Structuring digital plane by the 8-adjacency graph with a set of walks

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Abstract: In the digital plane $\mathbb{Z}^2$, we define connectedness induced by a set of walks of the same lengths in the 8-adjacency graph. The connectedness is shown to satisfy a digital analogue of the Jordan curve theorem. This proves that the 8-adjacency graph with a set of walks of the same lengths provides a convenient structure on the digital plane $\mathbb{Z}^2$ for the study of digital images.

Key–Words: Digital plane, 8-adjacency graph, walk, connectedness, Jordan curve theorem

1 Introduction

Discrete mathematics has many applications not only in mathematics itself but also in numerous other disciplines. This is caused by the rapid computerization, hence discretization, of most modern technologies used in our everyday life. For example, graph theory provides powerful tools for solving various types of problems, particularly those that occur in image processing. Indeed, digital topology, a theory that was founded for studying the topological and geometric properties of digital images, is based on graph theory rather than topology (cf. [4-5, 7-8]). One of the basic tasks of digital topology is to find a convenient structure on the digital plane $\mathbb{Z}^2$ allowing us to study and process digital images. In the classical approach to digital topology, adjacency graphs with the vertex set $\mathbb{Z}^2$ are used to provide such structures, namely the well-known 4- and 8-adjacency graphs. A problem connected with adopting this approach is that neither 4-adjacency nor 8-adjacency graph allows for an analogue of the Jordan curve theorem (recall that the classical Jordan curve theorem states that a simple closed curve separates the real, i.e., Euclidean, plane into precisely two components). This problem is usually solved by using a combination of the two adjacencies and most of the graphical software is based on employing such a combination.

In 1990, E.D. Khalimsky, R. Kopperman and P.R. Meyer [2] proposed a new approach to digital topology based on using a single structure on $\mathbb{Z}^2$, the so-called Khalimsky topology. This approach, which has been developed by many authors (see, e.g., [6] and [9]), is equivalent to the one based on using a particular graph, the connectedness graph of the Khalimsky topology, for structuring $\mathbb{Z}^2$. It the present note, we build on the classical approach to digital topology. We show that, to obtain a convenient structure on the digital plane, we may employ the 8-adjacency graph together with a set of paths of the same lengths. Such a structure proved to have an advantage over the Khalimsky topology.

In [10], graphs with path partitions are studied where the path partitions considered are nothing but certain sets of walks. It was shown in [10] that path partitions provide graphs with a special connectedness that allows for using these graphs as convenient background structures on the digital spaces for the study of digital images. In the present paper, in difference to [10], we employ sets of walks, which are more general than path partitions, and we restrict our considerations to the 8-adjacency graph on $\mathbb{Z}^2$.

2 Preliminaries

For the graph-theoretic terminology, we refer to [1]. We will work with (simple) graphs, i.e., pairs $G = (V, E)$ where $V \neq \emptyset$ is a set, the so-called *vertex set* of G, and $E \subseteq \{\{x, y\};\ x, y \in V,\ x \neq y\}$ is the so-called set of *edges*. Two vertices $x, y \in V$ are said to be *adjacent* if $\{x, y\} \in E$. A *walk* in G is a (finite) sequence $(x_n|\ i \leq n)$, i.e., $(x_0, x_1, ..., x_n)$, of vertices of V such that x_i is adjacent to x_{i+1} whenever $i < n$; the non-negative integer n is called the *length* of the walk $(x_n|\ i \leq n)$. A walk $(x_n|\ i \leq n)$ in G is called a *path* if $x_i \neq x_j$ whenever $i, j \leq n,\ i \neq j$, and it is called a *circle* if $x_i \neq x_j$ whenever $i, j < n,\ i \neq j$, and $x_0 = x_n$.

Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be graphs. We say that G_1 is a *subgraph* of G_2 if $V_1 \subseteq V_2$ and $E_1 \subseteq E_2$. If, moreover, $V_1 = V_2$, then G_1 is called a *factor* of G_2. A subgraph $G_1 = (V_1, E_1)$ of a graph $G_2 = (V_2, E_2)$ is said to be its *induced subgraph* if $E_1 = E_2 \cap \{\{x, y\};\ x, y \in V_1\}$. The *cartesian product* of G_1 and G_2 is the graph $G_1 \times G_2 = (V_1 \times V_2, E)$ where $E = \{\{(x_1, x_2), (y_1, y_2)\};\ (x_1, x_2), (y_1, y_2) \in V_1 \times V_2,,\ \{x_1, y_1\} \in E_1,\ \{x_2, y_2\} \in E_2\}$ and the *strong product* of G_1 and G_2 is the graph $G_1 \otimes G_2 = (V_1 \times V_2, E)$ where, for any $\{(x_1, x_2), (y_1, y_2)\}$ with $(x_1, x_2), (y_1, y_2) \in V_1 \times V_2$, $\{(x_1, x_2), (y_1, y_2)\} \in E$ if and only if one of the following three conditions is fulfilled:

(1) $\{x_1, y_1\} \in E_1$ and $\{x_2, y_2\} \in E_2$,
(2) $x_1 = y_1$ and $\{x_2, y_2\} \in E_2$,
(3) $\{x_1, y_1\} \in E_1$ and $x_2 = y_2$.

Thus, $G_1 \times G_2$ is always a factor of $G_1 \otimes G_2$.

3 Graphs with walk sets

Given a graph G and a positive integer n, we denote by $\mathcal{P}_n(G)$ the set of all walks of length n in G. For every set of walks (briefly, walk set) $\mathcal{B} \subseteq \mathcal{P}_n(G)$, we put
$\mathcal{B}^{-1} = \{(x_i|\ i \leq n) \in \mathcal{P}_n(G);\ (x_{n-i}|\ i \leq n) \in \mathcal{B}\}$,
$\hat{\mathcal{B}} = \{(x_i|\ i \leq m) \in \mathcal{P}_m(G);\ 0 < m \leq n$ and there exists $(y_i|\ i \leq n) \in \mathcal{B}$ such that $x_i = y_i$ for every $i \leq m\}$ (so that $\mathcal{B} \subseteq \hat{\mathcal{B}}$), and
$\mathcal{B}^* = \hat{\mathcal{B}} \cup \hat{\mathcal{B}}^{-1}$.

Let G_j be a graph and $\mathcal{B}_j \subseteq \mathcal{P}_n(G_j)$ for every $j \in \{1, 2\}$. The we put $\mathcal{B}_1 \otimes \mathcal{B}_2 = \mathcal{B}$ where $\mathcal{B} \subseteq \mathcal{P}_n(G_1 \otimes G_2)$ is the subset such that, for any $((x_0, y_0), ..., (x_n, y_n)) \in \mathcal{P}_n(G_1 \otimes G_2)$, $((x_0, y_0), ..., (x_n, y_n)) \in \mathcal{B}$ if and only if one of the following three conditions is satisfied:

(1) $(x_0, ..., x_n) \in \mathcal{B}_1$ and $(y_0, ..., y_n\} \in \mathcal{B}_2$,
(2) $x_0 = ... = x_n$ and $(y_0, ..., y_n\} \in \mathcal{B}_2$,
(3) $(x_0, ..., x_n) \in \mathcal{B}_1$ and $y_0 = ... = y_n$.

Definition 1 Let $G = (V, E)$ be a graph and $\mathcal{B} \subseteq \mathcal{P}_n(G)$. A sequence $C = (x_i|\ i \leq m)$, $m > 0$, of vertices of V is called a $\mathcal{B}$-*walk* in G if there is an increasing sequence $(i_k|\ k \leq p)$ of non-negative integers with $i_0 = 0$ and $i_p = m$ such that $i_k - i_{k-1} \leq n$ and $(x_i|\ i_{k-1} \leq i \leq i_k) \in \mathcal{B}^*$ for every k with $0 < k \leq p$. The sequence $(i_k|\ k \leq p)$ is said to be a *binding sequence* of C.

Definition 2 Let $G = (V, E)$ be a graph and $\mathcal{B} \subseteq \mathcal{P}_n(G)$. A set $A \subseteq V$ is said to be $\mathcal{B}$-*connected* in G if any two different vertices of G belonging to A can be joined by a $\mathcal{B}$-*walk* in G contained in A. A maximal $\mathcal{B}$-connected set in G is called a $\mathcal{B}$-*component* of G. A $\mathcal{B}$-walk in G is called a $\mathcal{B}$-circle if $x_i \neq x_j$ for all $i, j < m$ with $i \neq j$ and $x_0 = x_m$.

Given a graph $G = (V, E)$, a walk set $\mathcal{B} \subseteq \mathcal{P}_n(G)$, and a subset $X \subseteq V$, we say that X *separates* G into precisely two $\mathcal{B}$-components if the induced subgraph H of G with the vertex set $V - X$ consists of precisely two $\mathcal{B}'$-components where $\mathcal{B}' \subseteq \mathcal{P}_n(H)$ is the walk set given by $\mathcal{B}' = \mathcal{P}_n(G) \cap \{(x_i|\ i \leq n);\ x_i \in V - X$ for all $i < n\}$.

Proposition 3 *Let $G_j = (V_j, E_j)$ be a graph, $\mathcal{B}_j \subseteq \mathcal{P}_n(G_j)$, and $Y_j \subseteq V_j$ be a subset for every $j \in \{1, 2\}$. If Y_j is $\mathcal{B}_j$-connected in G_j for every $i \in \{1, 2\}$, then $Y_1 \times Y_2$ is $\mathcal{B}_1 \otimes \mathcal{B}_2$-connected in $G_1 \otimes G_2$.*

Proof: Let $Y_j = (y^j_i|\ i \leq p_j) \in \mathcal{B}^*_j$ for every $j \in \{1, 2\}$. For each $j \in \{1, 2\}$, there is a walk $(x^j_i|\ i \leq n) \in \mathcal{B}$ such that $y^j_i = x^j_i$ for all $i \leq p_j$ or $y^j_i = x^j_{p_j - i}$ for all $i \leq p_j$. Let $y \in \{y^1_i;\ i \leq p_1\} \times \{y^2_i;\ i \leq p_2\}$ be an arbitrary element. Then, for each $j \in \{1, 2\}$, there is a non-negative integer q_j, $q_j < p_j$, such that $y = (y^1_{q_1}, y^2_{q_2})$. Then, clearly, either $(y^1_{q_1 - i}|\ i \leq q_1)$ or $(y^1_i|\ q_1 \leq i \leq p_1)$ is an element of $\mathcal{B}^*_1$ with the first member $y^1_{q_1}$ and the last one x^1_0. Denote this element of $\mathcal{B}^*_1$ by $(z^1_i|\ i \leq r_1)$ and put $C_1 = ((z^1_i, y^2_{q_2})|\ i \leq r_1)$. Clearly, C_1 is an element of $(\mathcal{B}_1 \otimes \mathcal{B}_2)^*$ with all members belonging to $\{y^1_i;\ i \leq p_1\} \times \{y^2_i;\ i \leq p_2\}$, with the first member y, and with $z^1_{r_1} = x^1_0$. Clearly, either $(y^2_{q_2 - i}|\ i \leq q_2)$ or $(y^2_i|\ q_2 \leq i \leq p_2)$ is an element of $\mathcal{B}^*_2$ with the first member $y^2_{q_2}$ and the last one x^2_0. Denote this element of $\mathcal{B}^*_2$ by $(z^2_i|\ i \leq r_2)$ and put $C_2 = ((x^1_0, z^2_i)|\ i \leq r_2)$. Then C_2 is an element of $(\mathcal{B}_1 \otimes \mathcal{B}_2)^*$ with all members belonging to $\{y^1_i;\ i \leq p_1\} \times \{y^2_i;\ i \leq p_2\}$ such that $z^2_0 = y^2_{q_2}$ and $z^2_{r_2} = x^2_0$. Put $C = ((s^1_i, s^2_i)|\ i \leq 2q_2 + 1)$ where $(s^1_i, s^2_i) = (z^1_i, y^2_{q_2})$ for $i = 0, 1, ..., r_1$ and $(s^1_i, s^2_i) = (x^1_0, y^2_{i - r_1})$ for $i = r_1 + 1, r_1 + 2, ..., r_1 + r_2$. Then C is a $\mathcal{B}_1 \otimes \mathcal{B}_2$-walk in $G_1 \otimes G_2$ with all members belonging to $\{y^1_i;\ i \leq p_1\} \times \{y^2_i;\ i \leq p_2\}$, with the first member y, and with the last one (x^1_0, x^2_0). We have shown that any point of $\{y^1_i;\ i \leq p_1\} \times \{y^2_i;\ i \leq p_2\}$ can be connected with the point (x^1_0, x^2_0) by a $\mathcal{B}_1 \otimes \mathcal{B}_2$-walk in $G_1 \otimes G_2$ contained in $\{y^1_i;\ i \leq p_1\} \times \{y^2_i;\ i \leq p_2\}$. Thus, $Y_1 \times Y_2$ is $\mathcal{B}_1 \otimes \mathcal{B}_2$-connected in $G_1 \otimes G_2$ whenever $Y_1 \in \mathcal{B}^*_1$ and $Y_2 \in \mathcal{B}^*_2$.

Let $Y_j = (x^j_i|\ i \leq p_j)$ be a $\mathcal{B}_j$-walk in G_j for every $j \in \{1, 2\}$. For each $j \in \{1, 2\}$, let $(i^j_k|\ k \leq q_j)$ be the binding sequence of $(x^j_i|\ i \leq p_j)$, i.e., a sequence of non-negative integers with $i^j_0 = 0$ and $i^j_{q_j - 1} = p_j - 1$ such that $(x^j_i|\ i^j_k \leq i \leq i^j_{k+1})$ is an element of $\mathcal{B}_j^*$ whenever $k \le q_j$. For every $j \in \{1,2\}$, putting $C_k^j = \{x_i^j;\ i_k^j \le i \le i_{k+1}^j\}$, we get $\{x_i^j;\ i \le p_j\} = \bigcup_{k<q_j} C_k^j$. Therefore, $\{x_i^1;\ i \le p_1\} \times \{x_i^2;\ i \le p_2\} = \bigcup_{k_1<q_1} \bigcup_{k_2<q_2} (C_{k_1}^1 \times C_{k_2}^2)$ where $C_{k_1}^1 \times C_{k_2}^2$ is $\mathcal{B}_1 \otimes \mathcal{B}_2$-connected in $G_1 \otimes G_2$ whenever $k_j < q_j$, $j = 1,2$, by the previous part of the proof. Thus, for any $k_1 < q_1$, $(C_{k_1}^1 \times C_{k_2}^2 |\ k_2 < q_2)$ is a finite sequence of $\mathcal{B}_1 \otimes \mathcal{B}_2$-connected sets in $G_1 \otimes G_2$ with nonempty intersection of every consecutive pair of them. Hence, the set $\bigcup_{k_2<q_2} (C_{k_1}^1 \times C_{k_2}^2)$ is $\mathcal{B}_1 \otimes \mathcal{B}_2$-connected in $G_1 \otimes G_2$. Consequently, the set $\bigcup_{k_1<q_1} \bigcup_{k_2<q_2} (C_{k_1}^1 \times C_{k_2}^2)$ is $\mathcal{B}_1 \otimes \mathcal{B}_2$-connected in $G_1 \otimes G_2$. Therefore, $Y_1 \times Y_2$ is $\mathcal{B}_1 \otimes \mathcal{B}_2$-connected in $G_1 \otimes G_2$ whenever Y_1 is a $\mathcal{B}_1$-walk in G_1 and Y_2 is a $\mathcal{B}_2$-walk in G_2.

Let Y_j be a $\mathcal{B}_j$-connected set in G_j for every $j \in \{1,2\}$ and let $(x_1,x_2),(y_1,y_2) \in G_1 \otimes G_2$ be arbitrary points. Then, for each $j \in \{1,2\}$, there is a $\mathcal{B}_j$-walk $(z_i^j |\ i \le p_j)$ in G_j joining the points x_j and y_j which is contained in Y_j. The set $\{z_i^1 |\ i \le p_1\} \times \{y_i^2 |\ i \le p_2\}$ contains the points (x_1,x_2) and (y_1,y_2) and is a $\mathcal{B}_1 \otimes \mathcal{B}_2$-connected set in $G_1 \otimes G_2$ by the previous part of the proof. Thus, there is a $\mathcal{B}_1 \otimes \mathcal{B}_2$-walk C in $G_1 \otimes G_2$ joining the points (x_1,x_2) and (y_1,y_2) which is contained in $\{z_i^1 |\ i \le p_1\} \times \{y_i^2 |\ i \le p_2\}$. Since $\{z_i^1 |\ i \le p_1\} \times \{y_i^2 |\ i \le p_2\} \subseteq Y_1 \times Y_2$, C is contained in $Y_1 \times Y_2$, too, and so $Y_1 \times Y_2$ is $\mathcal{B}_1 \otimes \mathcal{B}_2$-connected in $G_1 \otimes G_2$. The proof is complete. □

4 8-adjacency graph with a set of walks

Recall that the 8-adjacency graph on $\mathbb{Z}^2$ is the graph $(\mathbb{Z}^2, A_8)$ where $A_8 = \{\{(x_1,y_1),(x_2,y_2)\};\ (x_1,y_1),(x_2,y_2) \in \mathbb{Z}^2, \max\{|x_1-x_2|,|y_1-y_2|\} = 1\}$. In the sequel, G will denote the 8-adjacency graph on $\mathbb{Z}^2$. It is evident that $G = Z_2 \otimes Z_2$ where Z_2 is the 2-adjacency graph on $\mathbb{Z}$, i.e., the graph $(\mathbb{Z}, A_2)$ where $A_2 = \{\{p,q\};\ p,q \in \mathbb{Z},\ |p-q| = 1\}$.

Let $\mathcal{B} \subseteq \mathcal{P}_2(Z_2)$ be the set given as follows:
$\mathcal{B} = \{(x_i|\ i \le 2) \in \mathcal{P}_2(Z_2)$; there exists an odd number $l \in \mathbb{Z}$ such that $x_i = 2l + i$ for all $i \le 2$ or $x_i = 2l - i$ for all $i \le 2\}$.

Using results of the previous section, we may propose a new structure on the digital plane convenient for the study of digital images. Such a structure is obtained as the 8-adjacency graph on $\mathbb{Z}^2$ (i.e., the strong product of two copies of the 2-adjacency graph on Z) with the walk set given by the strong product of two copies of the walk set $\mathcal{B}$.

Since the digital line $\mathbb{Z}$ is evidently $\mathcal{B}$-connected in the graph Z_2, the digital plane $\mathbb{Z}^2$ is $\mathcal{B}$- connected in the 8-adjacency graph G by Proposition 3.

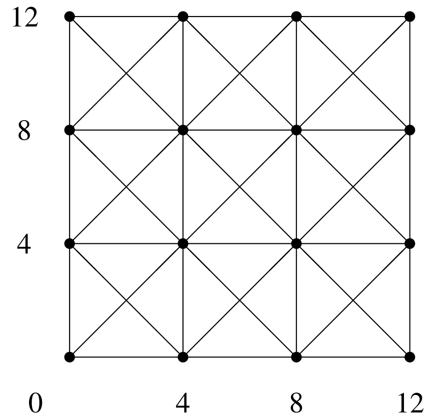

Figure 1: A section of the graph H

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We denote by H the factor of the 8-adjacency graph G whose edges are those $\{(x_1,y_1),(x_2,y_2)\} \in A_8$ that satisfy one of the following four conditions for some $k \in \mathbb{Z}$:
$x_1 - y_1 = x_2 - y_2 = 4k$,
$x_1 + y_1 = x_2 + y_2 = 4k$,
$x_1 = x_2 = 4k$,
$y_1 = y_2 = 4k$.
A section of the graph H is demonstrated in Figure 1 where only the vertices $(4k, 4l)$, $k,l \in \mathbb{Z}$, are marked out (by bold dots) and thus, on every edge drawn between two such vertices, there are 3 more (non-displayed) vertices, so that such an edge represents 4 edges in the graph H.

Definition 4 A $\mathcal{B} \otimes \mathcal{B}$-circle J in the graph G is said to be *fundamental* if it is a circle in H and, whenever $(4k+2, 4l+2) \in J$ for some $k,l \in \mathbb{Z}$, one of the following two conditions is true:
$\{((4k+2)-1,(4l+2)-1),((4k+2)+1,(4l+2)+1)\} \subseteq J$,
$\{((4k+2)-1,(4l+2)+1),((4k+2)+1,(4l+2)-1)\} \subseteq J$.

The fundamental circles are just the circles in H that turn at the bold vertices only as demonstrated in Figure 1.

Theorem 5 *If J is a fundamental circle in the graph G, then J separates G into precisely two $\mathcal{B} \otimes \mathcal{B}$-components, one finite and the other infinite, such that the union of any of them with J is a $\mathcal{B} \otimes \mathcal{B}$-connected set in G.*

Sketch of proof: For every point $z = (4k+2, 4l+2)$, $k,l \in \mathbb{Z}$, each of the following four subsets of $\mathbb{Z}^2$ is called a *fundamental triangle* (given by z):

$\{(r,s)\in\mathbb{Z}^2;\ 4k\le r\le 4k+4,\ 4l\le s\le 4l+4,\ s\le r+4l-4k\}$,
$\{(r,s)\in\mathbb{Z}^2;\ 4k\le r\le 4k+4,\ 4l\le s\le 4l+4,\ s\ge 4k+4l+4-r\}$,
$\{(r,s)\in\mathbb{Z}^2;\ 4k\le r\le 4k+4,\ 4l\le s\le 4l+4,\ s\ge r+4l-4k\}$,
$\{(r,s)\in\mathbb{Z}^2;\ 4k\le r\le 4k+4,\ 4l\le s\le 4l+4,\ s\le 4k+4l+4-r\}$.
Clearly, the edges of any fundamental triangle form a $\mathcal{B}\otimes\mathcal{B}$-circle in G. It may be shown that every fundamental triangle is $\mathcal{B}\otimes\mathcal{B}$-connected in G and so is also every set obtained from a fundamental triangle by subtracting some of its edges.
We will say that a (finite or infinite) sequence S of fundamental triangles is a *tiling sequence* if the members of S are pairwise different and every member of S, excluding the first one, has an edge in common with at least one of its predecessors. Given a tiling sequence S of fundamental triangles, we denote by S' the sequence obtained from S by subtracting, from every member of the sequence, all its edges that are not shared with any other member of the sequence. By the firs part of the proof, for every tiling sequence S of fundamental triangles, the set $\bigcup\{T;\ T\in S\}$ is $\mathcal{B}\otimes\mathcal{B}$-connected in G and the same is true for the set $\bigcup\{T;\ T\in S'\}$.
Let J be a fundamental circle in the graph G. Then J constitutes the border of a polygon $S_F\subseteq\mathbb{Z}^2$ consisting of fundamental triangles. More precisely, S_F is the union of some fundamental triangles such that any pair of them is disjoint or meets in just one edge in common. Let U be a tiling sequence of the fundamental triangles contained in S_F. Since S_F is finite, U is finite, too, and we have $S_F=\bigcup\{T;\ T\in U\}$. It may be shown that every fundamental triangle $T\in U$ is $\mathcal{B}\otimes\mathcal{B}$-connected in G. Thus, S_F is $\mathcal{B}\otimes\mathcal{B}$-connected in G, too. Similarly, U' is a finite sequence with $S_F-J=\bigcup\{T;\ T\in U'\}$ and we may show that every member of U' is $\mathcal{B}\otimes\mathcal{B}$-connected in G. It follows that S_F-J is $\mathcal{B}\otimes\mathcal{B}$-connected in G, too.
Further, let V be a tiling sequence of fundamental triangles which are not contained in S_F. Since the complement of S_F in $\mathbb{Z}^2$ is infinite, V is infinite, too. Put $S_I=\bigcup\{T;\ T\in V\}$. It may be shown that every fundamental triangle $T\in V$ is $\mathcal{B}\otimes\mathcal{B}$-connected in G, so that S_I is $\mathcal{B}\otimes\mathcal{B}$-connected in G, too. Similarly, V' is a finite sequence with $S_I-J=\bigcup\{T;\ T\in V'\}$ and we may show that every member of V' is $\mathcal{B}\otimes\mathcal{B}$-connected in G. Therefore, S_I-J is $\mathcal{B}\otimes\mathcal{B}$-connected in G, too.
It may easily be seen that every $\mathcal{B}\otimes\mathcal{B}$-walk $C=(z_i|\ i\le k)$, k a positive integer, in G connecting a point of S_F-J with a point of S_I-J meets J (i.e., meets an edge of a fundamental triangle which is contained in J). Therefore, the set $\mathbb{Z}^2-J=(S_F-J)\cup(S_I-J)$ is not $\mathcal{B}\otimes\mathcal{B}$-connected in G. We have shown that J separates G into precisely two components S_F-J and S_I-J, S_F-J finite and S_I-J infinite, with S_F and S_I $\mathcal{B}\otimes\mathcal{B}$-connected in G. □

5 Conclusion

We proposed a new structure of connectedness in the digital plane given by a sets of paths in the 8-adjacency graph. In Theorem 5, we showed that fundamental circles in the graph H (i.e., circles in the graph demonstrated in Figure 1) separate the digital plane $\mathbb{Z}^2$ into precisely two components (with respect to the connectedness given by the set of paths in the 8-adjacency graph) so that they may be considered to be digital analogues of the Jordan curves in the Euclidean plane. The fundamental circles may consist of horizontal, vertical and diagonal parts and their advantage over the digital Jordan curves in the Khalimsky topology determined in [2] is that they may turn at the acute angle $\frac{\pi}{4}$ - see Figure 1 (in the Khalimsky topology, the Jordan curves may never turn at the acute angle $\frac{\pi}{4}$). Since digital Jordan curves represent borders of objects in digital images (cf. [3]), the proposed connectedness structure in the digital plane given by the set $\mathcal{B}$ of walks in the 8-adjacency graph provides a richer variety of applications than the one provided by the Khalimsky topology.

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