On forced vibrations caused by earthquakes

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Abstract: We consider buildings of various shapes and their resistance against earthquakes. The 5-storey buildings are considered as a chainh of tightly connected oscillators. They are modelled by a linear system of ordinary differential equations which suffers forced oscillations. We provide results of numerical experiments where the parameters are given certain realistic values and the earthquake is interpreted as a periodic force with certain period. Four types of buildings are considered, the rectangle, the dumbbell-shaped, V-shaped and pyramidal ones.

Key-Words: forced oscillations, various shape buildings, earthquake (6 - 10 words)

1 Introduction

The problem of vibrations in mechanical systems is important from practical point of view and interesting as a descent object for research. Let us recall elementary harmonic oscillations. Free harmonic oscillations and their chracteristics, such as period and frequency, depend only on the elasticity coefficient in an equation. Taking into account damping makes consideration more realistic and leads to exponential solutions and exponentially decaying oscillatory ones.

Considering forced vibrations is another step towards generalization. The respective differential equation may have the form

$$x'' + 2\delta x' + \omega^2 x = F(t), \tag{1}$$

where x is a displacement from the equilibrium state, $2\delta x'$ is a description of damping, ω is the proper frequency of a system, $F(t)$ is the external force, depending on the time t.

The external force may be periodic with its own frequency θ. For a particular choice of $F(t) = A\sin\beta t$, the general solution of (1) has the form

$$x(t) = u(t) + B\sin\beta t + C\cos\beta t, \tag{2}$$

where $u(t)$ is a general solution of the homogeneous equation, and $B\sin\beta t + C\cos\beta t$ is a particular solution of (1). The solution $u(t)$ contains two arbitrary constants, but B and C can be computed. In presence of damping $u(t)$ becomes small and tends to zero, as $t \to +\infty$, but the second part of a solution is periodic wth the period $2\pi/\beta$. If damping is neglected, and periods of $u(t)$ and external force F are the same, the resonance phenomenon may occur and solutions $x(t)$ may become unbounded.

In this article we take our motivation from the paper [2], where forced vibrations of multi-storey buildings, caused by earthquakes, were considered. The authors used a linear model of forced vibrations, where the dimensionality of the respective system of ordinary differential equations is equal to the number of storeys of a building under consideration. An interesting feature of the paper [2] is that the authors consider buildings of various shapes, provided that buildings are vertically symmetrical and the mass of any storey in center symmetrical also. Generally they have considered five types of buildings, including two pyramidal forms like letters Δ and V.

2 Problem Formulation

We consider behavior of vertically symmetrical buildings under forced vibrations. We are motivated by the paper [2], where forced vibrations of multi-storey buildings, caused by earthquakes, were considered. The authors used a linear model of forced vibrations, where the dimensionality of the respective system of ordinary differential equations is equal to the number of storeys of a building. An interesting feature of this paper is that the authors consider buildings of various shapes, provided that buildings are vertically symmetrical and the mass of any storey is center symmetrical also. Generally they have considered five types of buildings, including two pyramidal forms like letters Δ and V.

The resistance of three-storey buildings to external forces of the form $E\omega^2\cos\omega t$ can be described by the system of the form ([2])

$$\left\{\begin{array}{l} m_1 x''_1 = -k_1 x_1 + k_2(x_2 - x_1) \\ -c_1(x'_1 - x'_2) + m_1 E\omega^2 \cos\omega t, \\ m_2 x''_2 = -k_2(x_2 - x_1) + k_3(x_3 - x_2) \\ -c_1(x'_2 - x'_1) - c_2(x'_2 - x'_3) + m_2 E\omega^2 \cos\omega t, \\ m_3 x''_3 = -k_3(x_3 - x_2) + k_4(x_4 - x_3) \\ -c_2(x'_3 - x'_2) - c_3(x'_3 - x'_4) + m_3 E\omega^2 \cos\omega t, \\ m_4 x''_4 = -k_4(x_4 - x_3) + k_5(x_5 - x_4) \\ -c_3(x'_4 - x'_3) - c_4(x'_4 - x'_5) + m_4 E\omega^2 \cos\omega t, \\ m_5 x''_5 = -k_5(x_5 - x_4) \\ -c_4(x'_5 - x'_4) + m_5 E\omega^2 \cos\omega t. \end{array}\right. \tag{3}$$

The stiffness parameters k_i are taken as a constant k for each floor with a value of 10000 units. The damping parameters c_i are also taken as a constant c for each floor with a value of 500 units. The amplitude of the external force (earthquake) is set to $E = 10$, the frequence $\omega = \frac{\pi}{3}$ or $\frac{\pi}{2}$ or π, which correspond to frequences of seismic waves [3]. The masses m_i of i-th floor depend on the shapes of buildings. The basic valiue is $m = 1000$ tonn, the increment $m_\delta = 200$.

We accept the assumptions from [2]:

(i) The floors have masses m_1 to m_5. Each floor is assumed to be a point mass concentrated in the centre of each floor.

(ii) A linear restoring force acts on each floor that is incorporated in the model by the stiffness factor k_1 to k_5.

(iii) There is a damping force which is directly proportional to the damping constants c_1 to c_5 between the floors.

(iv) A horizontal earthquake oscillation, $E\cos\omega t$ of the ground with amplitude E and acceleration $a = E\omega^2\cos\omega t$, produces a force $F = ma = mE\omega^2\cos\omega t$ on each floor of the building.

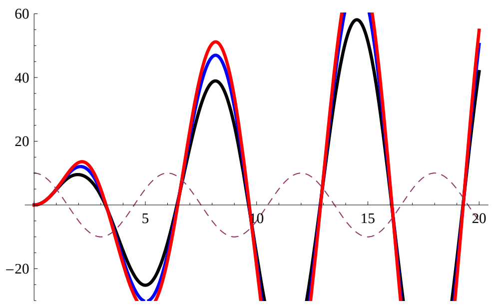
Time is measured in seconds. Frequency is measured in hertzs (Hz). Other parameters are measured in universal units suggested in [2]. We make also several simplifying assumptions. We do not take into account the duration of the vertical component of ground motion.

In all illustrations the deviations of floors (generally from the 3rd one to the 5th floor) with respect to the vertical axis of symmetry of a building are depicted. The dashed curve is for the seismic wave of a given frequency.

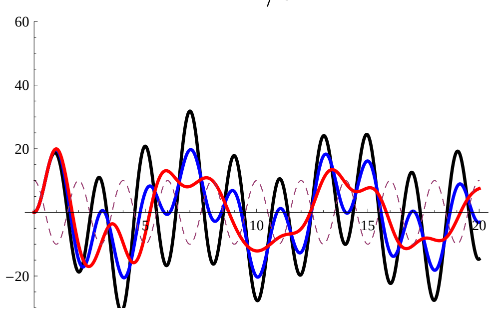
3 Recktangular form

The system (3) has the form

$$\left\{\begin{array}{l} m x''_1 = -k x_1 + k(x_2 - x_1) \\ -c(x'_1 - x'_2) + mE\omega^2 \cos\omega t, \\ m x''_2 = -k(x_2 - x_1) + k(x_3 - x_2) \\ -c(x'_2 - x'_1) - c(x'_2 - x'_3) + mE\omega^2 \cos\omega t, \\ m x''_3 = -k(x_3 - x_2) + k(x_4 - x_3) \\ -c(x'_3 - x'_2) - c(x'_3 - x'_4) + mE\omega^2 \cos\omega t, \\ m x''_4 = -k(x_4 - x_3) + k(x_5 - x_4) \\ -c(x'_4 - x'_3) - c(x'_4 - x'_5) + mE\omega^2 \cos\omega t, \\ m x''_5 = -k(x_5 - x_4) \\ -c(x'_5 - x'_4) + mE\omega^2 \cos\omega t. \end{array}\right. \tag{4}$$



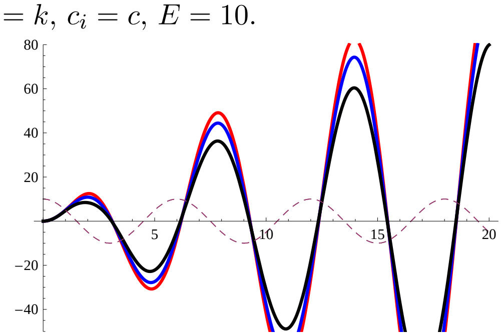
Oscillation of the floors 3,4,5 (black, blue, red), $\omega = \pi/3$



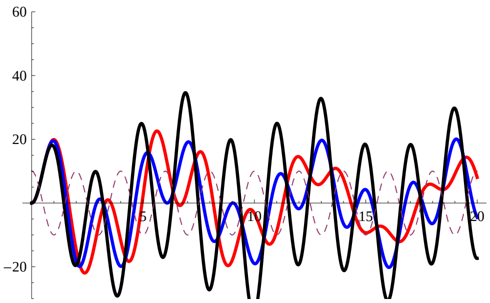
Oscillation of the floors 3,4,5 (black, blue, red), $\omega = \pi$

4 Dumbbell form

The modelling system of ODE is the system (3), where $m_1 = m_5 = 1000$, $m_2 = m_4 = 800$, $m_3 = 600$, $k_i = k$, $c_i = c$, $E = 10$.



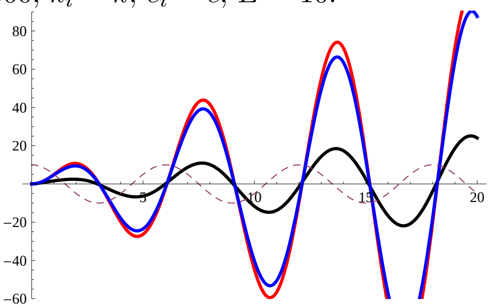
Oscillation of the floors 3,4,5 (black, blue, red), $\omega = \pi/3$



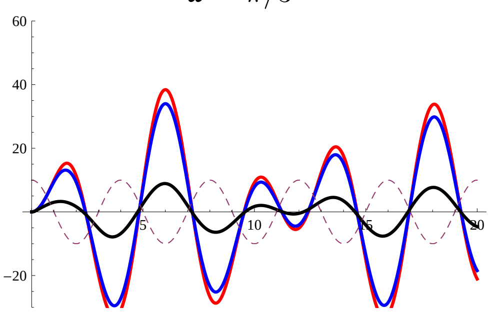
Oscillation of the floors 3,4,5 (black, blue, red), $\omega = \pi$

5 V-shaped

The modelling system of ODE is the system (3), where $m_1 = 200$, $m_2 = 400$, $m_3 = 600$, $m_4 = 800$, $m_5 = 1000$, $k_i = k$, $c_i = c$, $E = 10$.



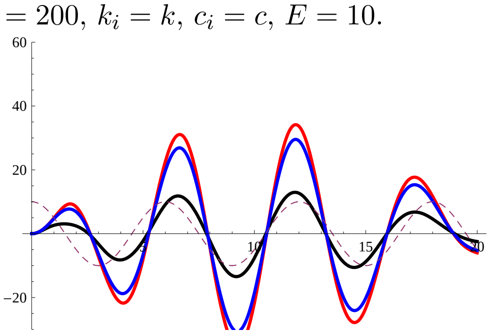
Oscillation of the floors 1,4,5 (black, blue, red), $\omega = \pi/3$



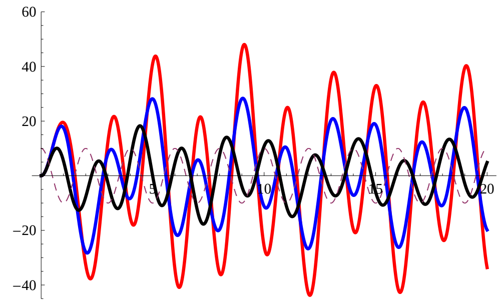
Oscillation of the floors 1,4,5 (black, blue, red), $\omega = \pi$

6 Pyramidal form

The modelling system of ODE is the system (3), where $m_1 = 1000$, $m_2 = 800$, $m_3 = 600$, $m_4 = 400$, $m_5 = 200$, $k_i = k$, $c_i = c$, $E = 10$.



Oscillation of the floors 1,3,5 (black, blue, red), $\omega = \pi/3$



Oscillation of the floors 1,3,5 (black, blue, red), $\omega = \pi$

7 Conclusion

For the rectangular form building, the displacement of the floors from the symmetry axis increases in time for small ($\omega = \pi/3$) frequency of seismic waves. For larger value ($\omega = \pi$) the displacements seem to be bounded.

For the dumbbell shape buildings behavior at $\omega = \pi/3$ is similar to the previous case. For larger value $\omega = \pi$ oscillations are bounded with the largest amplitude for the third floor. Vibrations of the higher floor are more intensive than ones for the rectangular building.

For the V-shaped buidings the tendency for resonance is obvious also as $\omega = \pi/3$. Even the first floor vibrates with relatively large amplitude. For larger value $\omega = \pi$ oscillations are bounded with largest displacements at approximately 40 units.

For the pyramidal form buildings there is no tendency for resonance at $\omega = \pi/3$. For larger value $\omega = \pi$ the highest floor oscillates intensively with the amplitudes over 40 units. This was not observed for other type buildings.

At the end let us remark that if the elastic model is accepted, also higher buildings can be considered of various shapes and for different values of parameters. The obstacle may be the lack of computer resources only.

References:

[1] Anil K. Chopra. Dynamics of structures. Theory and Applications to Earthquake Engineering, 4th edition. Prentice Hall, Boston, 2012.

[2] Shobha Bagai, Parul Madaan, Tarun Khajuria. A mathematical model for the effect of earthquake on high rise buildings of different shapes. Delhi University Journal of Undergraduate Research and Innovation, Vol. 2, Issue 1, 180 – 188, 2016.

[3] https://www.britannica.com/science/earthquake-geology/Properties-of-seismic-waves

Contribution of individual authors to the creation of a scientific article (ghostwriting policy)

Both authors have contributed equally to creation of this article.

Sources of funding for research presented in a scientific article or scientific article itself

There is not funding for this article.