Modelling and Vibration Analysis of a Railway Track

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Abstract: The components used in the rail track structure (e.g. Rails, fixing systems and supports) and rail failures significantly affect vibration behavior and hence the safety and comfort of trains. The design of the suspension system is a particularly important task due to the interaction between wheel and rail, which ensures safe and optimal driving conditions in high-speed trains.[1-2]. The vibration behavior of the train in the medium and high frequency range (40-1500 Hz), which is influenced by the rail structure, can be viewed as an indicator with regard to sound radiation, vibration sensitivity and the interaction of rail-track forces. In this study, natural frequencies of a high-speed railway track under various self–excited constrains are investigated by using mathematical modelling and fundamental vibration analysis methods. Dynamic behavior and in particular the resonance conditions of a specially designed suspension system are examined for vertical movements depending on various parameters The double joint connected chasse-bogie design, existing of mass-dampers and springs, is described with a two degrees of freedom mathematical model and calculated natural frequencies of the track with relevant motion modes are then graphically presented.

Keywords: rail corrugation, wear Friction, self-excited vibration, vibration mode, resonance, natural frequency, bogie.

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1. Introduction

The components used in the rail track structure (e.g. rails, fixing systems and supports) and rail failures significantly affect vibration behavior and hence the safety and comfort of trains.

The impact on the vibration behaviour caused by wheel-rail interaction and suspansion system design is therefore an especially important task to achieve travel safety and optimal travel conditions in high speed trains [1-2].

Different railway car-bogie designs and bogie constructions as seen in Figure 1 are commonly used in applications [3].

Figure 1. Different Type of Car Bogie Designs.

Before production, designer develops a suitable design that meets the requirements and research-engineer then creates relevant and suitable mathematical model in order to calculate the desired values and to test theoretically evaluated results.

The designer consciously tries to set the natural vibration frequencies (natural frequencies) in such a way that no disruptive resonance phenomena occur under normal operating conditions [4-5]. In this study, a mathematical model based on the bogie-chassis construction as shown in Figure 2 is selected and the natural frequencies of the system with relevant vibration modes are calculated. The self-excited vibration tendency of the model is analyzed using the complex eigenvalue method [6].

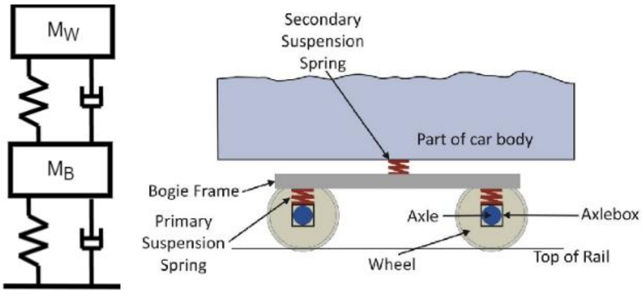

Figure 2. Mathematical model of bogie-chasse construction.

2. Basic Model of Suspention System

For various damping coefficients the vertical motions of susupension System of the track consisting of mass damper and spring can be represented with a seconnd degree of freedom model as shown in Figure 2.

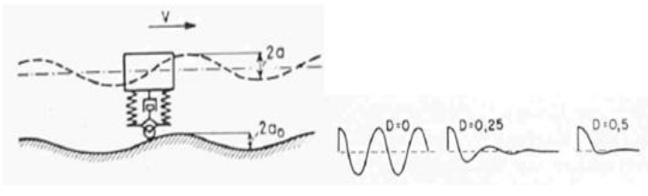

Figure 3. One dimensional model and free motions with different damping effects.

Considering the vertical motions the susupension of the track can be modelled as a second degree of freedom system existing of mass-damper–spring as represented in Figure 2. The vertical motion depending on different kind of excitations from railway can be described by following differential equation with M as mass, C damping coefficient, K as spring stiffness and F as ground forces [3].

$$M\ddot{x} + C\dot{x} + Kx = F \tag{1}$$

The natural frequency ω_n damping constant D and resonance frequency ω_r can be calculated as follows:

$$\omega_n = \sqrt{K/M} \tag{2}$$

$$D = \frac{c}{2M\omega_n} \tag{3}$$

$$\omega_r = \omega_n\sqrt{1-2D^2} \tag{4}$$

There are different kind of excitations which cause to vibrations on railway tracks; these can occur depending on wheel-rail interactions because of disturbances through rail failures, unbalanced mass effects and assembly-maintenance failures etc. [2-8]. Especially periodical excitations in form of frequency-spectrum influence the system parts with different Eigen-frequencies and amplitudes producing pulse train effect.

Assuming a harmonic disturbance with constant amplitude a0 free vibration tests of track with different damping ratios can be applied and frequency response can be investigated by using the following differential equation (with a=x and a_0=x_0):

$$M\ddot{x} + D\dot{x} + Kx = D\dot{x_0} + Kx_0$$

In frequency domain with:

$$\frac{X(s)}{X_0(s)} = \frac{Ds+K}{Ms^2+Ds+K}$$

Speed which is ϑ_k corresponds to natural frequency of track is taken as critical speed; Ratio of measured value a to input amplitude a0 for various travel speed ratios ϑ/ϑ_k and damping rates can be calculated with:

$$\frac{a}{a_0} = \sqrt{\frac{1+\left(2D\frac{\vartheta}{\vartheta_k}\right)^2}{\left(1-\left(\frac{\vartheta}{\vartheta_k}\right)^2\right)^2+\left(2D\frac{\vartheta}{\vartheta_k}\right)^2}} \tag{5}$$

The simulation result for different vehicle speed ratios ϑ/ϑ_k and damping rates is illustrated in Figure 4. In the same figure the change of the vibrating force with the driving speed ratio and for different damping rates is also given. It is evident from Figure 4 that resonance occurs for $\vartheta=\vartheta_k$. In order to avoid critical driving conditions, these graphics provide highlighted information [3-8].

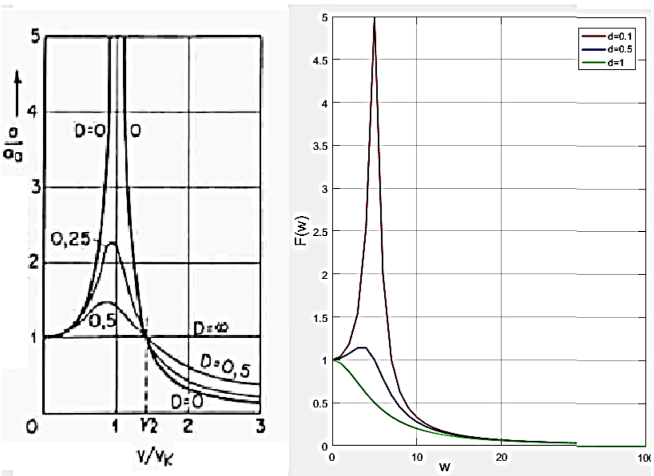

Figure 4. Change of Amplitude with speed depending on various damping Coefficients

3. Network Modelling

When analyzing a mechanical system composed of mass, spring, and dampers, it is important to model the configuration of these elements that make up the system.

This combination can be described with a Linear Mechanical Network Operator (LMNO) model [5]. As an example, a network model of a second-order system with transfer function is shown in Figure 5.

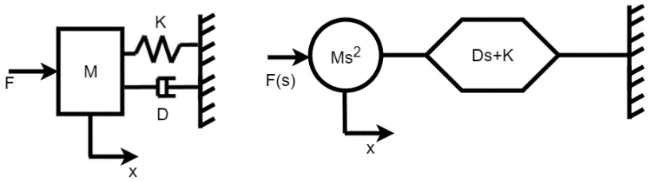

Figure 5. Representation Network Model of a Second Order System

$$X(s) = \frac{F(s)}{Ms^2+Ds+K} \tag{6}$$

Similarly, network modelling can be used and applied to solve vibration equations and to calculate natural frequencies with different motion modes of more complicated systems with several degrees of freedom.

The following example of the calculations and solutions for a reduced track system with three degrees of freedom is shown in Figure 6.

By considering the orbital disturbances Ya, Yb, Yc and Yd as system inputs, vertical movements in the z-direction with lateral and longitudinal rotational movements around the x (θ) and y (ϕ) axes as outputs, the network model of this system can be combined with dynamic Equations are described as given in Equation (7)

The mass spring damper connections in the network model are described by network operators Z.

Further analysis of the system as determination of the free vibrations and natural frequencies etc. can then be found by solving these equations.

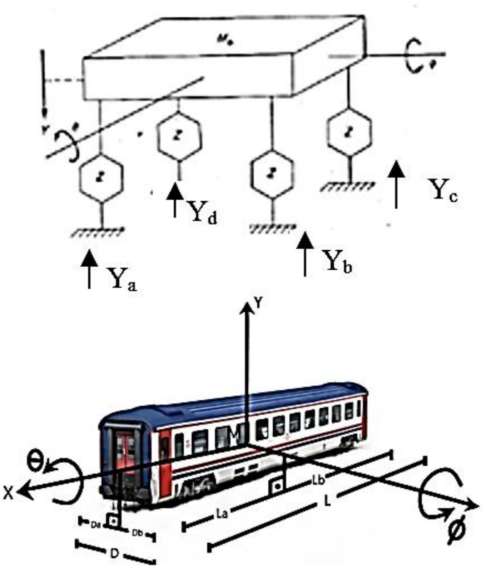

Figure 6. Three Degree of Freedom Network Suspension Model.

Network models of more complicated systems with several degrees of freedom can be described also with Network models as shown in Figure 7.

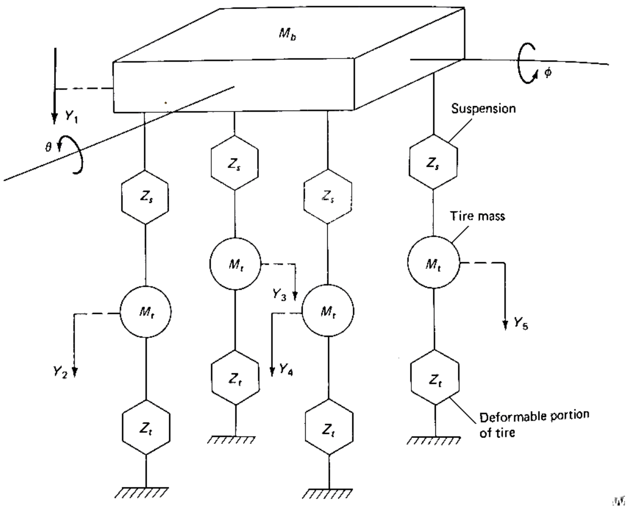

Figure 7. Seven degree of freedom track suspension network models

Assuming a three degree of freedom system with x_1, x_2 and x_3 as variables and L, M, N as inputs, system dynamic equations of motion and solutions can be described and obtained in following form:

$$\begin{aligned} Ax_1 + Dx_2 + Gx_3 &= L \\ Bx_1 + Ex_2 + Hx_3 &= M \\ Cx_1 + Fx_2 + Ix_3 &= N \end{aligned} \tag{7}$$

Q as characteristic determinant:

$$Q = \begin{vmatrix} A & D & G \\ B & E & H \\ C & F & I \end{vmatrix} \tag{8}$$

Solution of equations (8) can be evaluated as:

$$x_1 = \begin{vmatrix} L & D & G \\ M & E & H \\ N & F & I \end{vmatrix} + Q$$

$$X_2 = \begin{vmatrix} A & L & G \\ B & M & H \\ C & N & I \end{vmatrix} + Q$$

$$X_3 = \begin{vmatrix} A & D & L \\ B & E & M \\ C & F & N \end{vmatrix} + Q$$

4. Equations of Reduced Model Motions

This model can be further described with a reduced system at the center of mass and with free body model shown in Fig.8 The suspensions of each of the four Wheels consisting of spring and dampers can be expressed with network operators Zi and inputs Yi which present disturbances from road conditions.

The motions (z, θ, ϕ) of the car body with mass M and inertias J_ϕ, J_θ with reduced spring and dampers at real and front sides can be described with network operators.

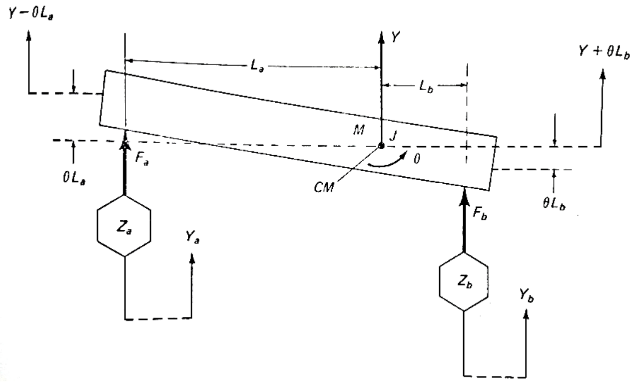

Figure 8. Reduced free body model.

In order to investigate the natural frequencies and neglecting damping effects, the suspension system can be thought consisting of only reduced springs on the front and back side.

The Network operators can be written then as Z_i=K_i and F_i=K_iY_i. Including translation and rotation the acting forces F_a and F_b at the front and back side can be expressed as following (K_a=Z_a; K_b=Z_b):

$$F_a = Z_a\big(Y_a - (Y - \theta L_a)\big)$$

$$F_b = Z_b\big(Y_b - (Y - \theta L_b)\big)$$

According to Newton Law with zero initial conditions; it can be written as:

$$\sum F = M\ddot{y} \rightarrow F_a + F_b = Ms^2Y(s)$$

$$\begin{aligned} (Ms^2 + Z_a + Z_b)Y & \\ - (Z_aL_a - Z_bL_b)\theta & \\ = Z_aY_a + Z_bY_b & \end{aligned} \tag{9}$$

Similarly sum of the Torques about center of mass with zero initial conditions:

$$\begin{aligned} \sum T = J\theta + K_\theta\theta(t) & \\ \rightarrow F_bL_b - F_aL_a & \\ = Js^2\theta(s) & \\ + K_\theta\theta(s) & \end{aligned} \tag{10}$$

$$\begin{aligned} (Js^2 + Z_bL_b + Z_aL_a + K_\theta)\theta(s) & \\ - (Z_aL_a - Z_bL_b)Y & \\ = Z_bL_bY_b - Z_aL_aY_a & \end{aligned}$$

Finally sum of the Torques about center of mass with zero initial conditions:

$$\begin{aligned} \sum T = J\ddot{\varphi} + K_\phi\phi(\mathrm{t}) \rightarrow F_cL_c - F_aL_d & \\ = Js^2\varphi(s) + K_\phi\phi(s) & \\ \big(Js^2 + Z_cL_c + Z_dL_d + K_\phi\big)\phi(s) & \\ - (Z_dL_d - Z_cL_c)Y & \\ = Z_cL_cY_c - Z_dL_dY_d & \end{aligned} \tag{11}$$

5. Network-model of the Track Suspension System

According to the Chasse-Bogie design like type a given in Figure 1, a simplified but more precise mathematical model of one of the four suspensions can be described with a two degrees of freedom second order system as shown in Figure 9.

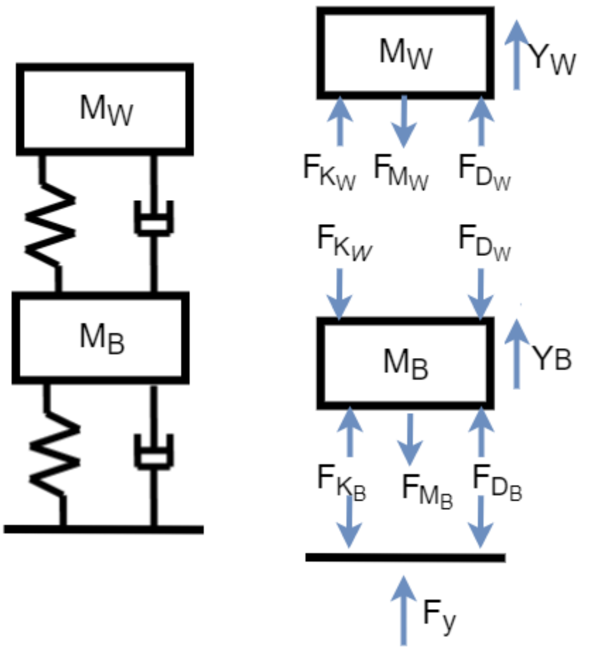

Figure 8. Bogie-car model with free-body diagram.

Describing the carriage and bogie masses as M_w and M_b; coordinates with Y_w and Y_b, the spring constants with K_w and K_b, and the damping coefficients with D_w and D_b following expressions can be written as:

$$F_{K_B} = K_B(Y - Y_B) \qquad F_{D_B} = D_B(\dot{Y} - \dot{Y}_b)$$
$$F_{K_W} = K_W(Y_B - Y_W) \quad F_{D_W} = D_B(\dot{Y}_B - \dot{Y}_W)$$
$$F_{M_W} = M_W \ddot{Y}_W \qquad F_{M_B} = M_B(\dot{Y} - \dot{Y}_B)$$

According to above expressions, road disturbance is Y as input and Y_W, Y_B are outputs and the equations of motion and Network Table of the system can be written as:

$$F = F_Y = F_{K_B} + F_{D_B} \tag{12}$$

$$M_W \ddot{Y}_W = F_{K_W} + F_{D_W} \tag{13}$$

$$M_B \dot{Y}_B = F_{K_B} + F_{D_B} - F_{K_W} - F_{D_W} \tag{14}$$

$$(M_W s^2 + D_W s + K)Y_W(s) - (K_W + D_W s)Y_b = 0 \tag{15}$$

$$-(K_W + D_W s)Y_W(s) + \left(M_B s^2 + (D_B + D_W)s + (K_B + K_W)\right)Y_B(s) = (K_B + D_B s)Y(s) \tag{16}$$

The resonance frequencies and damped motions of the two degrees of freedom system can be obtained from following table and equations:

Yw	YB	Y
$Mw.s2 + Dw.s + Kw$	$-(Kw + Dw.s)$	0
$-(Kw + Dw.s)$	$Mb.s2 + (Db + Dw)s + (Kb + Kw)$	$(Kb + Db.s)$

6. Simulations and Solutions of the System

The vertical motions of car body and bogie of the two degrees of freedom system with the given model and parameters is simulated in MATLAB-Simulink as shown in Figure 10. [9-11].Given Parameters are M_W=40t, M_B=10t, K_W=1500kN/m, K_B=2500kN/m, D_W=50kNs/m, D_B=10kNs/m

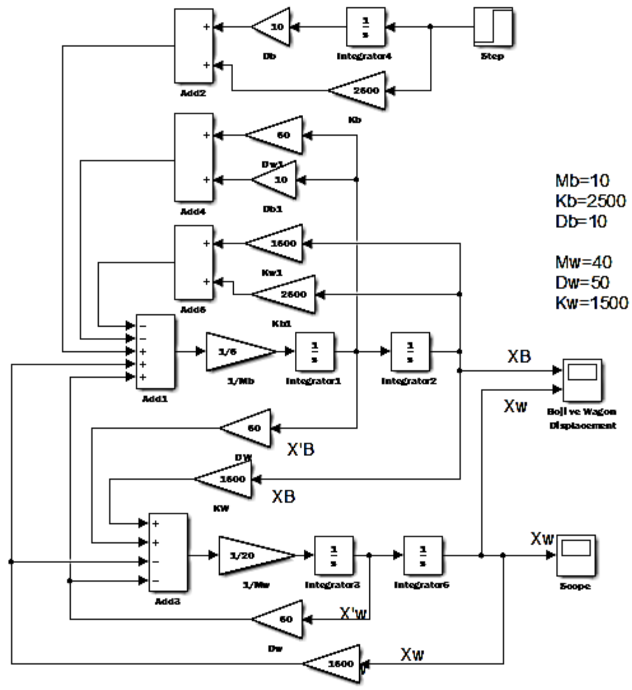

Figure 9. Simulink simulation of vertical suspension motion

Simulation results respect to time responses and natural frequencies of Bogie and car body as time and Bode Plots with above parameters are given in Figure 11 and 12 (vibration modes).

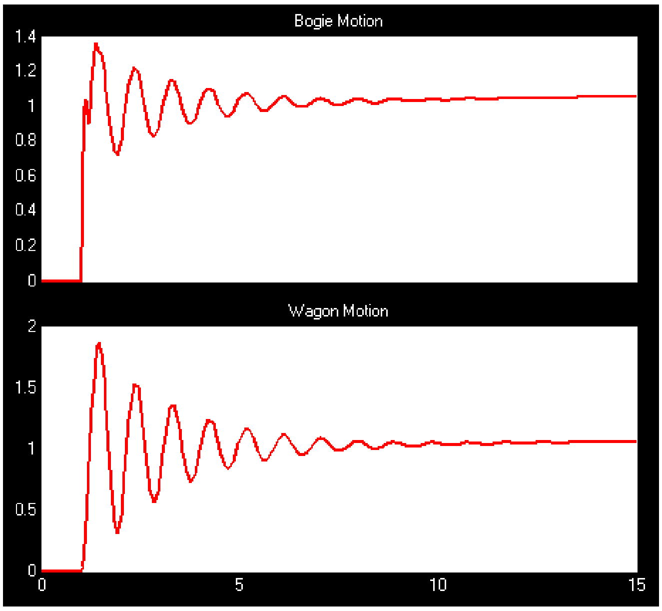

Figure 10.Time response of Bogie-Wagon

The natural frequencies can be calculated from above equations and following table by neglecting the damping effects.

Y_W	Y_B	Y
$M_W s^2 + K_W$	$-(K_W)$	0
$-(K_W)$	$M_B s^2 + (K_B + K_W)$	0

The roots of characteristic equation as natural frequencies can be found as ω_{n1}=4.66 rad/s and ω_{n2}=20.43 rad/s. These values can also be identified from simulated Bode Plot in Figure 12.

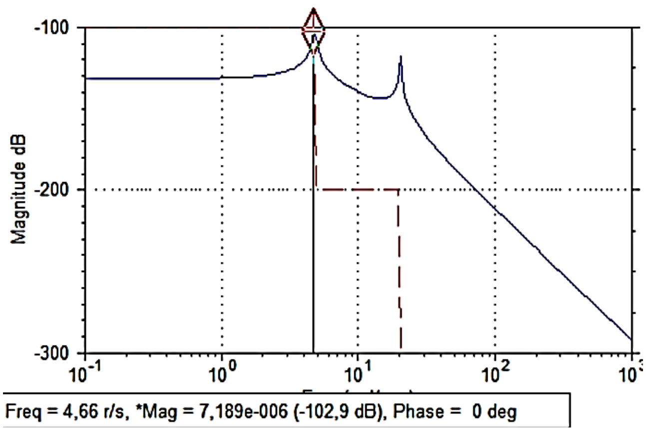

Figure 11. Bode Plot and Natural Frequencies of Vertical Motion

As mentioned in introduction free motions of suspension system according to various speed and damping coefficients can be investigated and described with above simulations. The calculation of natural frequencies has great importance to estimate resonance conditions and critical speeds for ride safety and comfort.

7. Investigation of Lateral and Longitudinal Natural Frequncies

Under consideration the reduced model shown in Figure 8 and the relevant equations given in table and with (9), (10), (11) the system natural frequencies for lateral and longitudinal motions can be obtained from:

$$\omega_{n_l} = \sqrt{K_l/J_\theta} \text{ and } \omega_{n_{lg}} = \sqrt{K_{lg}/J_\phi}$$

Values in formula above; K_l as reduced lateral spring constant, K_{lg} as reduced longitudinal spring constant, J_θ as reduced lateral inertia of moment, J_ϕ as reduced longitudinal inertia of moment.

Both inertias can be calculated from the specified railway and wagon data, since the lateral and longitudinal vibrations are less important than the displacements in the z direction, the details of the movements in these directions have not been further investigated.

8. Conclusions

According to the calculated natural frequencies the relevant critical speeds for the case of disturbances caused by rail connections (with const. length of 20m) can be obtained by using following expressions:

$$V = \frac{L}{T} = \frac{L}{2\pi/\omega_n} = \frac{L\omega_n}{2\pi} \qquad (17)$$

For the first natural frequency. ω_n = 4.66 rad/s seen in Figure 12 the corresponding critical speed will be calculated with (17) as V = 14.84 m/s or as V = 53.5 Km/h.

Assuming that the nominal speed of the car is around 120 km/h which corresponds to V = 33.33 m/s, the vibration frequency at this speed is $\omega = V\pi/20 = 10$ rad/s which is quite far from the resonance state.

The car vibrations reach the highest amplitude value around $\omega = 4.66$ rad/s, as can be seen from the curves of figure 13 obtained against harmonic inputs with different frequencies based on the simulation model given in Figure 10.

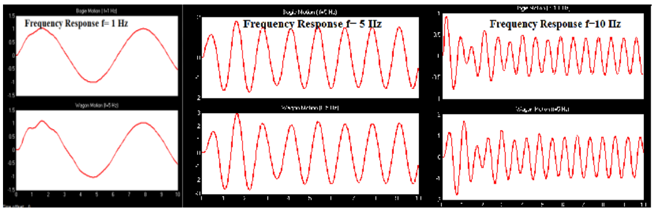

Figure 12. Frequency responses of car for ω=1.6 and 10 rad/s

To obtain best results for safe and comfortable travel conditions the above mentioned modelling methods with equations and simulations can provide practical and reasonable preliminary design properties for the constructor.

References

[1] Esveld, C., & De Man, A. (2003, January). Use of railway track vibration behaviour for design and maintenance. In IABSE Symposium Report (Vol. 87, No. 5, pp. 39-45). International Association for Bridge and Structural Engineering.

[2] Connolly, D. P., Kouroussis, G., Laghrouche, O., Ho, C. L., & Forde, M. C. (2015). Benchmarking railway vibrations–Track, vehicle, ground and building effects. Construction and Building Materials, 92, 64-81.

[3] Zweifel O. (1956) Berechnung des elastischen Verhaltens und der Eigenschwingungen von Eisenbahnfahrzeugen: ergänzte Fassung der Antrittsvorlesung in Schweizerische Bauzeitung Band: 74

[4] Eason, G., Noble, B., & Sneddon, I. N. (1955). On certain integrals of Lipschitz-Hankel type involving products of Bessel functions. Philosophical Transactions of the Royal Society of London. Series A, Mathematical and Physical Sciences, 247(935), 529-551.

[5] Hempelmann, K., Influence of operational track and vehicle parameters on corrugation growth, TU Berlin, June 1995.

[6] Man, A. P. D. (2004). Dynatrack: A survey of dynamic railway track properties and their quality.

[7] Markine, Man, A.D., Jovanović, S., & Esveld, C. (2000). MODELLING AND OPTIMISATION OF AN EMBEDDED RAIL STRUCTURE.

[8] Cochin, I. (1980). Analysis and design of dynamic systems. HarperCollins Publishers.

[9] Hung, H., Yang, Y., (2001). A review of researches on ground-borne vibrations with emphasis on those induced by trains. In: Proc. Nat. Sci. Council Part A: Phys. Sci. & Eng. p. 1-16.

[10] Avillez, J., Frost, M., Cawser, S., Skinner, C., El-Hamalawi, A., & Shields, P. (2012, August). Procedures for estimating environmental impact from railway induced vibration: a review. In ASME Proc. Noise Control And Acoustic Division Conference at InterNoise (pp. 381-392).

[11] Barke, D. W., & Chiu, W. K. (2005). A review of the effects of out-of-round wheels on track and vehicle components. Proceedings of the Institution of Mechanical Engineers, Part F: Journal of Rail and Rapid Transit, 219(3), 151-175.

[12] Popp, K., Kruse, H., & Kaiser, I. (1999). Vehicle-track dynamics in the mid-frequency range. Vehicle System Dynamics, 31(5-6), 423-464.