviour. In this paper different statistical satellite channel models are studied with emphasize on two main models, first is the Rice-Log normal model, due to its close representation for the satellite channel environment including shadowing and multi-path components that affect the propagated signal along its path, and second a three state model that take into account different fading conditions (clear area, moderate shadow and heavy shadowing). The communication system is enhanced through the usage of Residue-Number-System coding making benefit of its inherent features that is highlighted in the paper. The transmission system Bit Error Rate (BER), Peak-Average-Power Ratio (PAPR), and the channel capacity for each fading models are measured and analysed. These simulations are implemented using MATLAB tool and the results had shown the performance of transmission system over different channel models.

**Key-Words:** - Fading channels, MIMO communication, QoS, RNS Scheme, Statistical modeling.

## 1 Introduction

The advantages of using Multiple-Input-Multiple-Output (MIMO) technology with Residue Number System (RNS) coding in satellite communication, stands out through the ability of multiple antenna configuration in providing higher transmission data rate and high quality of service (QoS) by the utilization of the spatial multiplexing gain and improving the link reliability by antenna diversity respectively [1], [2], and at the same time the RNS coding scheme decreases the signal amplitude and thus decrease the expected signal distortion compared to the conventional MIMO systems, which further enhance the Signal-to-Noise Ratio (SNR) of the system [3].

In this environment, the key elements for enhancing the system are through the implantation of suitable signal propagation channel model to simulate the attenuating effects suffered by a propagating signal which cause its scattering, and the suitable selection of the RNS moduli set to represent transmitted data for parallel transmission over the channel, overcome the expected signal distortion and compete system noise.

The paper start in section two with description of the overall MIMO transmit-receive system, this is followed by sections three and four that provides a detailed description of the channel fading types and channel modelling configurations respectively. In section five the RNS features and implementation in the overall MIMO communication system is given. In section six system evaluation methods are provided and the simulation results are given in section seven. Finally in section eight, a conclusion will be provided.

## 2 MIMO Communication System

The satellite MIMO system shown in Fig (1), is a 2x2 MIMO system comprised of two dual polarized antennas at each side [4], [5].

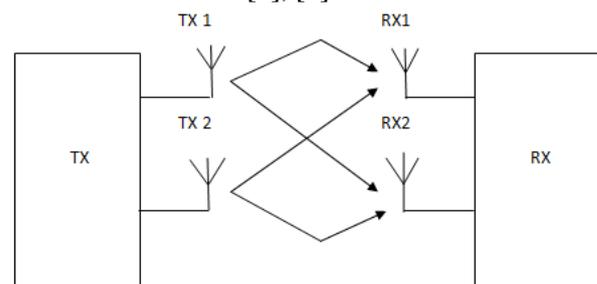


Fig.1 MIMO Communication Channel

The communication system shown in Fig (1) can be represented as seen in equation (1),

$$r(t) = H(t).x(t) + n(t) \tag{1}$$

Where;

$r(t)$  is the received signal

$x(t)$  is the transmitted signal,

$n(t)$  is the noise vector

And;  $H(t)$  is the channel matrix representation that can be modelled as an 2x2 MIMO communication channel as seen in equation (2),

$$H = \begin{bmatrix} h_{rr} & h_{ll} \\ h_{rl} & h_{lr} \end{bmatrix} \tag{2}$$

Where;

$h_{rr}$ , and  $h_{ll}$  are two co-polarizations,

$h_{rl}$  and  $h_{lr}$  are two cross-polarization.

The Multiple-Input-Multiple-Output (MIMO) configuration take advantage of the spatial diversity that is achieved through the spatial separation between the mounted antennas, and through transmitting the same information over multiple transmits antennas and collecting them at the receiving side, the system is able to achieve higher signal received level, and thus better signal Quality of Service (QoS).

At the receiver side a Maximum Likelihood Sequence Equalizer (MLSE) technique is utilized to mitigate the effect of Inter-Symbol Interference (ISI) and overcomes problems coming from most equalizers especially linear equalizers in terms of minimizing the average symbol error probability [6]. This is achieved through observing not only the segment of received signal containing the desired symbol, but as the whole received signal. The Maximum Likelihood approach determines the estimate of the transmitted signal vector  $x$  as;

$$K = |y - Hx|^2 \tag{3}$$

### 3 Channel Fading

In satellite MIMO communication system equation (1) could be represented as function of different fading components as seen in equation (4):

$$r(t) = s_1 + s_2 + n(t) + PL \tag{4}$$

$$= A_0 e^{j(2\pi f_0 t + \phi)} + \sum_{m=1}^M A_0 e^{j(2\pi f_0 t + \phi)} + s(t) + k$$

Where;

$s_1$  is the line-of-sight (LOS) component,

$s_2$  is multipath component causing fast fading,

$n(t)$  is Gauss white noise,

PL is the path loss constant.

In this system a multiple fading environments are encountered, which could be categorized as seen in Fig.2.

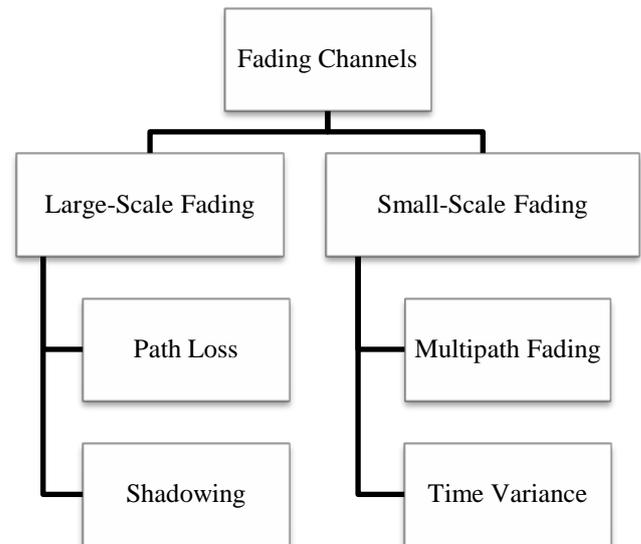


Fig.2 Classification of fading channels

Equation (4) that describes the channel is segmented into three components:

#### 3.1 Free space path loss

A simple communication channel, where the signal propagates without any obstacles present between the receiver and the transmitter or around the path between them. This parameter could be normalized as it is merely an offset value. The path loss expressed in **dB** is defined by:

$$PL(d) = 10 \log\left(\frac{P_t}{P_r}\right) = -10 \log_{10}\left(\frac{G_t G_r \lambda^2}{(4\pi d)^2}\right) \tag{5}$$

Where;

$P_t, P_r$ : The Transmitted and received power,

$G_t, G_r$ : Transmit and receive antenna gains,

$\lambda$ : The wavelength; and  $d$  is the path distance.

#### 3.2 Shadowing or large scale fading

It is the slow variation in the local mean of the received signal power, and it is caused by different degrees of attenuation that propagating signal suffers as it propagates through, reflects off or diffracts past large objects such as buildings, vegetation and terrain. Normally, the shadowing  $\chi$  is modeled as a lognormal random variable as;

$$\chi = 10^{x/10}; \text{ and } x \sim N(0, \sigma_s^2) \tag{6}$$

Where;

$\sigma_s$ : Shadowing standard deviation.

### 3.3 Multipath or small scale fading

It is the rapid fluctuation in the amplitude of the received signal power, which is caused by constructive or destructive addition of different path components that reach the receiver with different amplitude and phase representations, which result in either increased or decreased received power at the receiver. The literature [7] reveals three modeling methods (Additive White Gaussian, Ricean and Rayleigh) which are used for small scale fading in satellite channels and seen next.

The probability density function for Rayleigh and Rician Channels are given by:-

$$p(r) = \frac{r}{\rho^2} \exp\left(-\frac{r^2}{2\rho^2}\right) \quad ; \text{Rayleigh channel} \quad (7)$$

$$p(r|Z) = \frac{r}{b_0} \exp\left(-\frac{r^2+z^2}{2b_0}\right) I_0\left(\frac{r z}{b_0}\right); \text{Rician Channel} \quad (8)$$

Where;

$\rho$  is standard deviation  $r$ .

$b_0$  is the received power from multipath sources,

$I_0$  is the modified Bessel function of zero<sup>th</sup> order

## 4 Channel Fade Modeling

Using statistical approach that was initially based on terrestrial propagation models derived from early 1960's and are considered well defined approach. This was even updated for communication systems through the work of Suzuki [8] with his mixed distribution model, the Gilbert-Elliot [9] multistate model, and Loo [10] in 1985.

In this paper two models are studied representing two different statistical model approaches suitable for earth-to-space communication. The first is Rice-Lognormal modelling that represents "fading distribution" model approach, and the second is Three-state modelling that represents "state-oriented" model approach.

### 4.1 Rice-lognormal modelling (RLN)

This model which was presented by Corazza [11] and Vatalaro [12]; incorporates both multipath and shadowing components in one process, where the received signal is expressed as:

$$r = RS \exp^{j\theta} + x_1 + j y_1; \quad (9)$$

Where

$R, S$ : Ricean pdf, and lognormal pdf.

$x_1, y_1$ : Gaussian zero-mean random processes.

And, the probability density function is given by:-

$$p(r) = \int_0^\infty p(r|S)p(S)dS; \quad (10)$$

Where;

$p(r|S), p(S)$  are represented in equations (11), (12)

$$p(r|S) = \frac{2r\zeta(K+1)}{1+\zeta S^2} \exp\left[-\zeta \frac{KS^2+(K+1)r^2}{1+\zeta S^2}\right] \cdot I_0\left[2rS \frac{\zeta\sqrt{K(K+1)}}{1+\zeta S^2}\right] \quad (11)$$

$$p(S) = \frac{1}{\sqrt{2\pi}h\sigma_s S} \exp\left[-\frac{1}{2} \left(\frac{\ln S - h\mu_s}{h\sigma_s}\right)^2\right] \quad (12)$$

Where;

$a, \sigma$  are the Ricean parameters,

$\mu_s, \sigma_s$  are the lognormal mean and variance

$\sigma_1$  is the Gaussian random process' variance,

$S$  the lognormal distributed variable,

$$K = \frac{a^2}{2\sigma^2}; \zeta = \frac{\sigma^2}{\sigma_1^2}$$

### 4.2 Three-state channel modelling

This was started by Gilbert's model [9], and followed by Fontan [13], Karasawa [14] and Gillespie [15], were a three state models are derived for the satellite channel. These states are, A-state: LOS; B-state: moderate shadowing; C-state: deep shadowing, and are switched based on the different transmission media seen and will be simulated and analyzed in this paper.

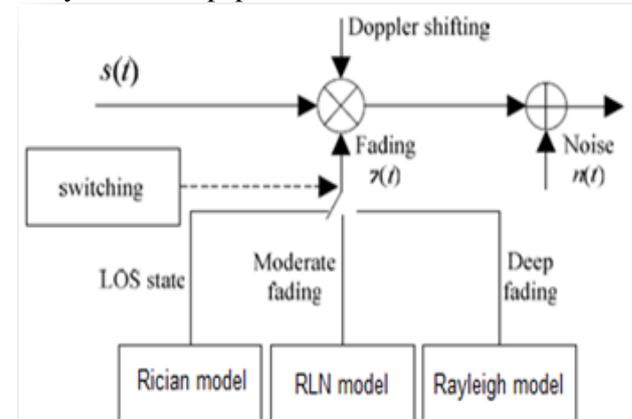


Fig. 3 Three-state simulation model

## 5 RNS Coding Scheme

Now introducing RNS coding scheme which is based on Chinese remainder theorem (CRT) of modular arithmetic for its operation and make use of its inherent features that it is a carry-free arithmetic which enable independent-parallel operations, and lack of order significance among residue digits, that prevent an error in one digit from corrupting other digits [16], [17].

Thus, the operations related to the individual residue digits of different moduli are mutually independent and so being suitable for multiple transmission/multiple receive communication system.

The scheme as shown in Fig (4), where in the first block the initial frequency band is divided into  $\nu$  sub-bands and the input signal is converted into  $\nu$  residuals through a set of pre-selected pairwise relative primes, given from equation (13);

$$r_i = N \pmod{m_i}; \tag{13}$$

Where;

$m_i$  is the relative prime number

$N$  is the integer number

The frequency-domain symbols in residue form in each residue sub-channel are modulated through the OFDM (Orthogonal Frequency Division Multiplexer) based on ‘‘Direct Mapping’’ scheme through IFFT that transfer the data to RNS-based OFDM symbols.

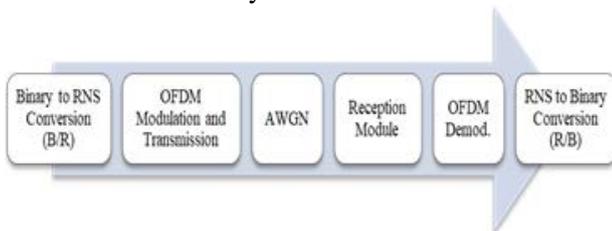


Fig. 4 RNS-OFDM Tx/Rx Block diagram

In this scheme, the parallel transmitted signals of  $\nu$  residue sub-channels are simultaneously sent to the channel in  $\nu$  frequency band portions. The reception module of the receiver is dedicated to receiving signals on the corresponding residue sub-channel. Then through FFT unit the signal is demodulated for each residue sub-channel, the input signals are recovered after R/B using the CRT method to recover the message  $N$  from the received residue digits, according to the equations (14) and (15):

$$N = \sum r_i T_i M_i \pmod{M_r}; \tag{14}$$

Where;

$M_i = M_r/m_i$ ; and  $M_r$  is the dynamic range

$$T_i M_i = 1 \pmod{m_i}; \tag{15}$$

Where;

$T_i$  is multiplicative inverse of  $M_i$

## 6 Evaluation Performance Methods

The performance of the system is evaluated through measuring the signal-to-noise ratio (SNR or  $E_b/N_0$ ) enhancement at a specific outage probability and the SNR reduction for achieving a desired average BER, capacity enhancement and sub-channel correlations.

### 6.1 Bit Error Rate (BER)

The probability of error for MPSK modulated transmission in AWGN is given by:

$$P_{ERR} = \gamma \sum_{k=1}^{\min\{2, \lceil \frac{M}{4} \rceil\}} Q\left(\sqrt{2\sigma} x \sin\left(\frac{(2k-1)\pi}{M}\right)\right) \tag{16}$$

$$\gamma = \frac{2}{\max\{\log 2 M, 2\}} \tag{17}$$

Where;

$M$  is the constellation size,  $\rho$  is the SNR per symbol,  $x$  is a chi-square distributed random variable  $\lceil M/4 \rceil$  denotes the smallest integer  $\geq M/4$ .

### 6.2 Channel Capacity

Another way to characterize the performance of MIMO channel is the Shannon channel capacity metric. Shannon in [18] defined capacity as the maximum data rate a channel can support at an arbitrarily low error probability. The capacity of MIMO system is give in equation (18) as;

$$C = \log_2 \left[ \det\left(\mathbf{I}_n + \frac{\rho}{N} \mathbf{H} \mathbf{H}^H\right) \right] \tag{18}$$

Where;

$\mathbf{I}_n$ :  $n \times n$  identity matrix,

$N$ : The number of transmit antennas and

$\mathbf{H}^H$ : Complex conjugate transpose of  $\mathbf{H}$ .

### 6.3 PAPR Performance Evaluation

The performance is evaluated by measuring the Peak-to-Average Power Ratio (PAPR) of the signal,  $x(t)$  [19], as seen in equation (19);

$$PAPR = 10 \log \left( \max |x_k|^2 / E[|x_k|^2] \right), \text{ in dB} \tag{19}$$

Where;

$E$  = Mean of the signal

It could also be illustrated through ‘‘Complementary Cumulative Distribution Function’’ (CCDF), which is the probability of PAPR exceeding a threshold and its mathematical expression as seen in equation (20);

$$CCDF(PAPR(x(n))) = P(PAPR(x(n))) > PAPR_0 \tag{20}$$

$$= (1 - e^{-PAPR_0})^N;$$

for  $N$  mutual independent signals

By taking into account  $M$  transmitted antennas; equation (20) would yield to equation (21) for MIMO-OFDM system;

$$CCDF(PAPR(x(n))) = P(PAPR(x(n))) > PAPR_0 \tag{21}$$

$$= (1 - e^{-PAPR_0})^{M_T N}$$

## 7 Simulations Results

Using MATLAB simulations on the model shown in Fig. 4 transmitting 10000 symbols using 512 QAM, the channel model performance is examined and evaluated in a MIMO- communication system, and RNS coding is implemented for optimizing the communication system performance.

### 7.1 Probability Density Function

Fig. 5.a and Fig 5.b; shows the PDF for Rician, Gaussian, Rayleigh and Lognormal channel fading models.

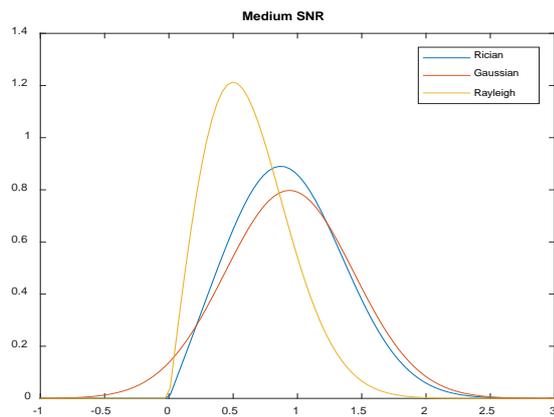


Fig. 5.a PDF for different distributions

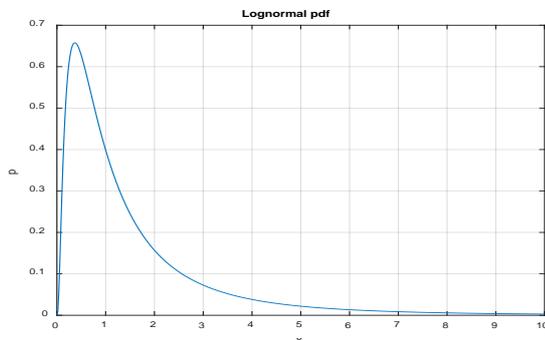


Fig. 5.b PDF for Lognormal distribution

### 7.2 Inter-Symbol Interference mitigation

In this section evaluate the channel equalizer performance by measuring the system performance with and without the equalizer as seen in table (1).

Table 1: ISI Mitigation effect

SNR(dB)	MIMO-RNS-OFDM System BER Performance	
	With Equalizer	Without Equalizer
5	0.1556	0.2349
10	0.0785	0.1449
15	0.02881	0.06194
20	0.0055	0.0125

As seen from table 1, the equalizer improves the performance of the signal by around 50% compared to that without using it.

Now, study the effect on different channel models on the communication system performance as seen in subsections 7.3 till 7.5.

### 7.3 BER performance analysis

The bit-error-rate (BER) versus SNR is measured for different channel models, as seen in Fig (6).

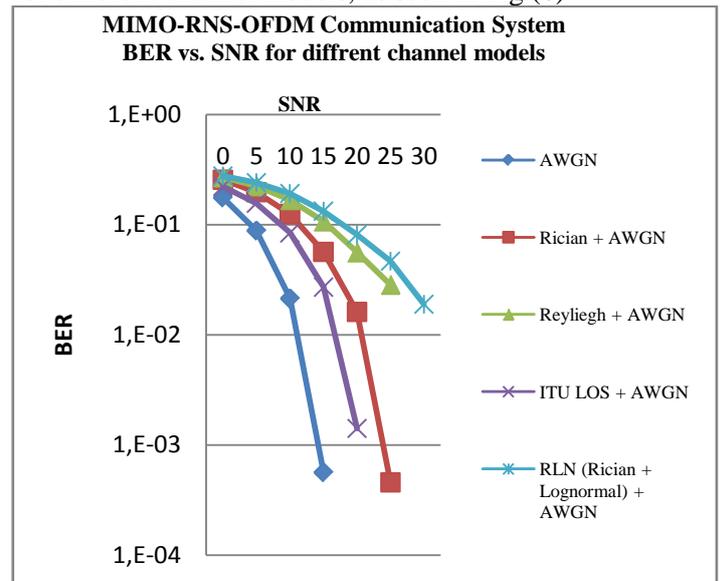


Fig. 6 BER vs. SNR for Varies Channels

The analysis showed that RLN as it takes into account both multipath and shadowing components provides that worst BER performance, while ITU LOS model provide the best BER performance.

### 7.4 PAPR and CCDF performance

In this section we study effect of fading channels on the PAPR, as seen in Fig 7.a and Fig 7.b.

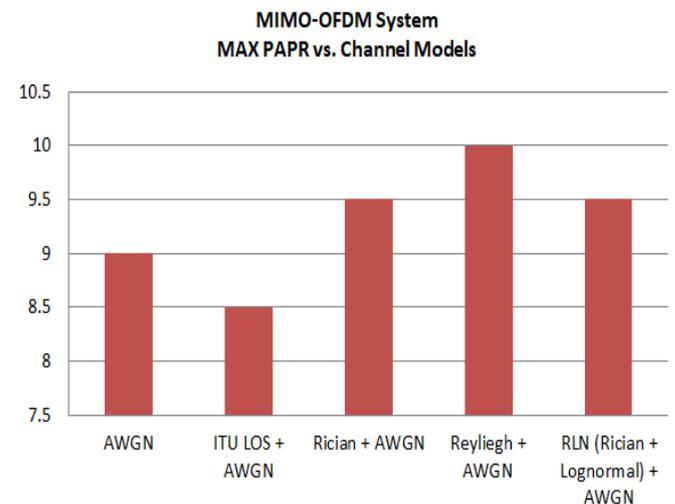


Fig. 7.a Max PAPR for different channel models

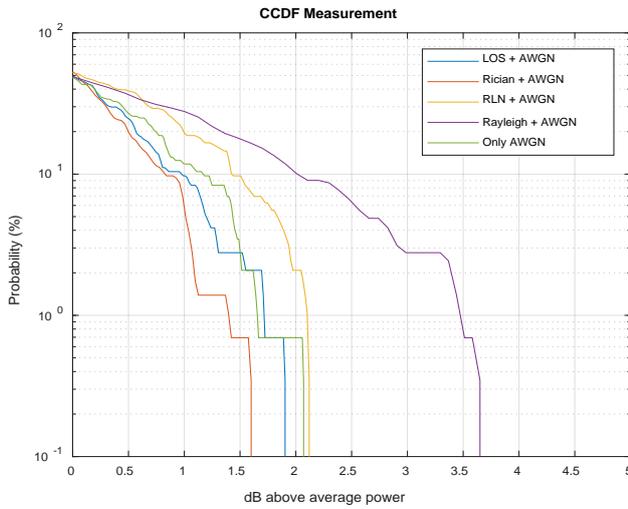


Fig. 7.b CCDF for different channel models

Where; from Fig. 7.a, and 7.b it is seen that the Rayleigh and RLN provide the highest (worst) PAPR in comparison to ITU LOS and AWGN fading channels.

### 7.5 Channel capacity evolution

Different fading channels are evaluated with respect to the system achieved capacity over different SNR, as seen in Fig. 8.

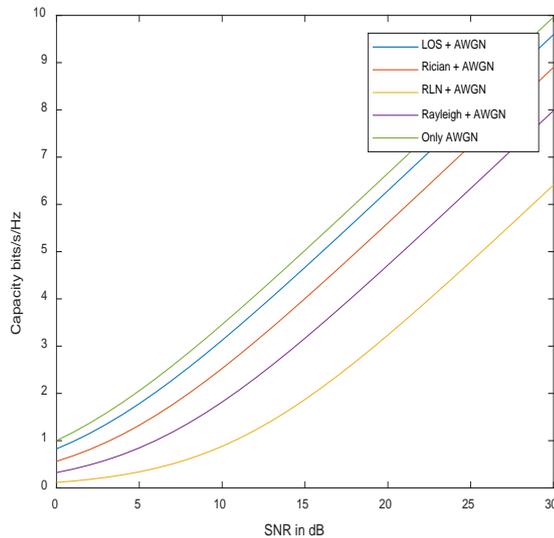


Fig. 8 Channel Capacity vs. fading models

The evolution provided in Fig. 8 shows that the capacity is optimum for low fading values as AWGN, LOS, and Rician models, while it is low for the high fading values seen in NLOS models as Rayleigh and when adding multipath components as in RLN model.

From the previous analysis conducted in subsections 7.3 till 7.5, it was noticed that RLN and Rayleigh models provide the worst fading conditions, and as the proposed system is designed for point-to-point communication, the next sub-sections will focus on Rician and RLN fading models as it is the most suitable models for such service.

The system performance is studied using RNS as a coding scheme to be able to enhance the system performance and optimize it as seen in sub-sections 7.6 till 7.8.

### 7.6 Fading levels over Rician Channel

Measuring the fading level effect over Rician channel, where we could see that system performance degradation in table (2) as k-factor that represent the LOS signal over the scattered signal decreases towards a Rayleigh model.

Table 2: Rician k-factor effect

k-factor value	BER @ SNR = 10 dB	
	MIMO-RNS system	MIMO system
k = 20	0.07608	0.2412
k = 10	0.1189	0.2702
k = 5	0.1497	0.3088
k = 2	0.1953	0.3543

Where; the decrease in k factor indicates an increase in the multipath fading component. Also, from table (2) it is clear the effect of RNS in improving the system performance and decreasing the BER.

### 7.7 Effect of selection RNS moduli set

For a 256 QAM scheme, over a RLN fading channel and changing the utilized RNS moduli set to measure its effect on the system performance as seen in Fig. (9).

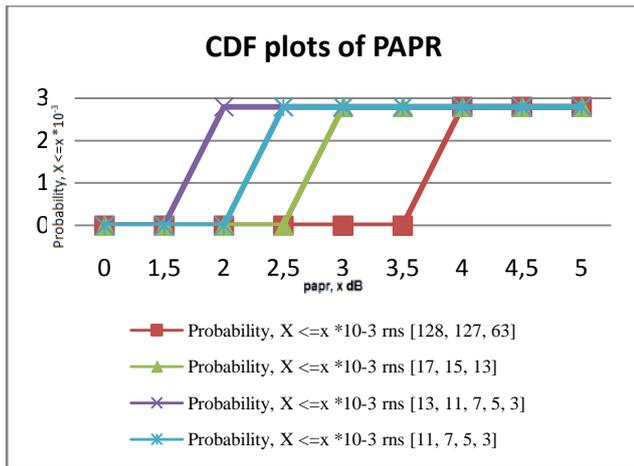


Fig. 9 PAPR for various RNS moduli set

It can be deduced from Fig. (9), that when the difference between RNS moduli and modulation index is increased (which mean utilizing lower RNS set), the PAPR is improved. Also, in all cases; the sequential RNS improves significantly the SNR, compared to the system without RNS coding scheme.

### 7.8 Performance over RLN channel

The MIMO-RNS system over RLN fading channel would have an effect not only on the BER performance, but also on the transmitted PAPR as seen in Fig (10). Through transmission of 10,000 symbols and appropriate selection of the RNS moduli sets [11, 7, 5, 3] to present a wider range of data representation with low signal amplitude, a PAPR gain and better SNR are obtained.

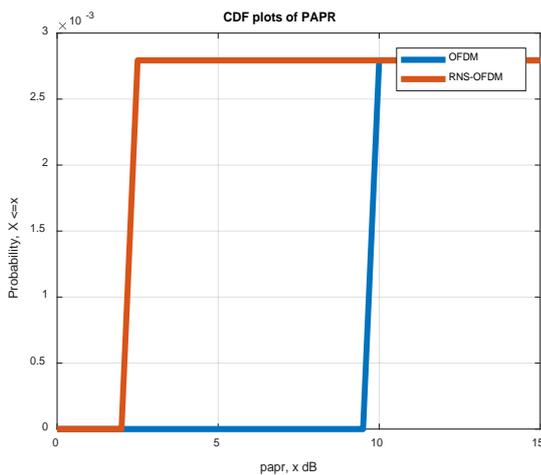


Fig. 10.a PAPR for system with & without RNS

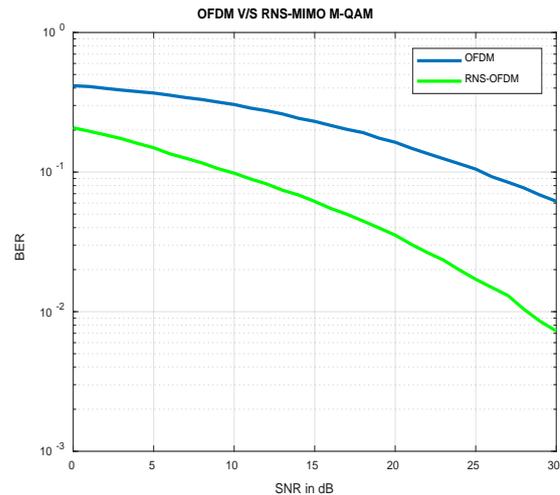


Fig. 10.b BER vs. SNR with & without RNS

Where, it could be seen the reduction provided in the PAPR when using RNS scheme in Fig. 10 (a), and the enhancement in the received SNR seen in Fig. 10 (b) compared to that without using the RNS schemes.

## 8 Conclusion

This paper investigated two channel fading models, RLN model that takes into account both shadowing and low multipath fading factors, and a three state fading model that simulate different fading conditions for satellite communication channel.

The characteristics of propagation in the channel are presented, and comparisons are made with the different statistical fading channel models using bit error rate (BER) as the figure of merit.

The simulation results over the different fading models had showed that that the degree of fading experienced by the satellite link depends on the percentage of shadowing and it is observed from the BER results that the propagation impairment of the fading channel is relatively low in LOS and Rician channels than that seen in RLN and Rayleigh fading channels, and the achieved system capacity for low fading values is higher than that foreseen for high fading models.

An RNS coding scheme was utilized in the MIMO communication system over a selected RLN fading channel, and demonstrated its ability to provide better system QoS through improving the PAPR of the transmitted signal, and decreasing the BER for a given SNR.

### References:

[1] Biradar, R. (2015, July). Study and Analysis of 2x2 MIMO Systems for Different Modulation Techniques Using MATLAB. Retrieved September 10, 2017, from <http://www.techrepublic.com/resource->

library/whitepapers/study-and-analysis-of-2x2-mimo-systems-for-different-modulation-techniques-using-matlab/

- [2] Tanmeet, K. (2013). "Performance Evaluation of MIMO Systems with varying number of transmitting antennas", *International Journal of Innovation research in electrical, electronics, instrumentation and control engineering*, 1(2), 6th ser., 38-40.
- [3] Abd-Elfattah, O. (2016, Jan). Data Conversion in Residue Number System. A thesis submitted to McGill University in partial fulfillment of the requirements for the degree of Master of Engineering. Montreal, Canada.
- [4] H. Ozcelik and C. Oestges, "Some remarkable properties of diagonally correlated MIMO channels," *IEEE Transactions on Vehicular Technology*, vol. 54, pp. 2143-2145, 2005.
- [5] C. E. Shannon, "A Mathematical Theory of Communication," *Bell System Technical Journal*, vol. 27, pp. 379-423(Part 1), 623-656(Part 2), 1948.
- [6] P.Sunil Kumar, Performance Analysis of  $2 \times n$  MIMO Systems in Rayleigh Channels with Maximum Likelihood Equalization Scheme. *International Journal of Advanced Research in Computer Science and Software Engineering*. Volume 3, Issue 6, June 2013
- [7] S. R. Saunders and A. Aragon-Zavala, *Antennas and Propagation for Wireless Communication Systems*, 2nd ed. Chichester: John Wiley, 2007.
- [8] H. Suzuki, "A statistical model for urban radio propagation," *Communications, IEEE Transactions on*, vol. 25, no. 7, pp. 673 – 680, jul 1977.
- [9] E. Elliot, "Estimates of error rates for codes on burst-noise channels." *Bell Systems Technical Journal*, vol. 42, pp. 1977–1997, September 1963
- [10] C. Loo, "A statistical model for a land mobile satellite link," *Vehicular Technology, IEEE Transactions on*, vol. 34, no. 3, pp. 122 – 127, aug 1985.
- [11] G. Corazza, C. Ferrarelli, and F. Vatalaro, "A rice-lognormal terrestrial and satellite channel model," in *Universal Personal Communications, 1994. Record., 1994 Third Annual International Conference on*, sep-1 oct 1994, pp. 155 –159
- [12] F. Vatalaro, "Generalised rice-lognormal channel model for wireless communications," *Electronics Letters*, vol. 31, no. 22, pp. 1899 – 1900, oct 1995.
- [13] F. Fontan, M. Vazquez-Castro, C. Cabado, J. Garcia, and E. Kubista, "Statistical modeling of the lms channel," *Vehicular Technology, IEEE Transactions on*, vol.50, no. 6, pp. 1549 –1567, nov 2001.
- [14] Y. Karasawa, K. Minamisono, and T. Matsudo, "A propagation channel model for personal mobile satellite services," 1994
- [15] T. Gillespie and C. Robertson, "An improved markov model for the urban sotm channel," in *Military Communications Conference, 2008. MILCOM 2008. IEEE*, nov. 2008, pp. 1 –8.
- [16] A. Omondi and B. Premkumar, *Residue number systems: theory and implementation*. Imperial College Press, 2007.
- [17] Idris Abiodun Aremu, "An overview of Residue Number System" *International Journal of Advanced Research in Computer Engineering & Technology (IJARCET)* Volume 6, Issue 10, October 2017, ISSN: 2278–1323
- [18] Ramoni Adeogun, "Capacity and Error Rate Analysis of MIMO Satellite Communication Systems in Fading Scenarios", *International Journal of Electrical and Computer Engineering (IJECE)* Vol. 4, No. 4, August 2014, pp. 614~622.
- [19] Paredes, M. (2015). Reduction of peak-to-average power ratio in OFDM systems. doi:10.14711/thesis-b650985