A New Exact Finite difference model for the solution of the Harmonic Oscillator Differential Equation

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Abstract: - In this work we used a combination of dynamically allocated step-size and an interpolating function to develop a new class of finite difference scheme for the exact solution of Harmonic Oscillator Differential Equation**.** The schemes are locally stable, convergent and consistent. The new model has been compared with an earlier exact discrete model for the same equation. The new scheme is very suitable for the numerical simulation of the tested equation as proposed.

Key-Words: - Harmonic Oscillator; Exact Numerical model; Nonstandard Techniques; Interpolation methods, Dynamic system;

1 Introduction

Generally the problem of motion in a resistive medium is a difficult one. Most differential equation for the Harmonic oscillator are based on the Newton's second law of motion and a lot of real life models are derived from this common model by considering other issues that affect the behaviour of the Oscillator like, frictions, external forces etc. acting upon the system. In this process we end up with much more realistic but much more complex harmonic equations which allow a detailed study of the behaviour and properties of the physical phenomena being modelled. A lot of dynamical systems with oscillatory properties can be approximately modelled by some form of Harmonic Oscillator equation or a variant of differential equation with harmonic terms.

Many a times such differential equation does not totally fit into one of the common ones whose analytic solution are readily available researchers therefore resort to methods of finding approximate solution. For example in classical mechanics, the problem of a simple harmonic oscillator, modified by the presence of a non-harmonic term, is customarily solved by perturbative methods. Researchers many times have had to consider the homogeneous model as a first step to finding close enough solution to such complex differential equations such solutions are then conclude using a combination of some other methods . An exact equation to the homogeneous part becomes very important in this case.

2 Literature Review

Among some past work on solution to such harmonic Oscillator is the work of [8] who obtained an exact solution of perturbed harmonic oscillator using hyper-geometric approach .

[9] considered the damped harmonic oscillator with a time-dependent damping constant and a time-dependent angular frequency, which is actually the generalized Caldirola-Kanai Hamiltonian. They investigate the exact solution of the oscillator system for some choices of the time-dependent damping constant and the time-dependent angular frequency.

Another notable work that considered exact solution is that of [10] In that paper, a modified harmonic balance method is used to investigate the strongly nonlinear oscillators. The approximate frequency and periodic solution for both small and large amplitude of oscillations show a good agreement with the numerical solution.

[11] also presented an analytic technique to determine approximate periods of a strongly nonlinear Duffing-harmonic oscillator. Working on the background that a set of difficult nonlinear algebraic equations always appear when harmonic balance method is imposed and that the power series solutions of these equations are invalid, he proposed idea avoids this limitation and the necessity of numerically solving such nonlinear algebraic equations with very complex nonlinearities. In this technique, different parameters for the same nonlinear problems are found, for which the power series solution yields desired results. Besides a suitable truncation formula is found in which the

solution measures better results than existing solutions.

[12] also consider the simple harmonic Oscillator . I*n that paper a* study was conducted on the oscillatory behaviour of a spring-mass system, considering the influence of varying the average spring diameter Φ on the elastic constant k, the angular frequency ω, the damping factor γ, and the dynamics of the oscillations. The results were applied on physical phenomena like clock, guitar, violin, bungee jumping, rubber bands and diving boards. [6] constructed a family of exact numerical schemes for the Homogeneous harmonic Oscillator equation using Nonstandard techniques alone. This work is of particular interest because the method does not allow for numerical instability.

2.1 A review of the Nonstandard rules ([6],[7],[1])

The nonstandard rules 2&3 [6] and their extensions in [7] and [1] are of special interest in order to create an hybrid of schemes from two methods . The rules and their implication are as shown below

Rule 2 ([6])

Denominator function for the discrete derivatives must be expressed in terms of more complicated function of the step-sizes than those conventionally used. This rule allows the introduction of complex analytic function ψ of h that satisfies the condition $\psi(h) \to h + 0(h^2)$ as $h \to 0$ in the denominator.

Rule 3 ([6])

The non-linear terms in the differential equation must in general be modelled (approximated) non-locally on the computational grid or lattice in many different ways.

Application and extension of the combination of these two rules will give us the following transformations for First and second order derivatives

$$\frac{dy}{dx} \equiv \frac{(y_{k+1} - y_k)}{\psi} \quad (1)$$

where $\psi(h) \to h + 0(h^2)\ as\ h \to 0$

$$\frac{dy}{dx} \equiv \frac{(y_{k+1} - \beta y_k)}{\psi} \quad \text{where} \quad \psi(h) \to h + 0(h^2),\ \beta(h) \to 1\ as h \to 0 \quad (2)$$

$$\frac{dy}{dx} \equiv \frac{(y_{k+1} - \beta y_{k-1})}{2\psi} \quad \text{where} \quad \psi(h) \to h + 0(h^2),\ \beta(h) \to 1\ as h \to 0 \quad (3)$$

$$\frac{d^2y}{dx^2} = \frac{y_{k+1} - 2y_k + y_{k-1}}{\varphi^2} \quad \text{where} \quad \varphi(h) \to h^2 + 0(h^n) as\ h \to 0\ for\ n \geq 3 \quad (4)$$

And the following non-local approximations for

$$y_{k+1} \equiv ay_{k+1} + by_k \quad a + b = 1 \quad (5)$$

$$y_k \equiv \frac{(y_{k+1} + \beta y_k)}{2} \text{ where } \beta(h) \to 1\ as\ h \to 0 \quad (6)$$

$$y_{k+1} \equiv \frac{(2y_k + \beta y_{k-1})}{3} \text{ where } \beta(h) \to 1\ as\ h \to 0 \quad (7)$$

Sample renormalization functions employed are

$$\psi = \sin(\propto h),\ \propto \in \text{R} \ \to h + 0(h^2)\ as\ h \to 0 \quad (8)$$

$$\psi = \frac{(e^{\lambda h} - 1)}{\lambda},\ \lambda \in \text{R},\ \to h + 0(h^2)\ as\ h \to 0 \quad (9)$$

$$\beta = cos(\propto h),\ \propto \in \text{R} \to 1 \quad as\ h \to 0 \quad (10)$$

$$\varphi = 4\sin^2\left(\frac{h}{2}\right) \quad \text{or} \quad h^2\psi \to h^2 + 0(h^4) \quad as\ h \to 0$$

3 Formulation of the basis function

The Harmonic Oscillator Equation under consideration is given by

$$\frac{d^2y}{dx^2} + 2\varepsilon\frac{dy}{dx} + y = 0 \quad (12)$$

$y(x) > 0$ is the distance/displacement of the body involved in the oscillation, x is a time variable. This is one of the simplest models for a harmonic motion considering the complex dynamics of the phenomena being modeled.

When several assumptions and affects are considered, the harmonic equation usually will become complex and sometimes and it may be difficult to obtain a solution that can be expressed as an explicit function of the variables involved. But since the solution for this equation is a function whose second derivative is itself with a minus sign. We have two possible functions that satisfy this requirement (sine and cosine) two functions that are essentially the same since each is just a phase shifted version of the other. When a trigonometric function is phase shifted, it's derivative is also phase shifted. Nothing else is affected, so we pick a sine function as the main component of our interpolation function.

Considering the general harmonic equation (12), it can be observed that the solutions to the simpler differential equation without the velocity term look like a trigonometric form; and solutions to the simpler differential equation without the acceleration term look like exponential functions. We also considered that a body that is in a motion moves a distance that can be described by a linear equation over time t. We also assumed a control parameter that can act as the residual (a slack or a surplus) of the unaccounted activities.

We therefor assumed a solution consisting of three components of linear exponentiation and trigonometric and the residual with requisite simulation parameters that can be determined from time to time depending on consideration of some physical measurement in a real life situation . We hereby proposed a an interpolating function of the type given below

$$y(x) = a_0 + \propto^x + a_1 x + a_2 \sin(\beta x^2 + k) \qquad (13)$$

The parameters can be selected carefully to complement or retract the response to the oscillatory nature of the dynamical system under consideration. This model can be made more complex to include special parameters to cover the level of variability and non-coherent behaviours that may vary speed and amplitude of oscillation. However the interpolating functions also have the tendency of being self-compensating depending on the value of the parameters.

3.1 The Exact Nonstandard scheme developed for Eq. (12) by ([6])

Mickens(1994) in his paper has developed an exact nonstandard scheme for the eq(12) given by

$$\left\{\frac{y_{k+1} - 2y_k + y_{k-1}}{\varphi^2}\right\} = 2\varepsilon\left\{\frac{y_k - \psi y_{k-1}}{\varphi}\right\} + \left\{\frac{2(1-\psi)y_k + (x^2 + x^2 - 1)y_{k-1})}{\varphi^2}\right\} \qquad (14)$$

with $\psi = \cosh \quad \varphi = 4\sin^2(\frac{h}{2})$

In this work we will construct a new class of numerical schemes with the same qualitative properties as the corresponding second order initial value equation representing some special class of Harmonic Oscillator Differential equations Eq (12). We posit that this discrete model will also produce solutions and curves that behave like the exact Nonstandard schemes constructed by [6]. This work is based on a combination of both Interpolation method and Non-standard method as explained above.

This new discrete models are significant because of the combined methodology and its total congruency to the early works on the exact schemes for Oscillator equations. The schemes have been tested in a numerical experiment which produced some interesting 3D graphs..

4. Derivation of the Standard Finite Difference Scheme Using Eq(13)

Thus we can derive a discrete model based on equation (13) as given below:

$$y(x) = a_0 + \propto^x + a_1 x + a_2 \sin(\beta x^2 + k)$$

$$y' = \propto^x \log \propto + a_1 + a_2 2\beta x \cos(\beta x^2 + k) \qquad (15)$$

$$y'' = a_2[2\beta x(-2\beta x \sin(\beta x^2 + k) + 2\beta \cos(\beta x^2 + k)] + \propto^x (log \propto)^2$$

$$y'' = a_2[-(2\beta x)^2 \sin(\beta x^2 + k) + 2\beta \cos(\beta x^2 + k)] + \propto^x (log \propto)^2 \qquad (16)$$

$$y''' = a_2[-(2\beta x)^3 \cos(\beta x^2 + k) + 4\beta x \sin(\beta x^2 + k)] + \propto^x (log \propto)^3 \qquad (17)$$

From (15), (16), (17)

$$a_1 = y' - \propto^x \log \propto - a_2 2\beta x \cos(\beta x^2 + k) \qquad (18)$$

$$a_2 = \frac{y'' - \propto^x (log \propto)^2}{[-(2\beta x)^2 \sin(\beta x^2 + k) + 2\beta \cos(\beta x^2 + k)]} \qquad (19)$$

The discrete form

$$y(x) = a_0 + \propto^x + a_1 x + a_2 \sin(\beta x^2 + k)$$

$$y(x_{n-1}) = a_0 + \propto^{x_{n-1}} + a_1 x_{n-1} + a_2 \sin(\beta x_{n-1}^2 + k)$$

$$y(x_n) = a_0 + \propto^{x_n} + a_1 x_n + a_2 \sin(\beta x_n^2 + k)$$

$$y(x_{n+1}) = a_0 + \propto^{x_{n+1}} + a_1 x_{n+1} + a_2 \sin(\beta x_{n+1}^2 + k)$$

$$y(x_{n+1}) - 2y(x_n) + y(x_{n-1})$$

$$= (a_o - 2a_o + a_o) + (\propto^{x_{n-1}} - 2 \propto^{x_n} + \propto^{x_{n+1}}) + a_1(x_{n-1} + x_{n+1} - 2x_n) + a_2[(\sin(\beta x_{n-1}^2 + k) + \sin(\beta x_{n+1}^2 + k) - 2\sin(\beta x_n^2 + k)] \qquad (20)$$

Let $P_n = [(\sin(\beta x_{n-1}^2 + k) + \sin(\beta x_{n+1}^2 + k) - 2\sin(\beta x_n^2 + k)]$

Substitute $x_{n-1} = a + nh - h$, $x_{n+1} = a + nh + h$, and $-2x_n = -2a - 2nh$ then

$$y(x_{n+1}) - 2y(x_n) + y(x_{n-1})$$

$$= \propto^{a+nh} (\propto^h + \propto^{-h} - 2) + a_2[P_n]$$

At the point of coincidence with the exact solution $y(x_{n+1}) \equiv y_{n+1}$,$y(x_n) \equiv y_n$ and $y(x_{n-1}) \equiv y_{n-1}$

$$y_{n+1} - 2y_n + y_{n-1} \equiv \quad \propto^{a+nh} \left(\propto^h + \propto^{-h} - 2\right) + a_2[P_n]$$

$$P_n = [(\sin(\beta x_{n-1}^2 + k) + \sin(\beta x_{n+1}^2 + k) - 2\sin(\beta x_n^2 + k)]$$

$$\beta x_{n-1}^2 + k = \beta x_n^2 + \beta(h^2 - 2hx_n) + k$$

$$\beta x_{n+1}^2 + k = \beta x_n^2 + \beta(h^2 + 2hx_n) + k$$

Let $= \beta(x_n^2 + h^2) + k$, $Q = 2\beta h x_n$ and $R = (\beta x_n^2 + k)$ (21)

Then

$P_n = [(\sin(P + Q) + \sin(P - Q) - 2\sin(R)]$

$P_n = 2\sin(P)\cos(Q) - 2\sin(R)]$ (22)

$P_n = 2\sin(\beta(x_n^2 + h^2) + k\ )\cos(2\beta h x_n) - 2\sin(\beta x_n^2 + k)]$ (23)

$$y_{n+1} - 2y_n + y_{n-1} = \propto^{a+nh} \left(\propto^h + \propto^{-h} - 2\right) + \frac{\{y'' - -\propto^x (log\propto)^2\}[P_n]}{[-(2\beta x)^2 \sin(\beta x^2 + k) + 2\beta\cos(\beta x^2 + k)]} \quad (24)$$

$$y_{n+1} - 2y_n + y_{n-1} =$$

$$\propto^{a+nh} \left(\propto^h + \propto^{-h} - 2\right) + \frac{\{y'' - \propto^x (log\propto)^2\}[2\sin(\beta(x_n^2 + h^2) + k)\cos(2\beta h x_n) - 2\sin(\beta x_n^2 + k)]}{[-(2\beta x)^2 \sin(\beta x^2 + k) + 2\beta\cos(\beta x^2 + k)]} \quad (25)$$

Applying nonstandard dynamic step size φ instead of h

The second derivative

$$\equiv \frac{y_{n+1} - 2y_n + y_{n-1}}{h^2} \equiv \frac{y_{n+1} - 2y_n + y_{n-1}}{\varphi^2}$$

The Standard scheme developed in equation (25) will be named NEW h

The hybrid scheme is obtained by substituting h for φ =sin(h) and $\varphi = \frac{(e^{\lambda h} - 1)}{\lambda}$ which will be named NEW SIN, NEW EXP respectively

5 Qualitative properties of the new scheme

Definition ([3])

Any algorithm for solving a differential equation in which the approximation y_{n+1} to the solution at x_{n+1} can be calculated iff $x_n, y_n and\ h$ are known is called a one step method. It is a common practice to write the functional dependence y_{n+1} on the quantities $x_n, y_n and\ h$ in the form y_{n+1}= y_n+$\phi(x_n, y_n, h)$

Where $\phi(x_n, y_n, h)\ is\ the$ incremental function

Theorem ([3])

Let the incremental function of the scheme defined in the one step scheme above be continuous and jointly as a function of its arguments in the region defined by $x \in [a, b] and\ y \in (-\infty, \infty)$, $0 \le h \le h_0$ Where $h_0 > 0$ and let there exists a constant L such that $\phi(x_n, y_n, h) - \phi(x_n, y_n^*, h) \le L|y_n - y_n^*|$ for all $(x_n, y_n, h) and\ (x_n, y_n^*, h)$ $in\ the\ region\ just\ defined$, Then the relation $(x_n, y_n, 0) = (x_n, y_n^*)$ is a necessary condition for the convergence of the new scheme

Definition ([4])

A numerical scheme with an incremental $\phi(x_n, y_n, h)$ is said to be consistent with the initial value problem $y' = f(x, y)$, $y(x_0) = y_0$ if the incremental function is identically zero at t_0 when $h = 0$,

Theorem ([4])

Let y_n= $y(x_n)$ and p_n= $p(x_n)$ denote two different numerical solution of the differential equation with the initial condition specified a

y_0= $y(x_0) = \xi$ and p_0= $p(x_0) = \xi^*$ respectively such that $|\xi - \xi^*| < \varepsilon$ $\varepsilon > 0$

If the two numerical estimates are generated by the integration scheme, we have

$$y_{n+1} = y_n + h\phi(x_n, y_n, h)$$

$$p_{n+1} = p_n + h\phi(x_n, p_n, h)$$

The condition that $|y_{n+1} - p_{n+1}| \le K\ |\xi - \xi^*|$ is the necessary and sufficient condition for the stability and convergence of the schemes.

5.1 Proof of Convergence

Let

$P_n = 2\sin(\beta(x_n^2 + h^2) + k)\cos(2\beta h x_n) - 2\sin(\beta x_n^2 + k)]$

$y' = f_n$, $y'' = f_n'$ and $y''' = f_n''$

$D_n = [-(2\beta x_n)^2 \sin(\beta x_n^2 + k) + 2\beta\cos(\beta x_n^2 + k)]$

$T_n = -\propto^x (log \propto)^2$

$K_n = \propto^{a+nh}\left(\propto^h + \propto^{-h} - 2\right)$

$$y_{n+1} = 2y_n + y_{n-1} + K_n + \left\{\frac{\{f_n' - T_n\}[P_n]}{[D_n]}\right\} \quad (26)$$

For small h the nonlocal approximation of $2y_n - y_{n-1} \cong y_n$

Simplify to obtain

$$y_{n+1} = y_n + K_n + \left\{\frac{\{f_n' - T_n\}[P_n]}{[D_n]}\right\}$$

$$y_{n+1} = y_n + \left\{K_n - \frac{T_n[P_n]}{[D_n]}\right\} + \left\{\frac{[P_n]}{[D_n]}\right\} f_n' \quad (27)$$

The incremental function can be written as

$$\phi(x_n, y_n, h) = \left\{K_n - \frac{T_n[P_n]}{[D_n]}\right\} + \left\{\frac{[P_n]}{[D_n]}\right\} f_n' \quad (28)$$

$\phi(x_n, y_n, h) = A + Bf_n'$

The value of A is fixed for every finite ($n << \infty$)

$\phi(x_n, y_n, h) - \phi(x_n, y_n^*, h) =$

$B[f'(x_n, y_n, h) - f'(x_n, y_n^*, h)]$

$= B[f'(x_n, y_n) - f'(x_n, y_n^*)]$

$= B[\frac{\partial f'(x_n,\bar{y})}{\partial y_n}(y_n - y_n^*)]$

L = $SUP_{(x_n,y_n)\in D} \ \frac{\partial f'(x_n,\bar{y})}{\partial y_n}$

then

$\phi(x_n, y_n, h) - \phi(x_n, y_n^*, h) = B[L(y_n - y_n^*)]$

Let M = | B.L2|

$$\phi(x_n, y_n, h) - \phi(x_n, y_n^*, h) \le M|y_n - y_n^*| \quad (29)$$

which is the condition for convergence

5.2 Consistency of the Schemes

$$y_{n+1} = y_n + \left\{K_n - \frac{T_n[P_n]}{[D_n]}\right\} + \left\{\frac{[P_n]}{[D_n]}\right\} f_n'$$

$y_{n+1} = y_n + \{A\} + \{B\}f_n'$

When $h = 0$ $K_n = 0$, $P_n = 0$, and $A = 0$ $B = 0$

$\Rightarrow$ $y_{n+1} = y_n$ and the incremental function is identically zero when $h = 0$ (30)

$\Rightarrow \phi(x_n, y_n, 0) \equiv 0$

5.3 Stability of the Schemes

Consider the equation

$y_{n+1} = y_n + \{A\} + \{B\}f_n'(x_n, y_n)$

Let $S_{n+1} = S_n + \{A\} + \{B\}f_n'(x_n, S_n)$

$$y_{n+1} - S_{n+1} = y_n - S_n + \{A - A\} + \{B\}[f_n'(x_n, y_n) - f_n'(x_n, S_n)] \quad (31)$$

$= y_n - S_n + B[\frac{\partial f'(x_n, S_n)}{\partial p_n}(y_n - S_n)]$

L = $SUP_{(x_n,y_n)\in D} \ \frac{\partial f'(x_n, S_n)}{\partial p_n}$

$y_{n+1} - S_{n+1} = y_n - S_n + B.L(y_n - S_n)$

$|y_{n+1} - S_{n+1}| = |y_n - S_n| + [B.L]|(y_n - p_n)|$

Let N = $|1 + [B.L]|$

$|y_{n+1} - S_{n+1}| \le N|y_n - S_n|$

Let $y_0 = y(x_0) = \xi$ and $S = S(x_0) = \xi^*$ then

$$|y_{n+1} - S_{n+1}| \le K|\xi - \xi^*| \quad (32)$$

6. Application of the Model to the Harmonic Oscillator Equation

From equation (12), we have

$$y'' = -2\varepsilon(y') - y \quad (33)$$

$$y''' = -2\varepsilon(y'') - y' \quad (34)$$

From the Nonstandard theory $y' = \frac{y_{n+1} - y_n}{\psi}$

$$f_n' = -2\varepsilon\left(\frac{y_{n+1} - y_n}{\psi}\right) - y_n$$

$$f_n' = -2\varepsilon\left(\frac{y_{n+1}}{\psi}\right) + 2\varepsilon\left(\frac{y_n}{\psi}\right) - y_n$$

The standard scheme equation (25) is

$y_{n+1} = y_n + \{A\} + \{B\}f_n'(x_n, y_n)$

$$y_{n+1} = y_n + \left\{K_n - \frac{T_n[P_n]}{[D_n]}\right\} - \left\{\frac{P_n}{D_n}\right\}\left\{\left(\frac{2\varepsilon}{\psi}\right)\right\} y_{n+1} + \left\{\left(\frac{2\varepsilon}{\psi}\right) - 1\right\}\left\{\frac{P_n}{D_n}\right\} y_n \quad (35)$$

$$\left\{\frac{\psi D_n + 2\varepsilon P_n}{\psi D_n}\right\} y_{n+1} = \left\{K_n - \frac{T_n P_n}{D_n}\right\} + \left\{1 + \frac{2\varepsilon P_n}{\psi D_n} - \frac{P_n}{D_n}\right\} y_n$$

$$y_{n+1} = \left\{\frac{\psi D_n}{\psi D_n + 2\varepsilon P_n}\right\}\left\{K_n - \frac{T_n P_n}{D_n}\right\} + \left\{\frac{\psi D_n}{\psi D_n + 2\varepsilon P_n}\right\}\left\{1 + \frac{2\varepsilon P_n}{\psi D_n} - \frac{P_n}{D_n}\right\} y_n \quad (36)$$

$$P_n = 2\sin(\beta(x_n^2 + h^2) + k) \cos(2\beta h x_n) - 2\sin(\beta x_n^2 + k)]$$

$$D_n = [-(2\beta x_n)^2 \sin(\beta x_n^2 + k) + 2\beta\cos(\beta x_n^2 + k)]$$

$$T_n = \propto^x (\log \propto)^2$$

$$K_n = \propto^{a+nh} (\propto^h + \propto^{-h} - 2)$$

The standard scheme will have $\varphi = h^2$ and the two hybrid schemes will be obtained by changing h to $\varphi = \sin^2(rh)$ and $\varphi = h\frac{(e^{\lambda h}-1)}{\lambda}$, $\psi = \sin(rh)$ or $\psi = \frac{(e^{\lambda h}-1)}{\lambda}$ $\quad r, \lambda \in \mathbb{R}$

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7. Result of Numerical Experiment

The algorithm of these new family of schemes have been coded into a software program. The schemes have been tested using step size h=0.01 and for about 100 iterations. The result of the numerical simulation is here presented in 3D graphs. NSTD EXACT is the exact solution as produced by [6] in 1994 . NSTD EXP is the solution with step-size substituted by exponential function NSTD SIN is the solution with step-size substituted by sine function. Likewise we have NEW SCH EXP which is the new scheme with exponential function as step-size and NEW SCH SIN is the new scheme with sine function as step-size

6.1 Graph of the new schemes for h=0.01 , $\varepsilon = 0.0001$ and initial value $y(0) = 1$

simulation parameters $r = 0.005, \lambda = 0.005$,$\propto = 0.5, \beta = -0.75,$

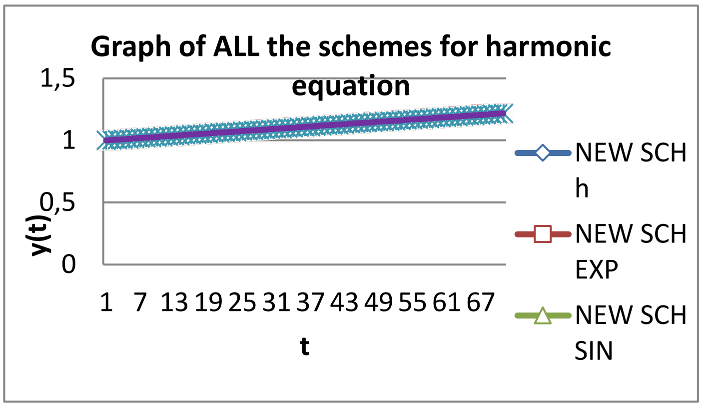

Fig 1: Graph of solution from all the new schemes and the exact solution

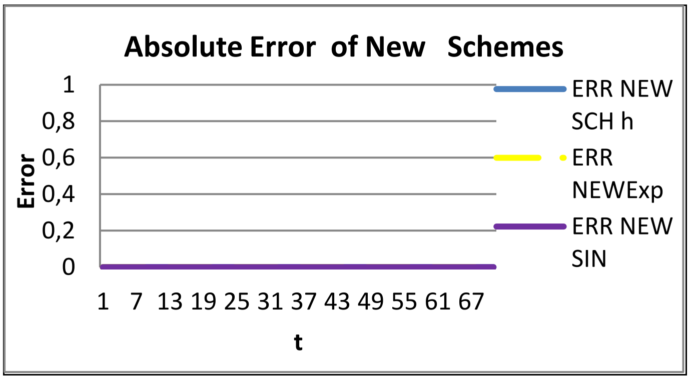

Fig 2: Graph of Error of deviation of schemes from the Mickens exact solution

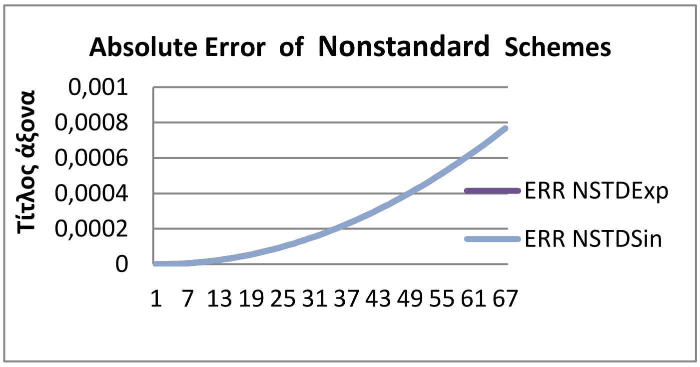

Fig 3: Graph of Error of deviation of schemes from the Mickens exact solution

6.2 Graph of the new schemes for h=0.001 , $\varepsilon = 0.001$ and initial value $y(0) = 1$

simulation parameters $r = 1, \lambda = -0.01$, $\propto = 0.5, \beta = 0.75$,

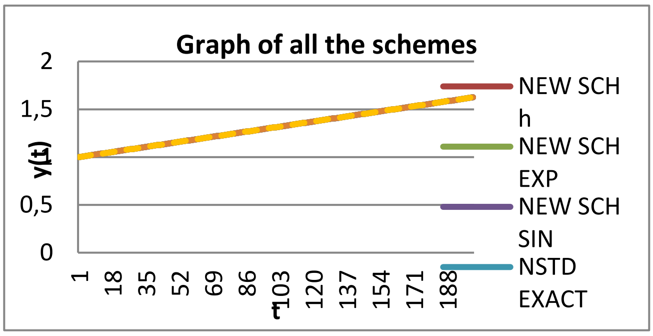

Fig 4: 3D Graph of solution from all the new schemes and the Exact solution

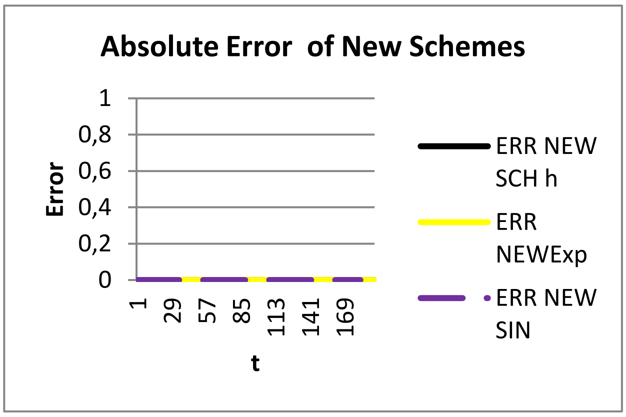

Fig 5: Graph of Error of deviation of New schemes from the Mickens exact solution

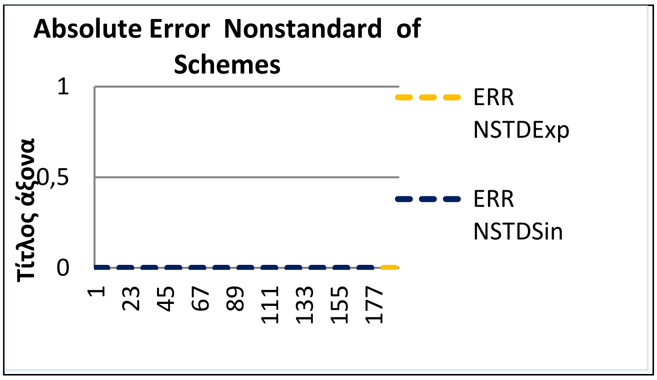

Fig 6: Graph of Error of deviation of schemes from the Mickens exact solution

8. Discussion and conclusion

We have experimented with the exact scheme obtain by Mickens in 1994 in comparison with the new schemes under the same parameter. We did for various step sizes . we observed monotonicity of solutions and total monotonicity of all solutions, monotone dependence on initial values. We observe total congruency as h becomes smaller for any finite number of iterations. The schemes converge faster depending on how small the value of $\propto$, large values of β makes the schemes to behave like very small value of β

The new Discrete Hybrid Nonstandard models NEWSH EXP and NEWSCH SIN produced the exact solution for the Harmonic Oscillator Equation as presented. The schemes have been proved to be stable, consistent and convergent analytically. The numerical experiment has shown that the absolute error of deviation for the Hybrid schemes are all zeros as h approaches zero (see Fig. 3 and 6) and this is consistent for all $h < 1$

These results shows that even though Mickens non-standard modelling rules remain a very powerful tool for discrete modelling of the true behaviour of dynamical systems, a carefully chosen interpolation function can produced an exact scheme with same property and also that interpolation schemes can be better if combined with some Nonstandard modelling techniques.

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