4 Conclusion

The model presented in this paper accounts for the costs due to raw material on the classical economic production model with planned shortages. The mathematical models was derived and explicit expressions for the optimal production and shortage quantities were obtained. A proof of the uniqueness of the optimal solution was presented. Numerical examples were given to illustrate the model and to examine the sensitivity of the optimal solution relative to changes in the parameters of the model.

For future work, we suggest incorporating the effects of quality of the raw materials in this model. Also, we suggest studying the effects of quality of the finished product where on these models by considering reworking and scraping the imperfect quality finished items. In other direction, we suggest considering the model presented in the paper in the supply chain context.

References:

[1] F.W. Harris, What quantity to make at once. In: The Library of Factory Management. Operation and Costs, *The Factory Management Series*, Vol. 5, pp. 47–52, A.W. Shaw Co., Chicago, 1913.

[2] L. A. Johnson, D. C. Montgomery, *Operations research in production planning, scheduling, and inventory control*, Vol. 6, New York: Wiley, 1974.

[3] M. K. Salameh, A. N. El-Kassar, Accounting for the Holding Cost of Raw Material in the Production Model. *In Proceeding of BIMA Inaugural Conference*, March 2007, Sharjah, pp. 72-81.

[4] N. Yassine, Joint Probability Distribution and the Minimum of a Set of Normalized Random Variables, *Procedia-Social and Behavioral Sciences*, 2016, Vol. 230, pp. 235-239.

[5] A. N. El-Kassar, M. K. Salameh, M. Bitar, EPQ model with imperfect quality raw material, *Mathematica Balkanica*, 2012, Vol. 26, pp. 123-132.

[6] A. N. El-Kassar, M. K. Salameh, M. Bitar, Effects of time value of money on the EPQ Model with the imperfect quality items of Raw material, *Proceedings of Academy of Information and Management Sciences*, New Orleans, 2012, Vol. 16, No. 1, pp. 11-18.

[7] T. Mikdashi, A.N. El-Kassar, A. Joudi, Effects of Probabilistic Imperfect Quality Items of Raw Material on the Optimal Production Lot Size, *Journal of the Academy of Business & Economics*, 2012, Vol. 12, No. 3, pp. 75-84.

[8] M. Khan, M. Y. Jaber, A. L. Guiffrida, S. Zolfaghari, A review of the extensions of a modified EOQ model for imperfect quality items, 2011, Vol. 132, No. 1, pp. 1-12.

[9] S. Pal, G. S. Mahapatra, G. P. Samanta, A Three-Layer Supply Chain EPQ Model for Price-and Stock-Dependent Stochastic Demand with Imperfect Item Under Rework, *Journal of Uncertainty Analysis and Applications*, 2016, Vol. 4, No. 1, 10.

[10] S. Pal, G. S. Mahapatra, A manufacturing-oriented supply chain model for imperfect quality with inspection errors, stochastic demand under rework and shortages, *Computers & Industrial Engineering*, 2017, 106, 299-314.

[11] M. Khan, M. Y. Jaber, Optimal inventory cycle in a two-stage supply chain incorporating imperfect items from suppliers, 2011, *International Journal of Operational Research*, Vol. 10, No. 4, pp. 442-457.

[12] S. S. Sana, A production-inventory model of imperfect quality products in a three-layer supply chain, *Decision Support Systems*, 2011, Vol. 50, No. 2, pp. 539-547.