Inventory Policy Selection Using Intuitionistic Fuzzy LINMAP model

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Abstract: Inventory constitutes the part and parcel of every business domain and therefore its control is one of the key areas for driving optimization initiatives in an organization. Different inventory control policies have received attention in recent times as they affect the overall cost, quality and service of organizations which faces threat from the dynamic business environment. Inventory policies are characterized on the basis of various attributes which may not be precise and lack proper information. Multiattribute decision making problems are often confronted with the innate problem of selection, evaluation or ranking of alternatives that are typically characterized by multiple and conflicting attributes. This paper is an attempt to develop a new methodology for solving multi-attribute inventory control policies using the concept of intuitionistic fuzzy sets (IFS).The theory of IFS provides a structure to deal with information of the real world which lack clarity and are imperfect and/or imprecise. This very concept can be seen as an alternative option to describe a fuzzy set in situations when the existing information is not enough to define a usual fuzzy set. The technique involves in developing a fuzzy linear programming for multidimensional analysis of preference (LINMAP) under intuitionistic fuzzy (IF) environment that reflects the relative preference of the factors that the decision maker adheres to. The DM's preferences are arranged through pair-wise comparisons of alternatives and the one that has the shortest distance to the positive ideal solution (PIS) is considered the best solution.

Key-Words : inventory policies, multi attribute decision making, LINMAP

1.Introduction

: The complex socio economic environment today has forced organizations to look for an edge that can make them successful. Inventory represents an arena that requires significant capital investment and blocks up the money that could have alternative use elsewhere. Therefore this prepares the ground for effective inventory management as it would offer the potential for significant cost savings. Looking at the complexities & uncertainties involved in industries, it is necessary to handle inventory control very carefully considering the various consequential effects on overall basis. Thus the inventory control personnel have to select an appropriate inventory policy which is very important in the current scenario of intense global competition and dynamic nature of industry. The selection process includes selecting and evaluation of right inventory policies, rating inventory policy performance, determining the optimum lead time, review period and reorder point, sourcing goods and service, timing purchases, selling terms of sale, etc. An extensive literature review reveals the proposals of quite a few inventory models over the past 30 years to estimate the optimal inventory level. Among them, three inventory policies viz. Economic order quantity EOQ, Just in time (JIT) and Vendor managed inventory (VMI) are considered in this paper. All three methods offer ways of reducing inventory costs. Had there been a single criterion in the decision making process, selection of the best inventory policy would be simple but in real situations, the purchasers have to face a different situation where they have to consider a number of criteria. This converts ranking and selection of inventory policies as a multi attribute decision making problem where the top priorities for selecting the best inventory policy are identified based on type of industry and its own capabilities. The best approach usually depends on the nature of the organization as well as the nature of the inventory itself. Therefore it becomes important to decide on how each of the criteria influence the decision making process - whether all criteria have equal importance or whether the influence varies according to the type of criteria. Thus, while formulating inventory policies a combination of some of selection criteria viz. the perish ability of the goods , the demand pattern , the length of the product or order cycle, carrying costs, capital necessities, the risks due to possible shortages/ price increases /price reduction/technological obsolescence/change in tastes/theft etc are considered. The present work is based on the influence of four selection criteria namely 'ordering cost', holding cost, shortage cost and 'demand' which are conflicting in nature and influence the decision making process of inventory policy selection .A typical MADM problem revolves around the problem of selecting, evaluation or ranking alternatives that are characterized by multiple, usually conflicting, attributes [8]. Srinivasan and Shocker [19] developed the classical Linear Programming Technique for Multidimensional Analysis of Preference (LINMAP) and has proved to be an effective and a simple method for solving multiattribute decision making (MADM) problems. The attributes being in exact are generally obtained analytically or by simulation technique or they are just linguistic subjective judgments defined by fuzzy sets. . LINMAP methodology ensures that all the decision data are known with precision or given as crisp values; which may not be true while depicting real time data.These may be vague or fuzzy in nature and as such it may not be appropriate to represent them by accurate numerical values [10]. Among the most popular theories to handle uncertainty include the fuzzy set theory introduced by Zadeh [24] in 1965 and it has achieved great success over the years. Recently, a lot of literature (5-7,9, 1718,20,22) investigate on MADM problems using the theory of the fuzzy set. The fuzzy set theory assigns a membership function to each element x in a universe of discourse - a membership degree ranging between zero and one and the non-membership degree equals one minus the membership degree, i.e., this membership degree combines the evidence for x and the evidence against x. In practice however the sum of the membership degree and the non membership degree of element in the universe corresponding to the fuzzy concept may be less than one. Hence the fuzzy set theory is not capable to incorporate the lack of knowledge with membership degrees; while Atanassov's intuitionistic fuzzy sets (A-IFSs) [1-3] can handle it by using an additional degree called hesitation factor which is required to construct really adequate models. Li and Yang [11] used linguistic variables to assess alternatives on qualitative attributes where these were transformed into positive triangular fuzzy numbers (TFNs) and thereby the LINMAP was developed for multi attribute group decision making (MAGDM).ElifAlaybeyoğlu and Y. Esra Albayrak [4] wanted to evaluate the pricing strategy and used the LINMAP under IFS to select the best. Deng Feng Li [13] used the (LINMAP) to develop a new methodology for solving multiattribute decision making (MADM) problems under Atanassov's intuitionistic fuzzy (IF) environments. Shu-Ping Wan and Deng-Feng Li[21]developed a new Atanassov's intuitionistic fuzzy (A-IF) programming method to solve heterogeneous multiattribute group decision-making problems with A-IF truth degrees in which there are several types of attribute values such as A-IF sets (A-IFSs), trapezoidal fuzzy numbers, intervals, and real numbers. Therefore, extending the LINMAP to suit the fuzzy or IF environments is of a great importance for scientific researches and real applications [12, 14]. Xia et. al. [23]also tried to capture fuzziness in decision information and processes by means of a fuzzy decision matrix. He used linguistic variables which were represented by trapezoidal fuzzy numbers (TrFNs).Similarly Li and Sun [16] transformed linguistic variables into TFNs and extended the LINMAP for solving MAGDM problems. Using IFSs to express the attribute values, Li [15]and Li et. al. [21] respectively used the LINMAP for MADM and MAGDM under IF environment.

Therefore, this paper is an attempt to extend the application of LINMAP in ranking inventory policies using intuitionistic fuzzy sets. Here intuitionistic fuzzy sets are used to illustrate the fuzziness in decision information and decision making process by means of intuitionistic fuzzy decision matrices. The weights of the criterions is not completely certain, and the criteria values of alternatives are Atanassov's intuitionistic fuzzy sets (A-IFSs).The LINMAP under IFSs is used to describe the DM's preferences given through pair-wise comparisons with hesitancy degrees. A new auxiliary linear programming model is framed for estimation of IF positive ideal solution (IFPIS) [10]and weights of attributes .The ranking of the inventory policies are done based on the distance of alternatives to the IF positive ideal solution .

The rest of the paper is organized as follows. In section 2, the concept of the Atanassov's IF set and a Euclidean distance between Atanassov's IF sets are introduced .Section 3 defines consistency and inconsistency indices and a linear programming model to solve a multiattribute decision making problem. The developed methodology is also illustrated with a real life example in Section 4. The concluding remark is given in Section 5

2. Basic concepts and definition

2.1 *Basic definition of Atanassov's IF set[1986,1999]*

Let X={$x_1, x_2, x_3 \ldots x_n$} be a finite universal set. An Atanassov's IF set Ain X isan object having the following form:

A= {($\mu_A(x), \nu_A(x)$)/x$\in$X} where the functions

$\mu_A : X \to [0, 1]$, $x_k \in X \to \mu_A(x_k) \in [0, 1]$

$\upsilon_A : X \to [0, 1]$, $x_k \in X \to \upsilon_A(x_k) \in [0, 1]$

define the degree of membership and degree of non-membership of the element $x_k \in X$ to the set $A \subseteq X$, respectively, and for every $x_k \in X$,

$0 \le \mu_A(x_k) + \upsilon_A(x_k) \le 1$.

Let $\pi_A(x_k) = 1 - \mu_A(x_k) - \upsilon_A(x_k)$ be thedegree of indeterminacy membership of the element $x_k$to the set A. It is called the Atanassov's IF index of the element $x_k$in the set A. Obviously, $0 \le \pi_A(x_k) \le 1$

2.2 ***Distance between Atanassov's IF sets***

Let $A = \{x_l, \mu_A(x_l), \upsilon_A(x_l) \mid x_l \in X\}$ and $B = \{x_l, \mu_B(x_l), \upsilon_B(x_l) \mid x_l \in X\}$ be twoAtanassov's IF sets in the set X where $\pi_A(x_l) = 1 - \mu_A(x_l) - \upsilon_A(x_l)$ *and* $\pi_B(x_l) = 1 - \mu_B(x_l) - \upsilon_B(x_l)$

An Euclidean distance between A and B is defined as follows

$$d(A,B) = \sqrt{\frac{1}{2}\Sigma[(\mu_A(x_1) - \mu_B(x_1))^2 + (\nu_A(x_1) - \nu_B(x_1))^2 + (\Pi_A(x_1) - \Pi_A(x_1))^2]} \ldots\ldots(1)$$

2.3 ***Multi attribute decision making problem under Atanassov's IF environment***

Suppose we havea set $A = \{A_1, A_2, \ldots, A_n\}$, j=1,2,…ndenoted as the set of alternatives from which the manager has to select the most preferred alternative on the basis of m attributes which may be both quantitative and qualitative.

The set of attributes is denoted by $X = \{x_1, x_2, \ldots, x_m\}$,i=1,2….m.

The ratings of alternatives on qualitative attributes are given using Atanassov's IF matrix $M_{ij}=\{\mu_{ij}, \upsilon_{ij}\}$, where $\mu_{ij} \in [0, 1]$ and $\upsilon_{ij} \in [0, 1]$ are the degree of satisfaction (or membership) and degree of non-satisfaction (non-membership) of $Aj \in A$ on qualitative attribute $x_i \in X$ and $0 \le \mu_{ij} + \upsilon_{ij} \le 1$.

Thus

$$M = ((\mu_{ij}, \nu_{ij}))_{mxn} = \begin{bmatrix} (\mu_{11}, \nu_{11}) & (\mu_{12}, \nu_{12}) & \ldots & (\mu_{1n}, \nu_{1n}) \\ (\mu_{21}, \nu_{21}) & (\mu_{22}, \nu_{22}) & \ldots & (\mu_{2n}, \nu_{2n}) \\ \ldots & \ldots & \ldots & \ldots \\ (\mu_{m1}, \nu_{m1}) & (\mu_{m2}, \nu_{m2}) & \ldots & (\mu_{mn}, \nu_{mn}) \end{bmatrix} \ldots(2)$$

Let a_{ij} be a value of alternative $A_j \in A$ on quantitative attribute $x_i \in X$ which needs to be normalized. Hence the formulae for relative degrees of membership and relative degrees of non-membership are used as defined by Li[10]

$$\mu_{ij} = \begin{cases} \alpha_i \dfrac{a_{ij}}{a_i^{\max}} \left(i \in F^1\right) \\ \delta_i \dfrac{a_i^{\min}}{a_{ij}} \left(i \in F^2\right) \end{cases} \ldots\ldots(3)$$

And $$\nu_{ij} = \begin{cases} \beta_i \dfrac{a_{ij}}{a_i^{\max}} \left(i \in F^1\right) \\ \gamma_i \dfrac{a_i^{\min}}{a_{ij}} \left(i \in F^2\right) \end{cases} \quad (4)$$

respectively, whereF^1 and F^2 are the set of attributes.

$a_i^{\max} = \max_{1\le j\le n}\{a_{ij}\}$, $a_i^{\min} = \min_{1\le j\le n}\{a_{ij}\}$

And $\alpha_i \in [0, 1]$, $\beta_i \in [0, 1]$, $\delta_i \in [0, 1]$ and $\gamma_i \in [0, 1]$ satisfying conditions $0 \le \alpha i + \beta i \le 1$ and $0 \le \delta i + \gamma i \le 1$. Obviously,

$$0 \le \mu_{ij} + \nu_{ij} = (\alpha_i + \beta_i)\frac{a_{ij}}{a_i^{\max}} \le 1 \quad (i \in F^1)$$

$$0 \le \mu_{ij} + \nu_{ij} = (\delta_i + \gamma_i)\frac{a_i^{\min}}{a_{ij}} \le 1 \quad (i \in F^2)$$

It is further assumed that each attribute $x_i \in X$ is has weight ω_i, denoted by the vector $\boldsymbol{\omega} = (\omega_1, \omega_2,..,\omega_m)^T$, which is unknown a priori $, 0 \le \omega_i \le 1$ and $\sum_{i=1}^{n} \omega_i = 1$

3. Intuitionistic Fuzzy LINMAP model and method

Consistency and Inconsistency Measurements

We have Atanassov's IF positive ideal solution (IFPIS) represented by an IF set as

$$M^+ = \left(\langle \mu_1^+, \nu_1^+ \rangle, \langle \mu_2^+, \nu_2^+ \rangle, ..., \langle \mu_m^+, \nu_m^+ \rangle\right)$$

Using (1), the square of the weighted Euclidean distance between the alternative A_i and the IFPIS M^+ can be calculated as follows:

$$S_j = \sum_{i=1}^{m} w_i (d(M_{ij}, M_i^+)^2 \quad ...(5)$$

Thus S_j(j=1,2, 3….n) can be explicitly rewritten as follows:

$$S_j = \frac{1}{2}\sum_{i=1}^{n} w_i[(\mu_{ij} - \mu_i^+)^2 + (\nu_{ij} - \nu_i^+)^2 + (\Pi_{ij} - \Pi_i^+)^2] \quad ....(6)$$

where $\pi_{ij}=1-\mu_{ij}-\nu_{ij}$ (i=1,2…m; j=1.2.3…n) and $\pi_i^+=1-\mu_i^+-\nu_i^+$ ....(7)

Let $\Omega=\{(k, j) \mid A_k \ge A_j, (k, j = 1, 2, . . . , n)\}$ represent the set of preference relation between alternatives. Using Eq. (5) the square of the weighted Euclidean distance between each pair of alternative $(k, j) \in \Omega$and the Atanassov's IFPIS M^+ which can be calculated as follows:

$$S_k = \sum_{i=1}^{m} w_i (d(M_{ik}, M_i^+))^2$$

$$S_j = \sum_{i=1}^{m} w_i (d(M_{ij}, M_i^+))^2$$

For each pair of alternatives $(k, j) \in \Omega$,if $S_j > S_k$ the alternative $A_k$is closer to the IFPIS than the alternative A_j. An index$(S_j - S_k)^-$is defined to measure inconsistency between the ranking order of alternatives A_k andA_j determined by S_j and S_k and the preferences given by the decision maker preferring A_k to A_j as follows

$$(S_j - S_k)^- = \begin{cases} S_k - S_j & (S_j < S_k) \\ 0 & (S_j \ge S_k) \end{cases} \quad (8)$$

Thus, the inconsistency index can be rewritten as

$$(S_j - S_k)^- = \max\{0, S_k - S_j\}$$

And a total inconsistency index of the decision maker is defined as

$$P = \sum_{(k,j)\in\Omega} (S_j - S_k)^- = \sum_{(k,j)\in\Omega} \max\{0, S_j - S_k\} \quad ..... (9)$$

In a similar way, an index $(Sj - Sk)^+$ is used to measure consistency between the ranking order of alternatives A_k and $A_j$and the preferences given by the decision maker preferring $A_k$to A_j which can be defined as follows:

$$(S_j - S_k)^+ = \begin{cases} S_j - S_k & (S_j \ge S_k) \\ 0 & (S_j < S_k) \end{cases} \quad (10)$$

Hence, a total consistency index of the decision maker is defined as

$$O = \sum_{(k,j)\in\Omega} (S_j - S_k)^+ = \sum_{(k,j)\in\Omega} \max\{0, S_j - S_k\} \quad …….. \quad (11)$$

3.1 LINMAP Model under intuitionistic environment

Linear programming is an important type of mathematical programming or optimization problems. To determine $(\boldsymbol{\omega}, M^+)$, the following mathematical programming is constructed as follows:

Max $\{O\}$

$$\text{s.t}\begin{cases} \mathrm{O - P \geq h} \\ \sum_{i=1}^{m} \omega_{\mathrm{i}} = 1 \\ \omega_{\mathrm{i}} \geq \varepsilon \quad (i = 1,2,\ldots\ldots,m), \end{cases} \tag{12}$$

For each pair of the alternatives $(k, j) \in \Omega$, let $\lambda_{kj} = \max\{0, S_j - S_k\}$, then, it easily follows $\lambda_{kj} \geq 0$ and $\lambda_{kj} \geq S_j - S_k((k, j) \in \Omega)$.

Thus Eq. (12) can be converted into the linear programming model as follows:

$$\max\left\{\sum_{(\mathrm{k,j})\in\Omega} \lambda_{kj}\right\}$$

$$\sum_{i=1}^{m}\omega_i \sum_{(\mathrm{k,j})\in\Omega}\left[\left(\mu_{ij}^2-\mu_{ik}^2\right)+\left(\upsilon_{ij}^2-\upsilon_{ik}^2\right)+\left(\pi_{ij}^2-\pi_{ik}^2\right)+2\left(\pi_{ij}-\pi_{ik}\right)+2\left(\upsilon_{ij}-\upsilon_{ik}\right)\right]$$
$$-\sum_{i=1}^{m}u_i\sum_{(\mathrm{k,j})\in\Omega}\left[4\left(\mu_{ij}-\mu_{ik}\right)+2\left(\upsilon_{ij}-\upsilon_{ik}\right)\right]-\sum_{i=1}^{m}\nu_i\sum_{(\mathrm{k,j})\in\Omega}\left[2\left(\mu_{ij}-\mu_{ik}\right)+4\left(\upsilon_{ij}-\upsilon_{ik}\right)\right]$$
$$\geq 2h$$

$$\sum_{i=1}^{m}\omega_i \sum_{(\mathrm{k,j})\in\Omega}\left[\left(\mu_{ik}^2-\mu_{ij}^2\right)+\left(\upsilon_{ik}^2-\upsilon_{ij}^2\right)+\left(\pi_{ik}^2-\pi_{ij}^2\right)+2\left(\pi_{ik}-\pi_{ij}\right)+2\left(\upsilon_{ik}-\upsilon_{ij}\right)\right]$$
$$-\sum_{i=1}^{m}u_i\sum_{(\mathrm{k,j})\in\Omega}\left[4\left(\mu_{ik}-\mu_{ij}\right)+2\left(\upsilon_{ik}-\upsilon_{ij}\right)\right]-\sum_{i=1((\mathrm{k,j})\in\Omega)}^{m}\nu_i\left[2\left(\mu_{ik}-\mu_{ij}\right)+4\left(\upsilon_{ik}-\upsilon_{ij}\right)\right]+2\lambda_{kj}\geq 0$$

$$((\mathrm{k,j})\in\Omega)$$
$$\lambda_{kj}\geq 0$$
$$u_{\mathrm{i}}\geq 0, v_i\geq 0 \quad (i=1,2,\ldots\ldots,m)$$
$$u_{\mathrm{i}}+v_i\leq\omega_{\mathrm{i}} \quad (i=1,2,\ldots\ldots,m)$$
$$\omega_{\mathrm{i}}\geq\varepsilon \quad (i=1,2,\ldots\ldots,m)$$
$$\sum_{i=1}^{m}\omega_{\mathrm{i}}=1$$

$$\text{where}\quad\begin{Bmatrix} u_i=\omega_i\mu_i^{+} & (i=1,2,\ldots\ldots,m) \\ v_i=\omega_i v_i^{+} & (i=1,2,\ldots\ldots,m)\end{Bmatrix} \tag{13}$$

When the problem is solved, the best values of μ_i^+ and v_i^+ are calculated using.(11)

3.2 Decision Process of Intuitionistic fuzzy LINMAP

Here, an intuitionistic fuzzy LINMAP method is proposed where the weight vector and the IFPIS is constructed. An algorithm and decision process of the multi attribute decision making with intuitionistic fuzzy set approach is given in the following.

Step 1: In the beginning attributes and alternatives are identified.

Step 2: The preference relation between alternatives is given by $\Omega = \{(k, j)|A_k \geq A_j, (k, j = 1, 2, \ldots, n)\}$

Step 3: The rating of alternatives $A_j \in A$ $(j = 1, 2, \ldots, n)$ on the qualitative attributes $x_i \in X_1$, is expressed with appropriate intuitionistic fuzzy sets;

Step 4: Compute a_{ij} of the alternatives $A_j \in A$ $(j = 1, 2, \ldots, n)$ on the quantitative attributes $x_i \in X_2$ using normalized formulae (3).

Step 5: The intuitionistic fuzzy decision matrix **M** = $(M_{ij})_{m\times n}$ is constructed.

Step 6: The linear programming model is obtained according to Eq. (13);

Step 7: The linear programming model is solved using the simplex method;

Step8:Now the weights ω_i, $\mathrm{M}_{\mathrm{i}}^{+} = \left\langle \mu_i^{+}, \upsilon_i^{+} \right\rangle$ $(i = 1,2,\ldots\ldots,m)$, weight vector $\boldsymbol{\omega}$= $(\omega_1,\omega_2,\ldots\ldots,\omega_m)^T$ and intuitionistic fuzzy positive ideal- solution $\mathrm{M}^{+} = \left(\mathrm{M}_1^{+}, \mathrm{M}_2^{+},\ldots\ldots, \mathrm{M}_{\mathrm{m}}^{+}\right)^{\mathrm{T}}$ are obtained.

Step 9: Also the distance S_j of each alternatives $A_j \in A$ $(j = 1, 2, \ldots, n)$ and intuitionistic fuzzy positive ideal- solution $\mathbf{M}^+$ are constructed according to Eq.(6)

Step 10: The best alternative from the alternative set A is determined as per the increasing order of the distances S_j $(j = 1, 2, \ldots, n)$, and the alternatives are ranked accordingly.

4. Numerical Example

Simdega Automotive Pvt. Ltd. has been established in 1988. It deals with the manufacture of automotive components that are primarily used for two and four wheelers. Traditionally this company relied on a cartel of suppliers as it had no other option. But things started getting better with liberalization in India and Simdega Automative Pvt. Ltd. started achieving better operating ratios. However the company's inventory management still presented tremendous loopholes as it was largely based on the preference given to discount prices for the inventory purchase. Over a period of time, the company realized that it's losing out to its competitors because of the inability to forecast demand as well as its rising inventory costs. Therefore the operations manager strongly believed that the company should review and refocus its existing inventory policy and in the process needed to streamline the company's inventory management system. Taking this aspect into consideration the operations manager was indecisive of which inventory policy should he chose? He wanted to determine the ranking order for the inventory policies based on criterions like cost, demand etc.

Solution:

We consider three inventory policies viz. EOQ, Heuristics and VMI. The ordering cost, holding cost and shortage cost are the quantitative attributes and demand is assumed to be the qualitative attribute whose ratings are expressed in terms of linguistic variables. This is represented in Table 1. Further the relations between linguistic terms and intuitionistic fuzzy sets are given in Table 2.

Table 1: Table representing attributes

Attribute values		Inventory Policies		
		A_1(Heuristics)	A_2(VMI)	A_3(EOQ)
Attributes	Ordering Cost	3.1	4.3	2.7
	Holding Cost	4.1	3.5	3.2
	Shortage Cost	2.8	2.6	2.9
	Demand	Medium	Low	High

Table 2 Relations between linguistic terms and intuitionistic fuzzy sets

Terms of linguistic variables	Intuitionistic fuzzy sets
Very high(VH)	(0.05,0.95)
High(H)	(0.25,0.70)
Medium (M)	(0.40,0.50)
Low(L)	(0.70, 0.25)
Very low(VL)	(0.95,0.05)

Using own evaluation criteria and judgement, the decision maker's preferences between the inventory policies are obtained as follows:

$$\Omega=\{(1,2),(3,2),(1,3)\} \quad (14)$$

According to Tables (1) and (2), the decision matrix is obtained as follows:

$$F = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \\ x_4 \end{array} \begin{array}{c} \begin{array}{ccc} A_1 & A_2 & A_3 \end{array} \\ \begin{pmatrix} 3.1 & 4.3 & 2.7 \\ 4.1 & 3.5 & 3.2 \\ 2.8 & 2.6 & 2.9 \\ (0.4,0.5) & (0.25,0.7) & (0.7,0.25) \end{pmatrix} \end{array} \quad (15)$$

For the attribute cost i.e. x_1, it is seen from the decision matrix that $a_1^{max}=4.3$. By comparison and judgement, we take $\alpha_1=0.8$ and $\beta_1=0.1$. Also using equation (3) and (4) we obtain the membership degrees and non- membership degrees of the alternatives $A_j(j=1,2,3)$ on the quantitative attribute cost as follows:

$$\mu_{11} = \alpha_1 \frac{a_{11}}{a_1^{\max}} = 0.8\frac{3.1}{4.3} = 0.576$$

$$\mu_{12} = 0.8\frac{4.3}{4.3} = 0.8$$

$$\mu_{13} = 0.8\frac{2.7}{4.3} = 0.50$$

$$\nu_{11} = 0.1\frac{3.1}{4.3} = 0.07$$

$$\nu_{12} = 0.1\frac{4.3}{4.3} = 0.1$$

$$\nu_{13} = 0.1\frac{2.7}{4.3} = 0.062$$

Now, a_{ij} of the alternatives A_j (j=1,2,3) on the quantitative attribute x_1 can be transformed into the intuitionistic fuzzy sets as follows:
$M_{11}=(\mu_{11},\nu_{11}) =(0.576, 0.07)$
$M_{12}=(\mu_{12},\nu_{12}) =(0.8, 0.1)$ and $M_{13}=(0.50,0.062)$ respectively.
Similarly, for the other attributes i.e. the holding cost and shortage cost of the quantitative attributes can be transformed into the intuitionistic fuzzy sets as follows with $\alpha_2=0.9$ and $\alpha_3=0.87$

$M_{21}=(0.9,0.6)$ $M_{31}=(0.84,0.096)$
$M_{22}=(0.768,0.05)$ $M_{32}=(0.78,0.089)$
$M_{23}=(0.70,0.046)$ $M_{33}=(0.87,0.1)$

Thus the decision matrix M given by (15) can be uniformly transformed into the intuitionistic fuzzy decision matrix as follows:

$$F = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \\ x_4 \end{array} \begin{array}{c} \begin{array}{ccc} A_1 & A_2 & A_3 \end{array} \\ \begin{pmatrix} (0.576,0.07) & (0.8,0.1) & (0.50,0.062) \\ (0.9,0.6) & (0.768,0.05) & (0.70,0.046) \\ (0.84,0.096) & (0.78,0.089) & (0.87,0.1) \\ (0.4,0.5) & (0.25,0.7) & (0.7,0.25) \end{pmatrix} \end{array} \quad (16)$$

According to equation (13) we can construct the LPP model as follows:
Max $\{\lambda_{12}+ \lambda_{32}+ \lambda_{13}\}$

Subject to

$0.41w_1-0.25w_2-0.46w_3+0.72w_4+1.95u_1-1.08u_2-0.508u_3-0.4u_4+1.14v_1-0.6 v_2 -0.30 v_3+ v_4 \geq 2h$

$-0.39w_1+0.46w_2+0.257w_3-0.235w_4-0.956u_1+0.54u_2+0.254u_3+0.2u_4-0.568 v_1+0.3 v_2+0.148 v_3-0.5 v_4+ \lambda_{12}\geq 0$

$-2.693w_1-0.2128w_2+0.3363w_3-1.28u_1-0.28u_2+0.382u_3+0.9u_4-4.36 v_1-0.14 v_2+0.224 v_3-0.9 v_4+ \lambda_{32}\geq 0$

$0.84679w_1+0.6777w_2-0.11689w_3-1.235w_4+0.324u_1+0.82u_2-0.128u_3-0.7u_4+0.192 \Upsilon_1+0.44 \Upsilon_2-0.076\Upsilon_3+0.4 \Upsilon_4+ \lambda_{13}\geq 0$

$u_i+v_i\leq w_i (i=1,2,3,4)$

$w_i\geq \acute{\varepsilon}$ (i=1,2,3,4)

$w_1+w_2+w_3+w_4=1$

$\lambda_{12}\geq 0, \lambda_{32}\geq 0$, + $\lambda_{13}\geq 0$

$u_i\geq 0 ,v_i\geq 0$ (17)

We consider h=0.5 and $\acute{\varepsilon}$=0.005 and use simplex method of LPP to obtain the optimal solution of the above problem whose important components are :

$W=(w_1,w_2,w_3,w_4)^T= (0.61,0.01,0.05,0.33)$ (18)

$U= (u_1,u_2,u_3,u_4)^T=(0,0.01,0.05,0.33)$ (19)

$V=(v_1,v_2,v_3,v_4)^T=(0.61,0.00,0.00,0.00,0.00)$ (20)

Using equations (13) and combining with equations (19) and (20) we can obtain the intuitionistic fuzzy positive ideal solution as follows:

$M^+=(M_1^+,M_2^+,M_3^+,M_4^+)^T=((\mu_1^+,\nu_1^+),((\mu_2^+,\nu_2^+),((\mu_3^+,\nu_3^+),((\mu_4^+,\nu_4^+))^T$

$= ((0,1), (1,0), (1,0), (1,0))^T$

Using equations (6) , (16) and (18), the square S_j of the weighted Euclidean distance between the alternative A_j(j=1,2,3) and the intuitionistic fuzzy positive ideal solution M^+ can be calculated as follows:

S_1=0.60070, S_2=0.6256, S_3=0.441 respectively .This gives the ranking order of the alternatives A_j(j=1,2,3)as follows:

$$A_3 > A_1 > A_2$$

Here by, it is obvious that the best alternative is A_3 i.e EOQ is the best inventory policy. However for different preference relation set Ω and specific values of the parameters h and έ different alternatives may be evaluated.

5. Conclusion

Most decision making problems which involves many attribute are flexible in optimizing decisions as it involves multiple decision makers and the weights of attributes are not provided a priori. Added to that the quantitative as well as qualitative factors are often assessed using imprecise data and human judgments. Thus to approximate such human subjective evaluation process, it would be desirable to apply an intuitionistic fuzzy MADM model. In particular, intuitionistic fuzzy sets are used in this paper to assess alternatives with respect to qualitative attributes. The model constructed helped to rank alternative decisions using the pair wise comparisons between alternatives. The applicability of the proposed method was illustrated with a real inventory policy selection example. In the application, the best solution is obtained as Economic Order Policy followed by Heuristics and VMI. This result is significant from the perspective of a company where inventory represents sufficient investment. The systematic framework for inventory selection in an intuitionistic fuzzy environment which is reflected here can be replicated to other areas of inventory management. The most reliable factor of the proposed approach is its systematic procedure added with the advantage to modify the imprecise data and parameters of a set of satisfactory compromise solution. However, the approach for solving the selection problem can be further improved and a group decision support system developed in a fuzzy and intuitionistic fuzzy environment for future research.

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