Taking the limsup above as $t \to \infty$, we obtain $c \leq \rho K d^{-1} c$, that is, a contradiction. Thus, $c = 0$ and the proof is complete.

4 Example

Here, we give a two dimensional example of the n-dimensional model developed in Section 3. Consider the two dimensional volatility model given by

$$\left.\begin{aligned} dS_1(t) &= \frac{-1}{0.25}\mu_1 S_1(t)dt + \mu_2 S_2(t)dt + \\ &\quad S_1(t)\big(\sigma_{11}dw_1(t) + \sigma_{12}dw_2(t)\big) \\ dS_2(t) &= \mu_1 S_1(t)dt - \frac{1}{0.25}\mu_2 S_2(t)dt + \\ &\quad S_2(t)\big(\sigma_{21}dw_1(t) + \sigma_{22}dw_2(t)\big) \end{aligned}\right\} \quad (8)$$

This can be written in matrix form with

$$x = (S_1,\ S_2)^T,\ A(t) = \begin{pmatrix} \frac{-\mu_1}{0.25} & \mu_2 \\ \mu_1 & \frac{-\mu_2}{0.25} \end{pmatrix},$$

$$B_i(\cdot) = \begin{pmatrix} \sigma_{1i} & 0 \\ 0 & \sigma_{2i} \end{pmatrix},\ \ i = 1, 2$$

where the processes $S_1(t),\ S_2(t)$ are correlated with $\sigma_{21} = \sigma_{12} \neq 0$ and

$$B_1(t) = \begin{pmatrix} \sigma_{11} & 0 \\ 0 & \sigma_{21} \end{pmatrix},\ \ B_2(t) = \begin{pmatrix} \sigma_{12} & 0 \\ 0 & \sigma_{22} \end{pmatrix}.$$

So that,

$$dx(t) = A(t)x(t)dt + \sum_{i=1}^{2} B_i\big(t, x(t)\big)dw_i(t) \quad (9)$$

$$x(0) = x_0.$$

4.1. Data Analysis and Result

To illustrate the unstable nature of the above model, we use eighteen years (1997-2014) closing stock price data from the Nigerian exchange market (NSE) extracted from [20]. The data on Table 1 shows a ten year stock price data traded on the floor of the NSE, it shows the initial stock prices, drift, volatility and trading days for 1998-2005, 2009, 2010, and 2013 which is 252, that of 2006, 2007 and 2012 is 251, while 1997, 2008 is 253 and 2014 is 96. This values where calculated using stock returns over a period of one year each by allocating each year its own unit period as given by the number of days for the stock year ([20])

Table 1: Values of initial stock prices, drift, volatility and trading days

Year	Initial stock price (S_0)	Drift μ	Volatility σ	Trading days n
1997	17.50	-0.4908	0.593452	253
1998	66.25	1.7862	0.906322	252
1999	24.80	1.8429	10.615442	252
2000	475.00	1.2322	1.162434	252
2001	28.19	-2.1212	1.147489	252
2002	18.63	0.1112	0.982237	252
2003	17.60	0.1483	0.747571	252
2004	45.40	1.0283	0.412699	252
2005	36.18	0.1204	0.784289	252
2006	40.91	0.0684	0.290624	251
2007	25.61	-0.3906	0.406075	251
2008	23.72	-0.0283	0.369683	253
2009	12.85	-0.3803	0.75789	252
2010	17.10	0.3567	0.42141	252
2011	16.75	0.0156	0.295142	252
2012	16.29	0.0407	0.396402	251
2013	20.08	0.2211	0.20114	252
2014	39.59	0.7039	0.44278	96

To obtain the values of the volatilities in the $B_i, i = 1, 2$ matrix in equation (9), we subdivide the volatility σ and trading days n columns in Table 1 into three groups as shown in Table 2, and compute the mean of each group to the get the $\sigma_{ji}, j = i = 1, 2$ as follows

Table 2: The mean values of volatility

Year	Volatility $\sigma 1$	Trading days $n1$	$\sigma 1 n1$
1997	0.593452	253	150.143356
1998	0.906322	252	228.393144
1999	10.615442	252	2675.091384
2000	1.162434	252	292.933366
2001	1.147489	252	289.167228
2002	0.982237	252	247.523724
Total		1513	3883.252204
Year	Volatility $\sigma 2$	Trading days $n2$	$\sigma 2 n2$
2003	0.747571	252	188.387892
2004	0.412699	252	104.000148
2005	0.784289	252	197.640828
2006	0.290624	251	72.946624
2007	0.406075	251	101.924825
2008	0.369683	253	93.529788
		1511	758.430105
Year	Volatility $\sigma 3$	Trading days $n3$	$\sigma 3 n3$
2009	0.75789	252	190.98828
2010	0.42141	252	106.19532
2011	0.295142	252	74.375784
2012	0.396402	251	99.496902
2013	0.20114	252	50.68728
2014	0.44278	96	42.50688
Total		1355	564.250446

$$\sigma_{11} = \frac{\sum \acute{o}1n1}{\sum n1} = \frac{3883.252204}{1513} = 2.5666,$$

$$\sigma_{21} = \sigma_{12} = \frac{\sum \sigma 2n2}{\sum n2} = \frac{758.430105}{1511} = 0.5019,$$

$$\sigma_{22} = \frac{\sum \sigma 3n3}{\sum n3} = \frac{564.250446}{1355} = 0.4164,$$

$$B_1 = \begin{pmatrix} 2.5666 & 0 \\ 0 & 0.5019 \end{pmatrix},$$

$$B_2 = \begin{pmatrix} 0.5019 & 0 \\ 0 & 0.4164 \end{pmatrix},$$

$$\sum B_i = B_1 + B_2 = \begin{pmatrix} 3.0685 & 0 \\ 0 & 0.9183 \end{pmatrix} \qquad (10)$$

To obtain the values of drift in the A matrix of equation (9), we subdivide the drift μ and trading days n columns in Table 1 into two groups as shown in Table 3, and compute the mean of each group to the get the μ_i, $i = 1, 2$ values as follows

Let

$$\mu_1 = \frac{\sum f\mu_i}{\sum f_i} = \frac{921.1992}{2269} = 0.4060,$$

$$\mu_2 = \frac{\sum f\mu_j}{\sum f_j} = \frac{38.4548}{2110} = 0.0182,$$

Having computed the values of the drift, we now have the matrix A as

$$A(t) = \begin{pmatrix} -1.6240 & 0.0182 \\ 0.4060 & -0.0728 \end{pmatrix}$$

The eigenvalues of matrix A are $\lambda = -0.0681, -1.6288$ and the fundamental matrix solution of equation (9) can be found following the methods in [17] as

$$X(t) = \begin{pmatrix} 0.0182e^{-0.0681t} & 0.0182e^{-1.6288\,t} \\ 1.5559e^{-0.0681t} & -0.0048e^{-1.6288\,t} \end{pmatrix}$$

To show the stability of equation (9), we use the conditions (i) and (ii) of Theorem 1 as follows. By condition (i) of Theorem 1, we get

$$\int_0^t \left\| \begin{pmatrix} 0.0182e^{-14.6843t} & 0.0182e^{-0.6140\,t} \\ 1.5559e^{-14.6843t} & -0.0048e^{-0.6140\,t} \end{pmatrix} \times \begin{pmatrix} 0.0115e^{-14.6843s} & 0.0437e^{-14.6843\,s} \\ 89.2267e^{-0.6140s} & -1.0438e^{-0.6140\,s} \end{pmatrix} \right\| ds,$$

Table 3: Mean values of drift

Year	Drift $\mu 1$	Days $n1$	$\mu 1 n 1$
1997	-0.4908	253	-124.1724
1998	1.7862	252	450.1224
1999	1.8429	252	464.4108
2000	1.2322	252	310.5144
2001	-2.1212	252	-534.5424
2002	0.1112	252	28.0224
2003	0.1483	252	37.3716
2004	1.0283	252	259.1316
2005	0.1204	252	30.3408
Total		2269	921.1992

Year	Drift $\mu 2$	Days $n2$	$\mu 2 n 2$
2006	0.0684	251	17.1684
2007	-0.3906	251	-98.0406
2008	-0.0283	253	-7.1599
2009	-0.3803	252	-95.8356
2010	0.3567	252	89.884
2011	0.0156	252	3.9312
2012	0.0407	251	10.2157
2013	0.2211	252	55.7172
2014	0.7039	96	62.5744
Total		2110	38.4548

It follows by the definition of the Euclidean norm, that

$$\int_0^t \| X(t)X(s)^{-1} \| \, ds = 1.6768 \leq K$$

To obtain condition (ii) of Theorem 1, we know from the calculations of the B_i matrices in equation (10) that

$$\left\| \sum B_i \left(t, x(t)\right) \xi_t \right\| = 3.2030 \leq \rho$$

It is now clear that not all the conditions on Theorem 1 are satisfied; that is $0 \leq \rho \nless k^{-1}$ i.e. $0 \leq 3.2030 \nless 1.6768$ and the system (8) is not asymptotically stable.

5 Discussion

5.1. Effect of Parameters on Return Distribution

There are statistical evidence of stock market data and many economic arguments of stock market which suggests that, the volatility of the stock is a time dependent quantity, which exhibit various random features and that stock return is not normally distributed; they have higher peaks and fatter tails than a normal distribution. This departure from normality is the main shortcoming of the Black-Scholes model. In this section, we analyse the effect of coefficient of correlation (ξ_t and σ on the stability of system (3) considered

5.2. Effect of ξ_t on Return Distribution

The coefficient of correlation ξ_t denotes the sources of randomness for the underlying Weiner process and the volatility. It captures the leverage effect, affecting the size of the tails; the skewness of the return distribution. That is, if $\xi_t < 0$ (ρ will be less than zero), then volatility increases and asset price return decreases, thus, leading to the spread of left tail and squeeze in right tail of the distribution, thus, creating a fat left-tailed distribution. If $\xi_t > 0$, volatility increases with increase in asset price return. This causes the right tail to spread with a squeeze in the left tail of the distribution, thus, creating a fat-tailed distribution. If $\xi_t = 0$, the skewness is close to zero , these effects of ξ_t on the skewness of the distribution are illustrated in

Figures 1-3 using varying values of ρ.

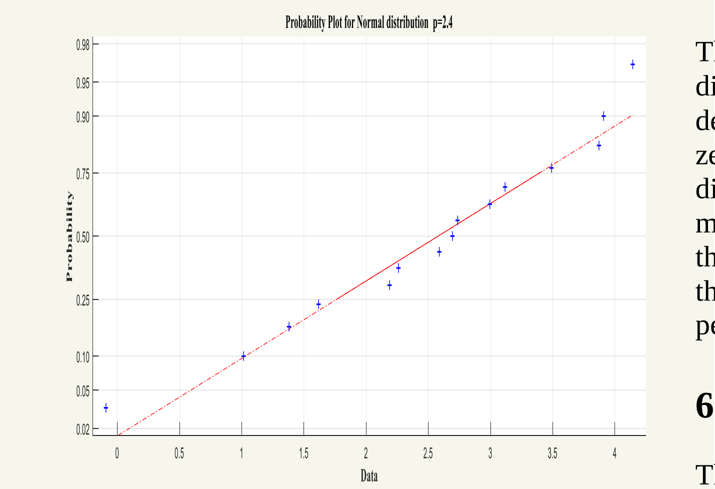

Fig. **1.** Probability plot when $\rho > 0$

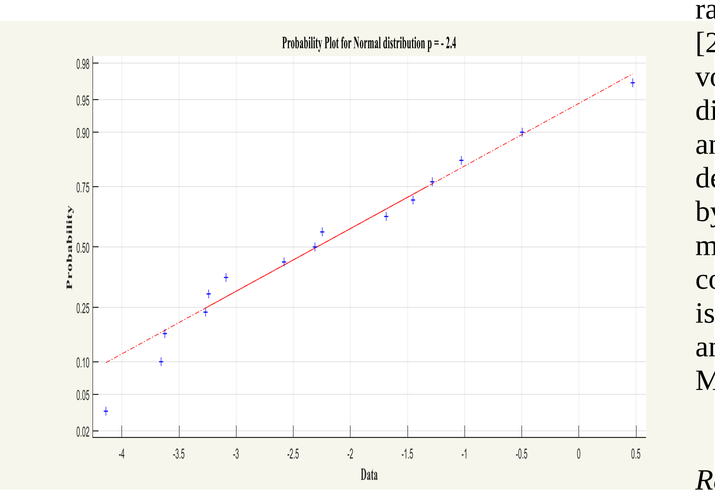

Fig. **2.** Probability plot when $\rho < 0$

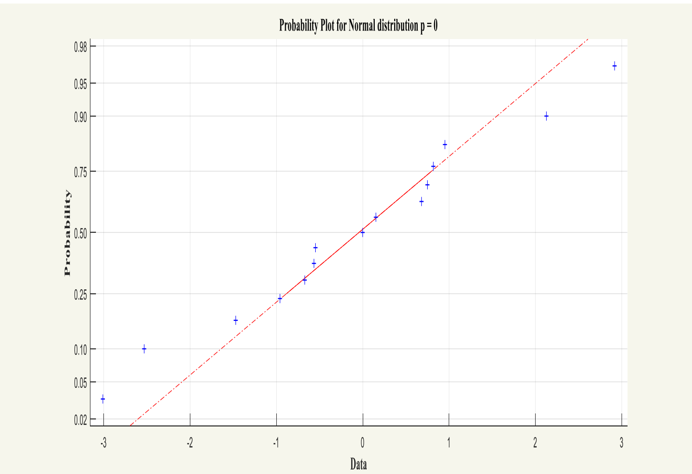

Fig. **3.** Probability plot when $\rho = 0$

5.2. Effect of σ on Return Distribution

The effect of σ is mainly on the peak of the distribution. When $\sigma = 0$, the volatility becomes deterministic because the processes $\sum B_i(\cdot)$ will be zero and ρ also zero. This will lead to a normal distribution of stock returns as in the Black Scholes model. Theoretically, increase in σ also increases the peak, creating fatty tails on both sides implying that, increase in market volatility σ gave higher peak of the distribution and vice versa.

6 Conclusion

The statistical analysis as described in [20] shows that volatility of stock is a time dependent quantity and also exhibits various random features. The randomness of stochastic volatility is addressed in [21] by assuming that both the stock price and the volatility are stochastic processes affected by the different sources of risk. In this work stability analysis on stock market forces were obtained by developing stochastic vector differential equation; by exploring the properties of the fundamental matrix solution of this equation and by placing continuity condition on the stochastic part. Example is given to illustrate the effectiveness of the model and simulation results presented graphically using MATLAB.

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