On the Bilinear Time Series Models Provided by GARCH White Noise: Estimation and Simulation

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Abstract: This work proposes the estimation of a sample of bilinear time series models mixed by a GARCH white noise, where GARCH model was followed by time varying coefficients, this study allows demonstrating some properties and remarks depending on the behavior of the estimators. Moreover, this work will be validated by a simulations study and digital illustrations using the Matlab software.

Key-Words: Bilinear models, GARCH models, the least squares approach, MLE, time varying coefficients.

1 Introduction

The applications of nonlinear time series models have undergone several developments over the past thirty years, and is not about economics and finance but also other fields like biology, medicine, chemistry and metrology...etc.And among these models we will suffice here to mention only two because they will be used in this paper, the first bilinear time series models, these models have appeared in the statistical literature because their applications of modeling of microeconomics and finance are unlimited. Bilinear models were created by Granger and Andersen 1978 see, and Subba Rao developed some theory of these models in (1981),and the works of Subba Rao and Gabr (1981) give some general discussions on stationary methods of estimation see [19]. Tong (1981) studies the ergodicity of special bilinear models see [20]. Quinn (1982) and Hannan (1982) discuss properties of some cases of these models. In addition, Bibi and Oyet have investigated this model in many worksin references [3, 4]. Where the discussion about one type of time varying bilinear models.

And the second are GARCH models (generalized autoregressive conditional heteroscedasticity) were introduced by Bollerslev (1968) these models are extension for ARCH models Engle (1982). GARCH models have been widely use in financial time series and particularly in analyzing the risks and their applications in forecasting are not limited see[]

The paper will be organized as follows, in section 2 we give some preliminaries for the bilinear models followed by some theory especially, where we amuse the value of Nilsen-Klimko theorem see [18]And necessary conditions of stability, the main goal of this study.In section 3 on estimateone bilinear model with GARCH white noise driven by time varying coefficients with MLE approach.We will finalize the work in Section 4, with numerical illustration and simulations inlaid with graphs for using the Matlab tool, where this simulation allows deducing some remarks and building comments.

2 Definitions and properties

We begin this section with some definitions and preliminaries identified by a little theoretical study

Definition 1. We define bilinear time series model in a space$(\boldsymbol{\Omega}, \boldsymbol{F}, \boldsymbol{P})$by a following stochastic equation

$$x_t = \sum_{i=1}^{p} a_i x_{t-1} + \sum_{j=1}^{q} c_j x_{t-j} + \sum_{i=1}^{r}\sum_{i=1}^{s} b_{ij} x_{t-i}\varepsilon_{t-j} + \varepsilon_t$$

Where is denoted by $BL(p,q,r,s)$, the sequences $(a_i)_{1\le i\le p}, (c_j)_{1\le j\le q}$ and $(b_{ij})_{1\le i\le r, 1\le j\le s}$ are the constants coefficients of the model, $(\varepsilon_t)_{t\in\mathbb{Z}}$ is white noise part not necessarily identically distributed in the general cases, habitually takes mean zero and variance $\sigma^2{}_t$, there are cases where we see that white noise is written as $\varepsilon_t = \eta_t \sigma_t$, such as the quantities σ_t are strictly positive numbers, $(\eta_t)_{t\in\mathbb{Z}}$ are independent and identically distributed random variables. As an example the white noise follow ARCH or GARCH models, we will suggest here in our paper that white noise will follow GARCH model that we will define in the following definition.

Definition 2. The process $(\varepsilon_t)_{t\in\mathbb{Z}}$ is called generalized autoregressive conditionally heteroskedastic with time varying coefficients and denoted by $GARCH\,(\mathrm{p},\mathrm{q})$ each model defined by the following stochastic equation

$$\varepsilon_t = \mathrm{z}_t h_t$$

Such as

$$h_t{}^2 = \gamma_0 + \sum_{j=1}^{q} \beta_{j,t}(\beta)\varepsilon_{t-j} + \sum_{i=1}^{p} \alpha_{i,t}(\alpha)\,\mathrm{h}^2{}_{t-i}$$

Where z_t independent identically distributed random variables with zero mean and variance are equals 1. Theparties $\{\beta_{j,t}(\beta)\}_{1\le j\le q}, \{\alpha_{i,t}(\alpha)\}_{1\le i\le p}$ represent the positive time varying coefficients, where consider in this paper that γ_0 is constant. α and β are vectors of $\mathbb{R}^n$ and $\mathbb{R}^m$ respectively. In this paper, we will project the study on a special case of bilinear time series models oriented by the GARCH white noise with time varying coefficients. And let be $\Im$ $\sigma-$field generated by the set of N observations $\{x_t, \mathrm{t}=1,...,\mathrm{N}\}$ and we will set the vectors $\beta=(\beta_1,\beta_2)$ and $\alpha=(\alpha_1,\alpha_2)$, so the parameter that we will estimate in this the following proposed model will be $\theta=(\mathrm{a},\mathrm{b},\gamma_0,\beta_1,\beta_2,\alpha_1,\alpha_2)$. In this paper, we go to estimate the following bilinear time series model where its white noise will be GARCH (1, 1) and we consider $E(\varepsilon_t^{\ 2})=\sigma_t^2$

$$x_t = ax_{t-s} + bx_{t-s}\varepsilon_{t-1} + \varepsilon_t$$

(1)Where $s\ge 1$ we consider $E(\varepsilon_t^{\ 2})=\sigma_t^2$, the necessary condition for the stability of the type of this model is $a^2+b^2E(\varepsilon_t^{\ 2})<1$, it is quite clear that the condition of stability is equivalent to the condition $|a|+\sigma_t|b|<1$, and by recurrence the model (1) can be written as

$$x_t = \sum_{j=1}^{[t/s]}\left\{\prod_{i=1}^{j-1}\left(a+b\varepsilon_{t-si-1}\right)\right\}\varepsilon_{t-sj}+\varepsilon_t \qquad (2)$$

Where we denote by $[.]$ for the integer value, and we assume that the white noise ε_t follows the law $N(0,h_t^{\ 2})$ so through this assumption it will be $E(\varepsilon^2_{\ t})=E\{E(\mathrm{h}^2_{\ t})\}$ where it produce that

$$E\{E(\mathrm{h}^2_{\ t})\}=E(\mathrm{h}^2_{\ t}) \\ =\gamma_0+\beta_t(\beta)E(\varepsilon^2_{\ t-1})+\alpha_t(\alpha)E(h^2_{\ t-1})$$

And by proposition that $\varepsilon^2_{\ t}=\varepsilon^2_{\ t-1}$ we will find

$$E(\varepsilon^2_{\ t})=\frac{\gamma_0}{1-\beta_t(\beta)-\alpha_t(\alpha)}$$

In this situation the condition of stability of the model will be

$$|a|+|b|\left\{\frac{\gamma_0}{1-\beta_t(\beta)-\alpha_t(\alpha)}\right\}^{0.5}<1$$

Proposition. The model presented by (2) is convergent through its stability condition.

Proof. It is enough to demonstrate that $\rho=E\{|x_t|\}<\infty$. And also it suffices to prove the proposition to take case s = 1, and with the application of Schwartz inequality we will find it results

$$\rho=E\left[\left|\sum_{j=1}^{\infty}\left\{\prod_{i=0}^{j-1}\left(a+b\varepsilon_{t-i-1}\right)\right\}\varepsilon_{t-j}\right|\right] \\ \le\sum_{j=1}^{\infty}\left\{E(\varepsilon_{t-j})^2\right\}^{0.5}\prod_{i=0}^{j-1}\left\{E(a+b\varepsilon_{t-i-1})^2\right\}^{0.5}$$

With evidence that

$$E(a+b\varepsilon_{t-i-1})^2=a^2+b^2E(\varepsilon^2_{\ t-i-1})+2abE(\varepsilon_{t-i-1}) \\ =a^2+b^2\sigma^2_{\ t}$$

We have $\{a^2+b^2E(\varepsilon^2_{\ t-i-1})\}^{0.5}=\delta<1$ and where $\{E(\varepsilon^2_{\ t-j})\}^{0.5}=\sigma_t$, and let be $\max(\sigma_t)=m$, then

$$\rho\le\sum_{j=1}^{\infty}M\prod_{i=0}^{j-1}\delta\le\frac{\delta M}{1-\delta}<\infty.$$

Which finalize the proof.

Among the best estimation approaches, the least square method (LS), this method is an explosion for knowledge and its applications are not finished until today, but its realization in bilinear time series subset with GARCH models is very few in the statistical references but with ARCH we can see Weiss (1986) and Pantulla (1988).

Definition. We define the predictor of a time series and denoted by $h_{t|t-1}(\theta)$, it represents the orthogonal projection of x_t on the observation up the time $t-1$ the difference

$$h_{t|t-1}(\theta)=x_t-\varepsilon_t(\theta)$$

Definition. We say $\hat{\theta}$ an estimator for θ if and only if $\hat{\theta}$ is solution of

$$\arg\min_{\theta\in\Omega} g_N(\theta)$$

Where $g_N(\theta)$ the penalty function defined by expression

$$g_N(\theta)=\frac{1}{N}\sum_{t=1}^{N}\varepsilon_t^2(\theta)$$

Least squares method based on the Taylor's formula where T represents transpose of matrix,then

$$\begin{aligned} g_N(\theta) &= g_N(\theta^0)+(\theta-\theta^0)\frac{\partial g_N(\theta^0)}{\partial\theta} \\ &+\frac{1}{2}(\theta-\theta^0)\frac{\partial^2 g_N(\theta^0)}{\partial\theta\partial\theta^T}(\theta-\theta^0)^T \\ &+\frac{1}{2}(\theta-\theta^0)\frac{\partial^2 g_N(\tilde{\theta})}{\partial\theta\partial\theta^T}(\theta-\theta^0)^T \end{aligned}$$

Where $\tilde{\theta}$ is an intermediate point between θ and θ^0. In sense of norm we have $\left\|\theta-\theta^0\right\|<\nu$, and $\nu>0$. We recall the famous theorem of Klimko and Nilsen on the existence and the necessary conditions of estimators.

Theorem. Let be $\{x_t, t\in\mathbb{Z}\}$ a stable process generated by expression of model (2), such as $h_{t|t-1}(\theta)$ almost surely twice continuously differentiable in an open subset $\Omega\subset\mathbb{R}^7$ and as we can say derivatives of order 1 and 2 for $h_{t|t-1}(\theta)$ are bounded, and which contains the true value θ^0 of the vector θ, and C_1 and C_2 are two positive constants,if the following assumptions are verified for all $i, j\in\{1,...,7\}$

H-1. $E_{\theta^0}\left\{\frac{\partial\varepsilon^2{}_t(\theta)}{\partial\theta_i}\right\}\le C_1, \forall i=1,...,7$.

H-2. $E_{\theta^0}\left[\frac{\partial^2\varepsilon^2{}_t(\theta)}{\partial\theta_i\partial\theta_j}-E_{\theta^0}\left\{\frac{\partial^2\varepsilon^2{}_t(\theta)}{\partial\theta_i\partial\theta_j}\mid F_{t-1}\right\}\right]\le C_2$.

H-3. $\frac{1}{2N}\sum_{t=1}^{N}E_{\theta^0}\left\{\frac{\partial^2\varepsilon^2{}_t(\theta)}{\partial\theta_i\partial\theta_j}\mid F_{t-1}\right\}$ converge as surely for matrix O_{ij} de 7×7 where and this matrix is strictly positive of constants.

H-4. Where $\left\|\theta-\theta^0\right\|<\eta$ and $\varphi_t(\theta)=\frac{\partial^2\varepsilon^2{}_t(\theta)}{\partial\theta_i\partial\theta_j}$ so

$$\lim_{N\to\infty}\sup_{\eta\to 0}(N\eta)^{-1}\sum_{t=1}^{N}\left\{\varphi_t(\theta^0)-\varphi_t(\tilde{\theta})\right\}<\infty.$$

Then, there exists an estimator $\hat{\theta}_N=(\hat{\theta}_{N,1},\hat{\theta}_{N,2},...,\hat{\theta}_{N,7})$ such as $\hat{\theta}_N\to\theta^0$ when $N\to\infty$.

Proof. See[20].

Bibi and Oyet in reference [6] found that the white noise of the model (1) can be written as the form

$$\begin{aligned}\varepsilon_t(\theta) &= x_t-ax_{t-s}-\sum_{j=1}^{t}(-1)^{j-1}b^j\left\{\prod_{i=0}^{j-1}x_{t-i-s}\right\} \\ &\times(x_{t-j}-ax_{t-j-s}) \end{aligned} \quad (3)$$

3. Estimation

The maximum likelihood method. MLE approachhas been applied in some references where the model will be GARCH but in a situation where their coefficients change over time almost there does not exist. The concept of time varying coefficients GARCH takes several dimensions in applications because there are physical phenomena rebelling on the laws of classical physics, where we note that the coefficients are not constants but take a time varying situation. In finance several models take the situation of variation of the coefficients over time,for example the most well-known models in the form of time-varying coefficients there are models with alternative coefficients, we will estimate the coefficients using MLE for the following model , we assume the observations

$\{x_t, t=1,...,N\}$. Starting point is the specification of the conditional density of the residuals ε_t, first we have the normal distribution was assumed, so that the conditional density assumes the form.Let be the model that was studied in the reference [12]

$$\Phi_\theta(\varepsilon_t | F_{t-1}) = \prod_{t=1}^{N} \frac{1}{\sqrt{2\pi y_t(\theta)}} \exp\left\{-\frac{\varepsilon^2_t}{2y_t(\theta)}\right\}$$

ε_t is subject to GARCH(1,1) with time varying coefficients, where $\varepsilon_t = h_t z_t$ such as z_t is Gaussian law with 0 mean and variance 1 and

$$h_t^2 = \gamma_0 + \beta_t(\beta)\varepsilon^2_{t-1} + \alpha_t(\alpha)h^2_{t-1}.$$

We assume here that the coefficients are alternative coefficients where

$$\beta_t(\beta) = \frac{1-(-1)^t}{2}\beta_1 + \frac{1+(-1)^t}{2}\beta_2, t \geq 1.$$

And

$$\beta_t(\alpha) = \frac{1-(-1)^t}{2}\alpha_1 + \frac{1+(-1)^t}{2}\alpha_2, t \geq 1.$$

The construction of $y_t(\theta)$ is given when we set $h^2_t = h^2_{t-1}$, where this construction makes it possible to estimate the coefficients of model (1), so when we put $y_t = h_t^2$ we will arrive

$$y_t = \frac{\gamma_0 + \beta_t(\beta)\varepsilon^2_{t-1}}{1-\alpha_t(\alpha)}$$

For estimating the parameters in model (1), we want to get a solution θ which maximize the logarithm likelihood function $G(\theta) = \ln \Phi_\theta(\varepsilon_t | F_{t-1})$, then

$$G(\theta) = \sum_{t=1}^{N} \underbrace{\left\{-\frac{1}{2}\ln y_t(\theta) - \frac{\varepsilon^2_t(\theta)}{2y_t(\theta)}\right\}}_{\Psi_t(\theta)} - N \ln\sqrt{2\pi}$$

From this situation, we will extract the algorithm that allows calculating the partial derivatives with the coordinates of θ.We will give the derivation and determine the derivatives 1 and 2 of $G(\theta)$ of relative θ, and to illustrate the techniques it suffices to make a derivation of $\Psi_t(\theta)$.

$$\frac{\partial \Psi_t}{\partial \theta_1} = \frac{\partial \Psi_t}{\partial a} = \left\{-\frac{1}{2y_t} + \frac{\varepsilon^2_t}{2y^2_t}\right\}\frac{\partial y_t}{\partial a} - \frac{\partial \varepsilon_t}{\partial a}\frac{\varepsilon_t}{y_t}$$

$$\frac{\partial \Psi_t}{\partial \theta_2} = \frac{\partial \Psi_t}{\partial b} = \left\{-\frac{1}{2y_t} + \frac{\varepsilon^2_t}{2y^2_t}\right\}\frac{\partial y_t}{\partial b} - \frac{\partial \varepsilon_t}{\partial b}\frac{\varepsilon_t}{y_t}$$

$$\frac{\partial \Psi_t(\theta)}{\partial \theta_3} = \frac{\partial \Psi_t(\theta)}{\partial \gamma_0} = -\frac{1}{2y_t}\frac{\partial y_t}{\partial \gamma_0} + \frac{\varepsilon^2_t(\theta)}{2y^2_t}\frac{\partial y_t}{\partial \gamma_0}$$

Where, $\frac{\partial y_t}{\partial \gamma_0} = \frac{1}{1-\alpha_t(\alpha)}$.

$$\frac{\partial \Psi_t(\theta)}{\partial \theta_4} = \frac{\partial \Psi_t(\theta)}{\partial \beta_1} = \frac{\partial y_t}{\partial \beta_1}\left\{-\frac{1}{2y_t} + \frac{\varepsilon^2_t(\theta)}{2y^2_t}\right\}$$

Such as $\frac{\partial y_t}{\partial \beta_1} = \frac{\varepsilon^2_{t-1}(\theta)}{1-\alpha_t(\alpha)}$, the same thing for

$$\frac{\partial \Psi_t(\theta)}{\partial \beta_2} = \frac{\partial y_t}{\partial \beta_2}\left\{-\frac{1}{2y_t} + \frac{\varepsilon^2_t(\theta)}{2y^2_t}\right\}.$$

$$\frac{\partial \Psi_t(\theta)}{\partial \alpha_1} = \frac{\partial y_t}{\partial \alpha_1}\left\{-\frac{1}{2y_t} + \frac{\varepsilon^2_t(\theta)}{2y^2_t}\right\}$$

Where we find $\frac{\partial y_t}{\partial \alpha_1} = \frac{\gamma_0 + \beta_t(\beta)\varepsilon^2_{t-1}}{(1-\alpha_1)^2}$, with the same way $\frac{\partial \Psi_t(\theta)}{\partial \alpha_2} = \frac{\partial y_t}{\partial \alpha_2}\left\{-\frac{1}{2y_t} + \frac{\varepsilon^2_t(\theta)}{2y^2_t}\right\}$.

Using the formula (3) we can calculate the two derivatives $\frac{\partial \varepsilon_t}{\partial a}$ and $\frac{\partial \varepsilon_t}{\partial b}$.

$$\frac{\partial \varepsilon_t(\theta)}{\partial a} = x_{t-s} + \sum_{j=1}^{t}(-1)^{j-1}b^j\left\{\prod_{i=0}^{j-1}x_{t-i-s}\right\}x_{t-j-s}.$$

$$\frac{\partial \varepsilon_t(\theta)}{\partial b} = \sum_{j=1}^{t}(-1)^{j-1}jb^{j-1}\left\{\prod_{i=0}^{j-1}x_{t-i-s}\right\} \times (x_{t-j} - ax_{t-j-s}).$$

We determine now the second derivatives, so we have

$$\frac{\partial^2 \Psi_t}{\partial a \partial a} = \left\{\frac{1}{2y^2_t} - \frac{\varepsilon^2_t}{y^3_t}\right\}\left(\frac{\partial y_t}{\partial a}\right)^2 + \left\{-\frac{1}{2y_t} + \frac{\varepsilon^2_t}{2y_t^2}\right\}\frac{\partial^2 y_t}{\partial a \partial a} + \frac{\partial^2 \varepsilon_t}{\partial a \partial a}\frac{\varepsilon_t}{y_t} + \left\{\frac{\varepsilon^2_t}{y_t^2}\frac{\partial y_t}{\partial a} + \frac{1}{y_t}\frac{\partial \varepsilon_t}{\partial a} - \frac{\varepsilon_t}{y_t^2}\frac{\partial y_t}{\partial a}\right\}\frac{\partial \varepsilon_t}{\partial a}$$

$$\frac{\partial^2\Psi_t}{\partial b\partial b}=\left\{\frac{1}{2y^2_t}-\frac{\varepsilon^2_t}{y^3_t}\right\}\left(\frac{\partial y_t}{\partial b}\right)^2$$
$$+\left\{-\frac{1}{2y_t}+\frac{\varepsilon^2_t}{2y_t^2}\right\}\frac{\partial^2 y_t}{\partial b\partial b}+\frac{\partial^2\varepsilon_t}{\partial b\partial b}\frac{\varepsilon_t}{y_t}$$
$$+\left\{\frac{\varepsilon^2_t}{y_t^2}\frac{\partial y_t}{\partial b}+\frac{1}{y_t}\frac{\partial\varepsilon_t}{\partial b}-\frac{\varepsilon_t}{y_t^2}\frac{\partial y_t}{\partial b}\right\}\frac{\partial\varepsilon_t}{\partial b}$$
$$\frac{\partial^2\Psi_t}{\partial\gamma_0\partial\gamma_0}=\left\{\frac{1}{2y^2_t}-\frac{\varepsilon^2_t(\theta)}{y^3_t}\right\}\left\{\frac{\partial y_t}{\partial\gamma_0}\right\}^2$$
$$=\left\{\frac{1}{2y^2_t}+\frac{\varepsilon^2_t}{y^3_t}\right\}\left\{\frac{1}{1-\alpha_t(\alpha)}\right\}^2$$
$$i,j=1,2:\frac{\partial^2\Psi_t}{\partial\beta_i\partial\beta_j}=\frac{\partial^2 y_t}{\partial\beta_i\partial\beta_j}\left\{-\frac{1}{2y_t}+\frac{\varepsilon^2_t}{2y_t^2}\right\}$$
$$+\left\{\frac{1}{2y_t^2}-\frac{\varepsilon^2_t}{y_t^3}\right\}\frac{\partial y_t}{\partial\beta_j}\frac{\partial y_t}{\partial\beta_i}.$$

We can generalize, if the parameters are a and b that is to say θ_1 and θ_2, so for $i,j\in\{1,2\}$ then it will be

$$\frac{\partial^2\Psi_t}{\partial\theta_i\partial\theta_j}=\left\{\frac{1}{2}\frac{1}{y_t^2}+\left(\frac{\partial y_t}{\partial\theta_j}\right)^{-1}\frac{\varepsilon_t}{y_t^2}\frac{\partial\varepsilon_t}{\partial\theta_j}-\frac{\varepsilon_t^2}{y_t^3}\right\}$$
$$\times\frac{\partial y_t}{\partial\theta_j}\frac{\partial y_t}{\partial\theta_i}+\left\{-\frac{1}{2y_t}+\frac{\varepsilon^2_t}{2y^2_t}\right\}\frac{\partial^2 y_t}{\partial\theta_i\partial\theta_j}$$
$$+\left\{-\frac{1}{y_t}\frac{\partial\varepsilon_t}{\partial\theta_j}+\frac{\varepsilon_t}{y_t^2}\frac{\partial y_t}{\partial\theta_j}\right\}\frac{\partial\varepsilon_t}{\partial\theta_i}+\frac{\varepsilon_t}{y_t}\frac{\partial^2\varepsilon_t}{\partial\theta_i\partial\theta_j}$$

With the same generalization, we give all 49 derivatives for each $i,j\in\{1,2,...,7\}$

Then, it is easy to obtain the partial derivatives of

$$L(\theta)=\left[\frac{\partial G(\theta)}{\partial\theta_i}\right]^T_{i=1,...,7}$$

$\mathrm{O}(\theta)$ is a matrix of second derivatives

$$\mathrm{O}(\theta)=\left[\frac{\partial^2 G(\theta)}{\partial\theta_i\partial\theta_j}\right]_{i,j=1,...,7}$$

The value of this program can also tell the true value θ^0, then we look for the solution of the following equation where we can find the estimated value $\hat{\theta}$, the construction of analgorithm is based to give a better approximation for the estimated value $\hat{\theta}$ through the proposed true value θ^0. This algorithm is known in numerical analysis by the Newton-Raphson approximation method see [**18**], so we assume firstly

$$L(\hat{\theta})=L(\theta)+\mathrm{O}(\theta)(\hat{\theta}-\theta)=0$$

To finding the approximate solution for the solution, we will ask this recursive expression

$$\hat{\theta}=\theta-\mathrm{O}^{-1}(\theta)L(\theta)$$

We apply now Newton-Raphson iterative

$$\theta^1=\theta^0-\mathrm{O}^{-1}(\theta^0)L(\theta^0)$$
$$\theta^2=\theta^1-\mathrm{O}^{-1}(\theta^1)L(\theta^1)$$
$$\vdots$$
$$\theta^m=\theta^{m-1}-\mathrm{O}^{-1}(\theta^{m-1})L(\theta^{m-1})$$

Where the repetition of the iterative ones each time can give a better approximation and then if m tends to infinity then θ^m will converge to the estimated value $\hat{\theta}$.

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4. Numerical illustration

This section is devoted to simulating a particular case of a bilinear model with time varying GARCH model, the generalized formula of this model will be

$$x_t=ax_{t-s}+bx_{t-s}\varepsilon_{t-s}+\varepsilon_t$$

This model is very successful in financial and economic applications, and has been discussed in detail by the reference [4], but using the least squares method but here the simulation will be according to MLE. Where ε_t oriented by GARCH white noise with their time varying coefficients.The tool that we have applied here is Matlab 2013, we will give some model simulations from changing number of simulations (NS) and sample size (N), the true value θ^0 and the estimated value is $\hat{\theta}$ .GARCH white noise coefficients take an alternative situation as we proposed above,we use here in the simulations some concepts like kurtosis (ku), skewness (sk) and estimated variance(var). The simulation presents some properties and remarks illustrated by the following tables where each gives a remark or observation related to the theory of bilinear models of time series.

Table 1

Number of simulation NS=250 + s=1		
Size N	True value $\theta^0 = (a, b, \gamma_0, \beta_1, \beta_2, \alpha_1, \alpha_2)$	Estimated value $\hat{\theta} = (\hat{a}, \hat{b}, \hat{\gamma}_0, \hat{\beta}_1, \hat{\beta}_2, \hat{\alpha}_1, \hat{\alpha}_2)$
100		(0.1704, 0.3702, 0.0536, -0.0037, 0.1990, 0.3281, 0.3299)
250		(0.1755, 0.3634, 0.0625, 0.0010, 0.1978, 0.3225, 0.3430)
500	(0.15 0.45, 0.05 ,0.01,0.2,0,3,0.4)	(0.1769, 0.3788, 0.0698, 0.0065, 0.2004, 0.3275, 0.3571)
1000		(0.1723, 0.3755, 0.0613, 0.0073, 0.1997, 0.3269, 0.3580)
2000		(0.1715, 0.3728, 0.0515, -0.0012, 0.1979, 0.3282, 0.3570)

Table 2

Number of simulation NS=500 + s=1		
Size N	True value $\theta^0 = (a, b, \gamma_0, \beta_1, \beta_2, \alpha_1, \alpha_2)$	Estimated value $\hat{\theta} = (\hat{a}, \hat{b}, \hat{\gamma}_0, \hat{\beta}_1, \hat{\beta}_2, \hat{\alpha}_1, \hat{\alpha}_2)$
100		**(0.1521, 0.3509, 0.0488, -0.0046, 0.1775, 0.2952, 0.3409)**
250		**(0.1675, 0.3631, 0.0532, 0.0011, 0.1841, 0.3202, 0.3522)**
500	(0.15,0.45,0.05,0.01,0.2,0,3,0.4)	**(0.1748, 0.3694, 0.0581, 0.0001, 0.1942, 0.3285, 0.3514)**
1000		**(0.1768, 0.3674, 0.0541, 0.0026, 0.1958, 0.3336, 0.3570)**
2000		**(0.1749, 0.3731, 0.0523, 0 .0017, 0.1977, 0.3330, 0.3577)**

Table 3

Number of simulation NS=1000 + s=1		
Size N	True value $\theta^0 = (a, b, \gamma_0, \beta_1, \beta_2, \alpha_1, \alpha_2)$	Estimated value $\hat{\theta} = (\hat{a}, \hat{b}, \hat{\gamma}_0, \hat{\beta}_1, \hat{\beta}_2, \hat{\alpha}_1, \hat{\alpha}_2)$
100		**(0.1737, 0.3664, 0.0418, -0.0087, 0.1821, 0.3131, 0.3363)**
250		**(0.1695, 0.3587, 0.0543, -0.0102, 0.1908, 0.3188, 0.3423)**
500	(0.15,0.45,0.05,0.01,0.2,0,3,0.4)	**(0.1755, 0.3669, 0.0571, -0.0035, 0.1949, 0.3277, 0.3512)**
1000		**(0.1746, 0.3739, 0.0530, -0.0021, 0.1962, 0.3290, 0.3564)**
2000		**(0.1725, 0.3763, 0.0512, -0.0010, 0.1986, 0.3293, 0.3584)**

Table 4

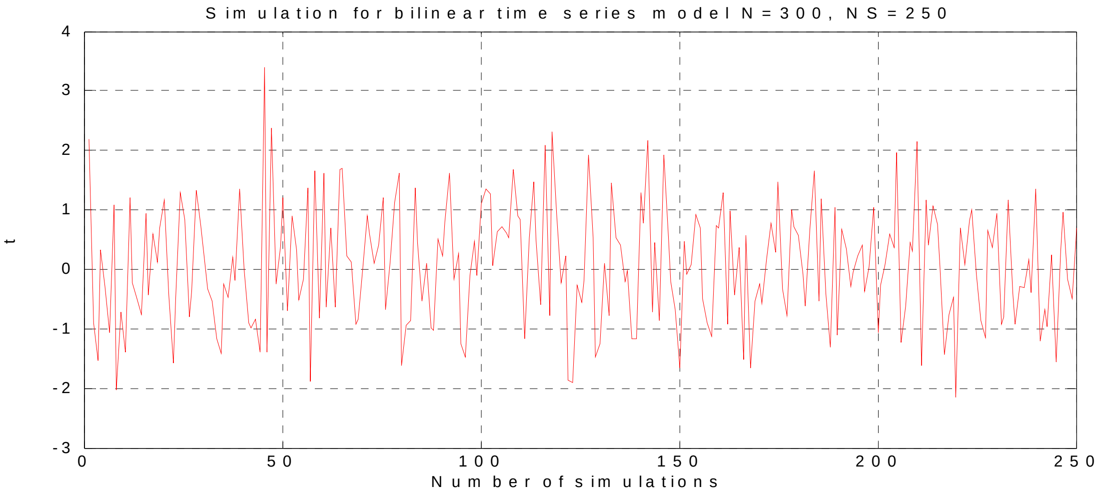
Size N	NS	Model order q	Coefficients (a,b)	Estimators $(\hat{a},\hat{b})$
250	250	s=1	(0.30, 0.02)	**(0.0197, 0.2966)**
		s=2		**(-0.0064, 0.2401)**
		s=3		**(0.0005, 0.3277)**

Table5

N	NS	Coefficients (a,b)	$\mathrm{var}(\hat{a},\hat{b})$	$ku(\hat{a},\hat{b})$	$sk(\hat{a},\hat{b})$
120	250	(0.2,0.58)	(0.0444, 0.0404)	(3.4780, 3.1526)	(0.2555, 0.5649)
480			(0.0133, 0.0139)	(3.2573, 3.2430)	(0.2979,0.3894)
900			(0.0063, 0.0066)	(2.9518, 2.9279)	(0.0730, 0.2837)

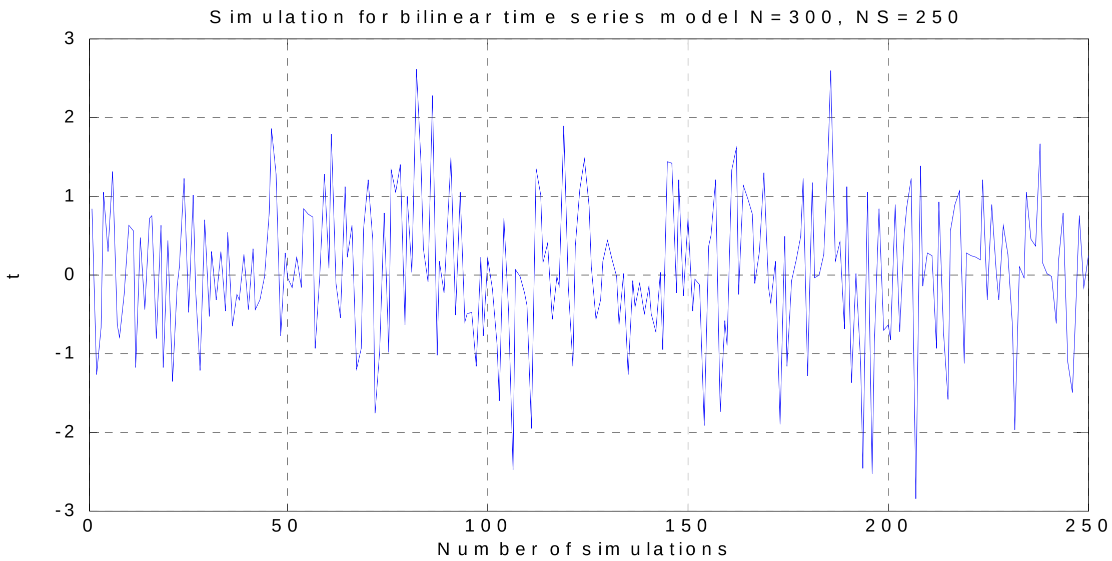
The graphs of model where s=1

Graph 1



Simulation for bilinear model q=1, a=0.2, b=0.45 with white noise GARCH(1,1) with time varying coefficients

Graph 2



Here the model coefficients will be the estimated values $\hat{a}, \hat{b}$ where the true values $a = 0.2$, $b = 0.45$ and white noise GARCH(1,1) with time varying coefficients

Comments and conclusion

Where we are going to simulate the model with the Matlab tool, we observe firstly that the significant criteria of convergence are verified, that is to say when N tends towards infinity then variance will tend to zero, Kurtosis tends towards 3 and Skewness approximate to zero where these results are illustrated in Table 5.On the other hand, if the sample size increased the estimators approximate towards the true values. Generally, also if the number of the simulations large therefore the estimators give a better approximation towards the true values see tables 1, 2 and 3.Moreover, we also observe among the model orders that the best approximation will be given where s = 1, this result is obvious according to the table 4.
There are also disturbances in the simulation because the coefficients of GARCH white noise coefficients take alternative situation. We observe in if the coefficients are very small then we will find that the true values and the estimated values are identical see tables. As a fundamental result the estimators when the model takes a bilinear expression with GARCH white noise, then the estimators will follow the best estimation standards.

The asymptotic behavior of estimators according to MLE with GARCH is very effective in the approximation, where there are cases where the true values of their estimators will be almost identical. When we change the model coefficients by their estimators then the two graphs will almost be identical, they illustrate the asymptotic behavior of the estimators.

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