

first and second-order derivatives in the loss function for fractional derivative calculation, the presented method involves a higher computational burden compared to classic gradient descent. Future endeavors should focus on investigating methods to mitigate computational demands, making the approach more feasible for real-world applications. These considerations underscore the potential for refinement and enhancement in the proposed gradient descent method, paving the way for its broader adoption and practical utility.

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Contribution of individual authors to the creation of a scientific article (ghostwriting policy)

Robab Kalantari and Khashayar Rahimi developed a novel method for gradient descent, conducted simulations, performed optimization, and were responsible for the writing and implementation of the proposed approach.

Saman Naderi has some edit in writing.