is replaced by a unary predicate distinguishing an element presenting the constant. Additionally, we should replace each nulary predicate by a unary one. This transformation defines an algebraic isomorphism of theories.

fP-to-Graph — a transformation from a theory of a finite pure predicate signature with predicates of arity $\geqslant 1$ to an extension of special graph theory *GRE* of signature $\{\Gamma^2\}$, cf. Stage *FG* in Theorem 5.10.1 in [7]. This transformation is a Cartesian extension of a theory, thus, it defines a Cartesian interpretation.

Graph-to-2u — a transformation from a theory of signature $\{\Gamma^2\}$ which is an extension of special graph theory *GRE* to a theory of signature with two unary functions; cf. Case 2 in Lemma 5.10.1 in [7]. This transformation is a Cartesian extension of a theory, thus, it defines a Cartesian interpretation.

Graph-to-1b — a transformation from a theory of signature $\{\Gamma^2\}$ which is an extension of a special graph theory *GRE* to a theory of signature with one binary function; cf. Case 1 in Lemma 5.10.1 in [7]. This transformation is a Cartesian extension of a theory, thus, it defines a Cartesian interpretation.

Enrich — a transformation from a theory of a signature matching one of the three cases (4.3) of a minimal finite rich signature to a theory of a given finite rich signature; cf. Stage *GL* in Table 5.8.1 in [7]. This transformation is an isomorphism of theories (notice that, a problem to assign values to constants possible in the target signature σ_2 is regularly solvable because the extension of Graph theory *GRE* preceding the stage Enrich has at least one distinguished element).

Fig. 2 represents a scheme of successive actions of the elementary transformations. We use circled digits and letters to point out some intermediate points in order to explain different variants of traversal through the scheme. `Entry1` of the scheme requires, as an input, a theory of a finite signature and yields an output theory of the demanded finite rich signature σ_2. We define Redu as a composition of transformations of theories along the passage 1-x-e in the scheme shown in Fig. 2. Each elementary stage is a Cartesian $\exists\cap\forall$-presentable interpretation. Thereby, the full passage that is a composition of these separate stages is also an a Cartesian $\exists\cap\forall$-presentable interpretation.

Theorem 4.1 is proved. □

Theorem 4.2 is easily deduced from Theorem 4.1 by applying elementary methods of c.e. Boolean algebras together with the properties of correspondence (0.2) considered in Lemma 0.1. □

Give a complementary statement.

Lemma 4.3. *Finite-to-finite signature reduction procedure we have described in Theorem* 4.1 *and Theorem* 4.2 *represents a particular case of the operation of a Cartesian extension of a theory.*

Moreover, interpretation I involved in the extension have the following properties:

(a) *I preserves all model-theoretic properties within the layer ACL*,

(b) *I preserves all model-theoretic properties within the real layer AreaL*,

(c) *in general case, the finite-to-finite signature reduction procedure does not preserve, both locally and globally, model-theoretic properties of $\exists\forall$-axiomatizability, $\forall$-axiomatizability, and $\exists$-axiomatizability; i.e., in some cases, these properties are not preserved by the procedure* Redu.

PROOF. Immediately, from proofs of Theorem 4.1 and Theorem 4.2. □

4.2 *Infinite-to-finite signature reduction procedure*. A theory of an arbitrary enumerable signature is transformed into a theory of any pre-specified finite rich signature. This type of signature reduction procedure is defined via an `Entry2` in Fig. 2. It is realized by the stages acting along the passage 2-u-e including those listed in Subsection 4.1 together with two additional transformations whose short specifications are presented below:

anysig-to-iP — a transformation from a theory of an arbitrary enumerable signature (either finite or infinite) into a theory of an infinite pure predicate signature with predicates of arity $\geqslant 1$; it is analogous to stage finsig-to-fP, but with addition a countable set of new trivially defined (dummy) predicates. A thin point is that, if an input theory is c.a. and is given by its weak c.e. index, the output theory is presented by a normal c.e. index. This transformation defines an algebraic isomorphism of theories.

iP-to-Graph — a transformation from a theory of an infinite pure predicate signature with predicates of arity $\geqslant 1$ to an extension of graph theory *GRE* of signature $\{\Gamma^2\}$ (main stage of the infinite-to-finite signature reduction procedure); cf. Stage *IG* in Theorem 5.9.1 in [7]. This transformation defines a quasiexact interpretation of theories.

We note an effective version of the infinite-to-finite signature reduction procedure:

Theorem 4.3. *Given a c.a. theory T and a finite rich signature σ_2. Effectively in a weak c.e. index of T, one can construct a c.a. theory S of signature σ_2 together with a quasiexact interpretation $I:T\rightarrowtail S$; in particular, the interpretation I defines a computable isomorphism $\mu:\mathcal{L}(T)\to\mathcal{L}(S)$ preserving model-theoretic properties of the infinitary semantic layer MQL.*

PROOF. Stage iP-to-Graph preserves layer *MQL* included in the layer *ACL*, cf. Fig. 1, that is preserved by the other stages in the passage 2-u-e in Fig. 2. □

4.3 *Transformation of theories for the universal* *construction*. A c.a. theory of an arbitrary enumerable signature given by its weak c.e. index is transformed into a f.a. theory of any pre-specified finite rich signature yielding its Godel number. This type of transformation is defined via an `Entry3` in Fig. 2. It is realized by the stages acting along the passage 3-w-e including those listed in Subsection 4.1 and Subsection 4.2 together with the following additional transformation:

CA-to-FA — a transformation from a computably axiomatizable theory of signature $\{\Gamma^2\}$ extending graph theory *GRE* into a finitely axiomatizable theory of a finite pure predicate signature (main stage of the universal construction); cf. Stage *GF* in Table 5.8.1 and Theorem 6.1.1 in [7]. An input theory is given by its c.e. index, while the output theory is presented by its Godel number. A standard release of this transformation defines a quasiexact interpretation of theories, thus, preserving the layer *MQL*. Notice that, there are simplified versions of the stage CA-to-FA preserving some proper sublayers of *MQL*.

We formulate the universal construction designed from the stage CA-to-FA:

Theorem 4.4. *Given a c.a. theory T and a finite rich signature σ_2. Effectively in a weak c.e. index of T, one can construct a f.a. theory F of signature σ_2 together with a quasiexact interpretation $I:T\rightarrowtail S$; in particular, the interpretation I defines a computable isomorphism $\mu:T\to F$ preserving the infinitary semantic layer MQL of model-theoretic properties (having a simplified version of the stage* CA-to-FA*, the layer of controlled properties will be smaller).*

PROOF. Stages iP-to-Graph and CA-to-FA preserve layer *MQL* included in layer *ACL*, cf. Fig. 1, that is preserved by the other stages in the passage 1-w-e in Fig. 2. □

REMARK 4.5. The works [11] and [4] describe a comparison method for semantic layer that is based on a representative list of model-theoretic properties. Applying this method, we can check that the power of the infinite-to-finite signature reduction procedure coincides with that of the universal construction, cf. Theorem 4.3 vs. Theorem 4.4. Any version of the universal construction can be considered as an advanced release of the infinite-to-finite signature reduction procedure. Thus, it would be unreal to expect that the former one can be more powerful in comparison with the latter one. This observation gives an informal substantiation to the fact that the power of an available standard version of the universal construction, [7], is actually maximum possible.

We should note that simple versions of the universal construction could also be useful. Indeed, high complexity of the universal construction represents a certain psychological barrier while studying results obtained on the base of this construction. The fact of availability of a mini-version of the universal construction that is more accessible for studying could reduce this barrier. Hereafter, we suppose that a fixed release of the universal construction is accepted, denoted by $\mathbb{U}$, that can control the following sublayer

$$MQL \subseteq MQL \qquad (4.4)$$

of the infinitary layer *MQL*. An example of a mini-version of the universal construction that is more simple in studying can be found in [12].

REMARK 4.6. Statement of Remark 4.5 establishes a close connection between the main stage of the infinite-to-finite signature reduction procedure and that of a standard release of the universal construction. This gives a good possibility to introduce a clear (understandable) definition to the concept of a quasiexact interpretation. First, we have to design a proof to the stage iP-to-Graph describing in detail properties of the involved interpretation I ensuring preservation of model-theoretic properties in infinitary semantic layer. After that, we have to extract a common description of the interpretation I such that it would be appropriate to both stages iP-to-Graph and CA-to-FA.

5 Virtual isomorphisms between the undecidable predicate calculi

The concept of a virtual extension of a theory can be found in [4] and [13]. We present the following statement:

Theorem 5.1. *Let σ_1 and σ_2 be arbitrary finite rich signatures. There are sequences $\varkappa_1$ and $\varkappa_2$ of formulas of the form (1.1)(b) in appropriate signatures satisfying (1.4) such that $PC(\sigma_1)\langle\varkappa_1\rangle \approx_a PC(\sigma_2)\langle\varkappa_2\rangle$.*

By using presentation of invertible multi-dimensional quotient interpretations via Cartesian-quotient extensions of theories, Lemma 6.8(a) in [6], we obtain that the main result of the work [14] represents a weak version of Theorem 5.1 with $\varkappa_1, \varkappa_2 \in KD$ and $\approx$ instead of $\approx_a$.

By applying Theorem 5.1, we can establish the following fact:

Corollary 5.2. *Let σ_1 and σ_2 be finite rich signatures. There exists a computable isomorphism $\mu: \mathcal{L}(PC(\sigma_1)) \to \mathcal{L}(PC(\sigma_2))$ that preserves model-theoretic properties from the semantic layer ACL (thereby, μ preserves all available model-theoretic properties). Moreover, μ preserves globally the following general-model properties: decidability, computable axiomatizability, to be a theory of* *a given degree of unsolvability, finite axiomatizability, and Π_n-axiomatizability, for any fixed $n \geqslant 2$.*

The part stating preservation globally of Π_n-axiomatizability, $n \geqslant 2$, gives the positive answer to an open question asked by V. L. Selivanov in 2007.

6 Semantic types of theories and operations on them

From the point of view of a semantic layer L, any computably axiomatizable theory T can be characterized by a 3-tuple $\big(\mathcal{L}(T),\gamma,\xi\big)$, where $\big(\mathcal{L}(T),\gamma\big)$ is its Tarski-Lindenbaum algebra with a Gödel numbering, while ξ is a mapping from the Stone space $St(\mathcal{L}(T))$ into the power-set $\mathcal{P}(L)=\{K|K\subseteq L\}$ which is defined as follows: for any T' in $St(\mathcal{L}(T))$ that is a complete extension of T, we put $\xi(T')=\{p\in L\,|\,T'$ *has the property* $p\}$. As a matter of fact, so defined 3-tuple $₤(T)=(\mathcal{L}(T),\gamma,\xi)$ represents a full abstract exposition of T in terms of the semantic layer L. We call this tuple *generalized Tarski-Lindenbaum algebra of the theory T under the semantic layer L*.

Generalizing the situation, we introduce a special class of objects to present isomorphism types of the generalized Tarski-Lindenbaum algebras under a semantic layer L of model-theoretic properties. Namely, consider an arbitrary 3-tuple of the form $\mathfrak{B}=(\mathcal{B},\nu,\xi)$, where $(\mathcal{B},\nu)$ is a c.e. Boolean algebra, while ξ is a mapping from Stone space $St(\mathcal{B})$ into the power-set $\mathcal{P}(L)$. So defined 3-tuple $(\mathcal{B},\nu,\xi)$ is called an *abstract semantic L-type*, or simply a *semantic type*.

Let $\mathfrak{B}_1=(\mathcal{B}_1,\nu_1,\xi_1)$ and $\mathfrak{B}_2=(\mathcal{B}_2,\nu_2,\xi_2)$ be two abstract semantic types under a semantic layer L. The types $\mathfrak{B}_1$ and $\mathfrak{B}_1$ are called *computably isomorphic* or *equivalent*, written $\mathfrak{B}_1\equiv_L\mathfrak{B}_2$, if there is a computable isomorphism $\mu:(\mathcal{B}_1,\nu_1)\to(\mathcal{B}_2,\nu_2)$ such that for any ultrafilter $\mathcal{F}_1\in\mathrm{St}(\mathcal{B}_1)$ and corresponding ultrafilter $\mathcal{F}_2\in\mathrm{St}(\mathcal{B}_2)$, $\mathcal{F}_2=\mu(\mathcal{F}_1)$, the equality $\xi_1(\mathcal{F}_1)=\xi_2(\mathcal{F}_2)$ takes place.

Let T be a theory and L be a semantic layer. Consider generalized Tarski-Lindenbaum algebra $(\mathcal{L}(T),\gamma,\xi)$ of T under the semantic layer L. If $\mathfrak{B}$ is a semantic type satisfying $(\mathcal{L}(T),\gamma,\xi)\equiv_L\mathfrak{B}$, we say that T *has the semantic type* $\mathfrak{B}$ *under* L, or that the *semantic type* $\mathfrak{B}$ *is presented* (*realized*) *in* T *under* L. By $₤(T)$, we denote the semantic type of a theory T under the full semantic layer *AL*, while $₤_L(T)$ stands for the semantic type of T under a semantic layer $L\subseteq AL$.

One can see that the concept of a semantic type together with the equivalence relation for such objects are in exact correspondence with the relation of semantic similarity of theories under a semantic layer. Namely, the following statement takes place:

Lemma 6.1. *Let T and S be theories of enumerable signatures and L be a semantic layer. Then, the following assertions are equivalent*:

(a) *T and S are semantically similar under L,*

(b) $₤(T)\equiv_L ₤(S)$.

Proof. Immediately, from definitions. □

Let $\mathfrak{B}$ be an abstract semantic type. The type $\mathcal{B}$ is said to be *finitely axiomatizable* or *$\mathcal{F}$-type*, if $\mathfrak{B}$ is realized in a finitely axiomatizable theory; $\mathcal{B}$ is *computably axiomatizable* or *$\mathcal{E}$-type*, if $\mathfrak{B}$ is realized in a computably axiomatizable theory.

Lemma 6.2. *Any $\mathcal{E}$-type under the layer $MQL\subseteq MQL$, cf.* (4.4)*, is an $\mathcal{F}$-type under MQL.*

Given a semantic layer $L\subseteq AL$. Let a semantic type $\mathfrak{B}$ be presented in computably axiomatizable theory with a c.e. index n. In such case, the number n is called an *$\mathcal{E}$-index* of this type $\mathfrak{B}$, symbolically $\mathfrak{B}=\mathscr{E}\{n\}$. Similarly, if a type $\mathfrak{B}$ is presented in finitely axiomatizable theory defined by a Gödel number n, the number n is called an *$\mathcal{F}$-index* of this type $\mathfrak{B}$, symbolically $\mathfrak{B}=\mathscr{F}\{n\}$.

Define the operation of a direct product of two semantic types.

Let two semantic types $\mathfrak{T}_1=(\mathcal{B}_1,\nu_1,h_1)$ and $\mathfrak{T}_2=(\mathcal{B}_2,\nu_2,h_2)$ be given under a layer L. Define some new semantic type

$$\mathfrak{T}=(\mathcal{B},\nu,h)=(\mathcal{B}_1,\nu_1,h_1)\otimes(\mathcal{B}_2,\nu_2,h_2)=\mathfrak{T}_1\otimes\mathfrak{T}_2$$

under the layer L as follows. We put $(\mathcal{B},\nu)=(\mathcal{B}_1,\nu_1)\otimes(\mathcal{B}_2,\nu_2)$, while the assignment function h is determined by the rule

$$h(\mathcal{F})=\begin{cases} h_1(\mathcal{F}), \text{ if } \mathcal{F}\in\mathrm{St}(\mathcal{B}_1),\\ h_2(\mathcal{F}), \text{ if } \mathcal{F}\in\mathrm{St}(\mathcal{B}_2).\end{cases}$$

So defined operation $\mathfrak{T}_1\otimes\mathfrak{T}_2$ is called *direct product* of semantic types $\mathfrak{T}_1$ and $\mathfrak{T}_2$, while the function h is called *direct product* of functions h_1 and h_2, using for this entry $h=h_1\otimes h_2$.

Now, we introduce a natural operation of the *direct product* of a *sequence of semantic types*. Let $\mathfrak{B}_n=(\mathcal{B}_n,\nu_n,\xi_n)$, $n\in\mathbb{N}$, be a sequence of semantic types under a semantic layer L, and P be a complete theory of an enumerable signature that is used as an additional parameter in the operation.

Define a new semantic type

$$(\mathcal{B},\nu,\xi)=\bigotimes\nolimits_{n\in\mathbb{N}}^{[P]}\mathfrak{B}_n=\bigotimes\nolimits_{n\in\mathbb{N}}^{[P]}\big(\mathcal{B}_n,\nu_n,\xi_n\big)$$

as follows. We put $(\mathcal{B},\nu)=\bigotimes_{n\in\mathbb{N}}(\mathcal{B}_n,\nu_n)$, while the assignment operation ξ is determined by the following rule

$$\xi(\mathfrak{F})=\begin{cases} \xi_n(\mathfrak{F}),\ \textit{if}\ \mathfrak{F}\in\mathrm{St}(\mathcal{B}_n),\ n\in\mathbb{N},\\ \mathrm{prop}(P),\ \ \textit{if}\ \mathfrak{F}=\hat{\mathfrak{F}}=\textit{Filter}\{\neg\varphi_i\,|\,i\in\mathbb{N}\},\end{cases}$$

where $\mathrm{prop}(P)$ is the set of model-theoretic properties associated with the complete theory P. The idea of the assignment function in the operation $\otimes$ for a sequence of semantic types is based on the following algebraic relation for the Boolean algebras

$$\mathrm{St}(\bigotimes_{i\in\mathbb{N}}\mathcal{B}_i)=\bigcup_{i\in\mathbb{N}}\mathrm{St}(\mathcal{B}_i)\cup\{\hat{\mathfrak{F}}\}.$$

7 Generalized Tarski-Lindenbaum algebra of undecidable predicate calculi

Now, we turn to principal statements characterizing the globalization structure of first-order predicate calculus of a finite rich signature under finitary and infinitary semantic layers in the form of some explicit formulas.

Remember notations introduced in this article:

- *AL* is the layer consisting of all model-theoretic properties of both model and algebraic types, cf. Preliminaries,
- *ACL* is the Cartesian semantic layer playing the role of a *working release* of the *finitary semantic layer*, cf. Section 2,
- *MQL* is the model quasiexact layer alternatively called the *infinitary semantic layer* cf. Section 2,
- $\mathcal{MQL}$ is a fixed sublayer of *MQL* supported by an accepted release of the universal construction as it was established in (4.4),
- $PC(\sigma)$ is the predicate calculus of signature σ considered as a first-order theory (defined by an empty set of axioms),
- $(\mathcal{L}(PC(\sigma)),\gamma,\xi)$ is the generalized Tarski-Lindenbaum algebra of predicate calculus $PC(\sigma)$; where γ is a fixed Gödel numbering of the set of sentences of signature σ, while $\xi:\mathrm{St}(PC(\sigma))\to\mathcal{P}(AL)$ is the mapping assigning model-theoretic properties to complete extensions of the theory $PC(\sigma)$,
- $\mathscr{F}_{\{n\}}$ is the *finitely axiomatizable* semantic type with an index n,
- $\mathscr{E}_{\{n\}}$ is the *computably axiomatizable* semantic type with an index n,
- The concept of an f-dense theory: a theory P of a finite signature σ is said to be *f-dense* under a semantic layer D if the following properties are satisfied: (a) theory P is complete and decidable, (b) for any $\Phi\in SL(\sigma)$ satisfying $P\vdash\Phi$, a sentence $\Psi\in SL(\sigma)$ and a computable isomorphism μ can be found, satisfying the following properties: $P\nvdash\neg\Psi$, $\vdash\Psi\to\Phi$, and $₤([\Psi]^\sigma)\equiv_D ₤(GRE)$ by means of μ; moreover, both a Gödel number of Ψ and an index of the isomorphism μ are found effectively from a Gödel number of the sentence Φ,
- The concept of an *inf*-dense theory is a generalization of the concept of an f-dense theory with using computably axiomatizable theories instead of finitely axiomatizable ones (details do not matter in this work).

We formulate the principal statement of the paper.

Theorem 7.1. [GLOBALIZATION THEOREM FOR FIRST-ORDER LOGIC] *Let σ be a finite rich signature, and*

$$₤\big(PC(\sigma)\big)=\big(\mathcal{L}(PC(\sigma)),\gamma,\xi\big)$$

be the semantic type of the predicate calculus of signature σ. Let L and K be semantic layers s.t. $L\subseteq ACL$ and $K\subseteq\mathcal{MQL}$, P be an f-dense theory under the layer L, and R be an inf-dense theory under the layer K. An extra demand $K\subseteq L$ is also accepted in Part (C) involving both layers L and K.

The following assertions take place:

(A) [FINITARY GLOBALIZATION] *The following presentation takes place*:

$$₤(PC(\sigma))\equiv_L \mathfrak{B}^{ACL}_{fin}=_{dfn}\bigotimes^{[P]}_{n\in\mathbb{N}}\mathscr{F}_{\{n\}},\qquad(7.1)$$

(B) [INFINITARY GLOBALIZATION] *The following presentation takes place*:

$$₤(PC(\sigma))\equiv_K \mathfrak{B}^{\mathcal{MQL}}_{inf}=_{dfn}\bigotimes^{[R]}_{n\in\mathbb{N}}\mathscr{E}_{\{n\}},\qquad(7.2)$$

(C) [INTERFERENCE] *Any computably axiomatizable semantic type under L is finitely axiomatizable under K. Moreover, there are total computable functions $q(n)$ and $v(n,t)$, such that q is a permutation of the set $\mathbb{N}$, and the following similarity relations are held for all $n\in\mathbb{N}$*:

$\mathscr{E}_{\{n\}}\equiv_K\mathscr{F}_{\{q(n)\}}$; *moreover, the function $(\lambda t)v(n,t)$ presents an isomorphism corresponding to this similarity relation.* (7.3)

Thereby, for an arbitrary f-dense under K theory S (that must automatically be inf-dense under K), the following similarity relation is satisfied

$$\bigotimes^{[S]}_{n\in\mathbb{N}}\mathscr{E}_{\{n\}}\equiv_K\bigotimes^{[S]}_{n\in\mathbb{N}}\mathscr{F}_{\{q(n)\}},\qquad(7.4)$$

such that corresponding Hanf's isomorphism μ maps member $\mathscr{E}_{\{n\}}$ onto member $\mathscr{F}_{\{q(n)\}}$ for all $n\in\mathbb{N}$, while a particular ultrafilter in the left-hand side of (7.4) is mapped onto a particular ultrafilter in the right-hand side,

(D) [Finitary Add/Omit Members] *Given an arbitrary $\mathcal{F}$-type $\mathfrak{B}'$ under the layer L and an integer $k_0 \geqslant 0$. We have*

$$\mathfrak{B}^{ACL}_{fin} = \bigotimes^{[P]}_{n<\omega} \mathscr{F}\{n\} \equiv_L \mathfrak{B}' \otimes \bigotimes^{[P]}_{k_0 \leqslant m<\omega} \mathscr{F}\{m\}; \quad (7.5)$$

more precisely: having omitted a few product members and attached an extra member in the sequence involved in the operation (7.1)*, it is possible to define a computable isomorphism μ between the latter semantic type and the changed one, such that, a particular ultrafilter from the left-hand side of* (7.5) *is linked by μ with that available in the right-hand side of* (7.5),

(E) [Infinitary Add/Omit Members] *Given an arbitrary $\mathcal{E}$-type $\mathfrak{B}''$ under the layer K and an integer $k_0 \geqslant 0$. We have*

$$\mathfrak{B}^{MQL}_{inf} = \bigotimes^{[R]}_{n<\omega} \mathscr{E}\{n\} \equiv_K \mathfrak{B}'' \otimes \bigotimes^{[R]}_{k_0 \leqslant m<\omega} \mathscr{E}\{m\}; \quad (7.6)$$

more precisely: having omitted a few product members and attached an extra member in the sequence involved in the operation (7.2)*, it is possible to define a computable isomorphism μ between the latter semantic type and the changed one, such that a particular ultrafilter from the left-hand side of* (7.6) *is linked by μ with that available in the right-hand side of* (7.6),

(F) [Effectiveness] *Transformations presented in the parts of this theorem are realized effectively in Gödel's numbers and/or c.e. indices of the objects involved in the construction. We can effectively find Gödel numbers and/or c.e. indices of all further objects appeared in the construction, such as c.e. index of a function, Gödel number or c.e. index of a semantic type, c.e. index of a computable sequence of semantic types, etc.*

8 Main applications of the Globalization Theorem

We show that the localized statements are immediate consequences of the globalization formulas.

Theorem 8.1. *Statement presenting the finite-to-finite signature reduction procedure, cf. Theorem* 4.1 *and Theorem* 4.2*, is an immediate consequence of Globalization Theorem* 7.1 (A).

Theorem 8.2. *An accepted version of the universal construction controlling the layer* (4.4)*, cf. Theorem* 4.4*, is an immediate consequence of Globalization Theorem* 7.1(C).

One more important application of Globalization Theorem.

The *pseudo-indecomposability* property was introduced due to Hanf [15], who considered this concept for the class of pure Boolean algebras. As an important application of the globalization formulas, we are going to study a version of this property expanded on the class of semantic types.

A semantic type $\mathfrak{B}$ under a semantic layer L is said to be *pseudo-indecomposable*, if for any $b \in \mathfrak{B}$, either $\mathfrak{B}[b]$ or $\mathfrak{B}[-b]$ is equivalent to $\mathfrak{B}$. A type $\mathfrak{B}$ is said to be *effectively pseudo-indecomposable* if it is pseudo-indecomposable and, effectively in the Godel number of an element $a \in |\mathfrak{B}|$, one can solve which of the two types $\mathfrak{B}[b]$ or $\mathfrak{B}[-b]$ is isomorphic to $\mathfrak{B}$; moreover, it is possible to get an index of corresponding isomorphism.

Theorem 8.3. *Let σ be a finite rich signature and*

$$\mathfrak{B}^* = Ł\big(PC(\sigma)\big) = \big(\mathcal{L}(PC(\sigma)), \gamma, \xi\big)$$

be the generalized Tarski-Lindenbaum algebra of the predicate calculus of signature σ under the layer AL. Then, $\mathfrak{B}^*$ is effectively pseudo-indecomposable under any semantic layer $L \subseteq ACL$.

Proof. Immediately from the decomposition (7.1) in Theorem 7.1. □

Conclusion

William Hanf in [16] has solved the known problem of Alfred Tarski about the isomorphism type of the Tarski-Lindenbaum algebra of predicate calculus of a finite rich signature. Historical background of the Tarski problem is discussed in the works [17], [16], [18], [19], [14], [20], and [7]. Results of this review solve a generalized Tarski problem characterizing the Tarski-Lindenbaum algebra of any predicate calculus of a finite rich signature with the description of model-theoretic properties of complete extensions of the predicate calculus. As an immediate consequence, we can obtain most of the currently available results on expressive power of first-order logic. A new extended and advanced edition of the book [7] is prepared to publication that includes both the new results based on the two levels of expressiveness of first-order logic and a new clearer definition to the concept of a quasiexact interpretation (by the scheme presented in Remark 4.6) together with an advanced exposition of the universal construction

Cartesian extensions and finite-to-finite signature reduction procedures are examples of methods of *finitary first-order combinatorics*, while infinite-to-finite signature reduction procedures and an available version of the universal construction of finitely axiomatizable theories are examples of methods of *infinitary first-order combinatorics*. From this point of view, methods and results of [16] and [21] correspond to the infinitary level of expressiveness of first-order logic. On the other hand, methods and results of [18], [19], [14], and [22] correspond to the finitary level of expressiveness of this logic. These two groups of works are based on different approaches, and both deserve to be studied independently, possibly, supplemented by their comparison and benchmarking.

Summing up, it is possible to say that the definition of the concept of a model-theoretic property together with its application to the globalization formulas can be regarded as an attempt to solve the general question of expressive power of formulas of first-order logic. The results can be of interest in pure logic and model theory as well as in applied logic and some branches of computer science.

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