Team formation and selection of strategies for computer simulations of baseball gaming

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Abstract: - In computer simulation of baseball gaming, we deal with selection of strategies by applying Nash equilibrium (NE) and Pareto efficiency (PE). NE and PE, each supports that selection of strategies is in a non-cooperative or a cooperative aim, respectively. During a baseball game, each one of these aims has a relevant meaning on the manager's decision making, as we showed from the results of computer simulation. In order to apply these techniques for making strategic choices, the utility function for the strategy profiles selection is constructed based on empirical baseball data. A complementary issue on baseball computer simulation is the team formation: By applying the Hungarian method (HM) guarantees that the selection of each player is done by regarding her contribution to the team's improved performance, as an assembling of abilities, beyond his individual qualification [1]. The results from computer simulation tests hint that the use of HM for team formation combined with the use of NE or PE for the selection of strategies, lead to the team's enhanced performance in a match. Furthermore, the performance of teams diminishes when only use NE or PE without using the HM.

Key-Words: - Baseball gaming, team formation, Hungarian method, strategic choices, Nash equilibrium, Pareto efficiency.

1 Introduction

Analysis of baseball game has been made from diverse perspectives: medical and health [2], psychological-emotional [3], specific players' performance [4], or on the best team formation regarding the available players and their skills [1]. Baseball is the same cooperative game from manager's perspective, as well as non-cooperative game from players' perspective [5]. The non-cooperative games formal account is used on multi-robot formal planning [6]; as well, as part of the methodology that support multi-player games mathematical modeling [7, 8]. On a broad perspective the formal modeling of games is relevant on engineering for computer process simulation [7], or in economy for price-based coordination on hierarchical systems [9], among a lot of other fields. Complementary, concepts form engineering like the many valued quantum computation is applied for modeling the games' dynamics [10]; or concepts of economy like the modeling of the gross-domestic product is taken as an ordinary differential game of pursuit, or as a hereditary game [11]. Furthermore, sophisticated analysis on convergence and computational complexity on Lyapunov (repeted) games is graph theory analyzed [12].

Focus on baseball gaming, new forms of calculating player's valuations have changed practices on management or entertainment in MLB [3]. Neuromuscular control and stiffness regulation strategies in healthy collegiate baseball players are basically for adaptations to the throwing arm of baseball players [2]. The performance space of MLB pitchers using a Data Envelopment Analysis (DEA) is on a large dataset to identify their revealed preferences or strategies by using historical and modern observations [4]. A network DEA model for evaluating the relative efficiency of each member of a set of organizational units is applied to MLB with the aim of detecting inefficiencies that are missed by the simpler DEA models [13]. However, insufficient analysis of the selection of strategies has been conducted so far. Game Theory formal methods are essential to this analysis. Equilibrium is the mathematical formalism to deal with complex decision-making that involves the aims and intentions of members of groups having specific skills and/or tasks to deploy in a collective project. Analysis of strategies by means of applying equilibriums is the point of this paper.

Multi-player game modeling is highly complex, and the strategic analysis of games such as baseball should include a huge number of parameters for automated decision-making support. In a baseball match, selection of the best team to play is essential to achieve good results. According to the Hungarian method, the optimal team is obtained by assigning the baseball positions not necessarily to the players having the highest score for those positions, but in such a way that, from the all players-positions assigned, the emerging team guarantees the optimum playing performance [1]. The team formation must consider not only each player's individual skills, but also each one's contribution in the assigned position for the best team performance. To deal with the assignment of baseball positions the Hungarian method [14, 15] combined with Britz and Maltitz's methodology [1], offers the best choice. In addition, for baseball strategic analysis we use the Nash equilibrium and/or the Pareto efficiency. The combination of the best team in the field with the selection of strategies by some or both of these techniques supports the analytical comparison of the performance of the teams, and this is the purpose of this proposal.

The Hungarian method (HM) is a prime method to solve the problem of the optimized assignment of a set of individuals to a set of tasks, such that the total cost of this assignation is minimal, by following the approaches of Kuhn [14], Munkres [15] and Jacobi [16]. Kuhn's work was based on the work of Hungarian mathematicians D. König and E. Egervary, hence the name of the algorithm. The problem is how best to assign a group of resources, sometimes people, to a set of tasks. Mapping this problem into our analysis means determining how to assign a set of players to a set of baseball positions in such a way that the team formed achieves the best performance. In [1] HM was applied to assess the abilities of baseball players in all practical aspects of the sport and to form a team in which all the players are assigned to positions such that the collective team skill is maximized.

Baseball is a multi-player game, played on a diamond-shaped field, two teams confronted during the nine ordinary innings of the match; an inning is complete when both teams have played the offensive and defensive role; the offensive role goal is to score runs while the defensive role is to record 3 outs of the adversary; extra innings are allowed when the match score is tied at the ninth inning. The team that scores more runs at the end of the match is the winner [17, 18].

Beyond baseball team formation, HM usefulness has been proved on a diversity of applications. A feasible envy-free and bidder-optimal outcome for settings with budgets on matching markets, uses HM to find out solutions in polynomial time [19]. In [20], a clustering algorithm uses HM to solve the problem of minimal-weight cycle cover that is late use as the basic building block of their clustering algorithm. A genetic algorithm uses HM to minimize the total distance that a salesman should use to travel by finding the shortest route [21]. HM is also applied to solve manufacturing scheduling problems, so generate efficient schedules for not complicated machine constraints and scheduling horizons are long enough compared with lengths of jobs [22].

In our present analysis, the use of HM for assigning baseball player-positions pays especial attention to the statistics of pitching, batting and fielding to measure the abilities of the players such that the emerging team guarantees the optimum playing performance. In addition we use the Nash equilibrium and the Pareto efficiency for selection of strategies in baseball gaming. According to the results of computer simulations, the team selected by HM and using the Nash equilibrium produces the best team performance compared with using any other combination of techniques. Based on the correct and real occurrence of the plays in a baseball match [5], we apply the team selection module by the HM [1] along with the selection of strategies by NE or PE, as illustrated in **Fig 1**.

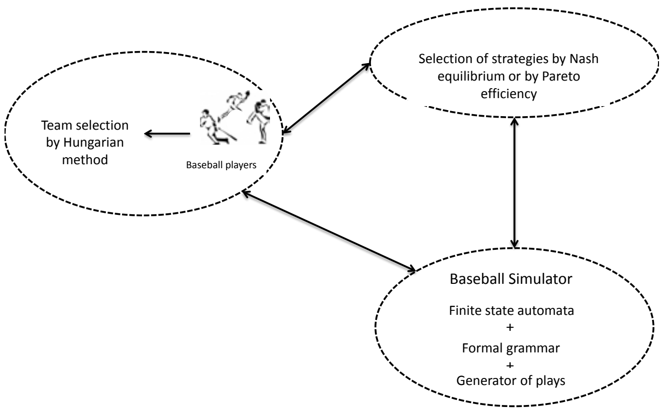

Fig. 1. Integrated approach for improving baseball gaming performance

The rest of the paper is organized as follows: Section 2 concerns the Game Theory formal methods we use for the analysis of team performance. Section 3 describes the selection of strategies by the Nash equilibrium or the Pareto efficiency. Section 4 shows the comparison of baseball teams' performance, using the HM for team formation jointly with the selection of strategies. Section 5 presents a discussion and the paper closes with conclusions.

2 Baseball Gaming and Positions Assignment

2.1 Baseball Formal Modelling

In [5, 8], the automation of baseball gaming comprises the basic and compound defense or offence plays by i player; baseball basic plays are weighted and the total is ordered regarding the frequency of their occurrence from MLB (Major League Baseball) statistics, e.g., *strike* occurs more frequently than *hit*, being precision weighted from our own computer simulation matches; the formal grammar rules set the generation of any simple or complex baseball gaming description, including a whole match; the baseball formal language is read by the associated finite state machine (FSM), hence any simple or complex baseball expression is formally correct; the occurrence of plays is in a realistic manner such that the higher the frequency of occurrence of a play in real human matches, the higher the probability that the play is included in the formal account and simulation of the match; the FSM for baseball is modeled like a shape-of-field: the home, 1st, 2nd and 3rd bases are modeled as the FSM states.

The generator of baseball plays produces correct strings of sequence of moves by regarding the baseball rules; as well, each baseball plays should occurred according to the average frequency of occurrence in real life games, so consistent with reality. The generator of strings works once having the baseball play to perform, it has to concatenate with the previous plays at the right end of a string, also indicating the player who performs. The empty string (ε) is for the beginning of a match simulation. Formal grammar, FSM and the generator of random plays are the algorithmic basis for this baseball automation that attains similar scores to human teams' matches in real life.

2.2 Player-position for the best team

To select the most effective baseball team, in [1] a methodology for tests to evaluate each player's skills is proposed, as summarized in **Table 1.**

Table 1. Britz and Maltitz's methodology

Methodology to measure baseball players' skills
Step 1: Normalize the statistical data to determine the lowest and upper value for each test $\hat{t}$, $\hat{t} = 1 \dots t$. The relative score for each observation i, $i = 1 \dots n$, in test $\hat{t}$ is a transformation of the absolute value.
Step 2: Once all the tests scores have been normalized, we obtain a matrix $T_{n \times t}$ with the relative scores; n is the number of players and t is the number of tests.
Step 3: We define a weight vector for each baseball position such that it ponders its relationship with each test $\hat{t}$. These vectors comprise the matrix $W_{t \times k}$, where k is the number of baseball positions.
Step 4: With the matrices T and W, we obtain the relationship between each player and each position that is given by the cost matrix $C_{n \times k} = T_{n \times t} \times W_{t \times k}$.

The solution is obtained by finding the combination of values in C that maximizes the efficiency, subject to certain constraints:

- Select exactly one value for each column, to ensure that each position is assigned to a player.
- Select at most one value for each row, to ensure that no player is assigned to more than one position.

The team efficiency is defined mathematically as follows (1):

$$\arg\max_{x_{ij}} \sum_{i,j=1}^{n} c_{ij}\, x_{ij} \quad (1)$$

$x_{ij} \in \{0,1\}$, when $x_{ij} = 1$if player i is assigned to the baseball position j and 0 otherwise. Subject to $\sum_{i=1}^{n} x_{ij} \le 1, \quad j = 1, \dots, n$ and to $\sum_{j=1}^{n} x_{ij} = 1, i = 1, \dots, n$.

For baseball player-positions assignment, the HM uses the cost matrix C obtained by Britz and Maltitz's methodology to find the optimal assignment such that it maximizes the baseball team efficiency. The main steps of the algorithm for player-positions assignment are in **Table 2**.

Table 2. The algorithm of Hungarian method for baseball player-positions assignment

Algorithm
Step 1: For each row of the cost matrix C, subtract the smallest element from each element of that row.
Step 2: For each column, subtract the smallest element from each element of that column.
Step 3: Draw lines through columns and rows so that all the zeros of the matrix C are covered with the minimal number of lines.
Step 4: If $k = min(nrow, ncol)$ is the number of lines covered, the zeros provide a unique complete assignation set. Otherwise, an optimal assignment of zeros is not even possible, so go to step 5.
Step 5: Find the smallest element not covered by any lines. Subtract this element from each uncovered row, and then add it to each covered column. Return to step 3.

The next section describes the selection of strategies to combine with the best team formation explained above.

3 Equilibrium for Selection Strategies

The mathematics for the formal description of the Nash and the Pareto efficiency, both used for baseball selection of strategies, follows.

3.1 Normal Formal Game

Let $P = \{1, \ldots, n\}$ be the set of players, $i \in P$, ${a_x}^i \in \Sigma^i$ be an element of the set of simple plays, and ${s_x}^i$ be a strategy of player i , ${s_x}^i \in \mathrm{S}_i$; let $G = (S_1, \ldots, S_n; u_1, \ldots, u_n)$ be the *game in normal form* [23] where:

- A strategy is a sequence of actions ${s_x}^i = {a_1}^i \ldots {a_n}^i$.
- A strategy profiles is $(s_1, \ldots, s_n)$ an n-tuple of strategies one strategy per player.
- S_i is the set of strategies for the i_{th} player.
- $\{S_1, \ldots, S_n\}$ is the set of all the S_i strategies.
- $\{u_1, \ldots, u_n\}$ is the set of all payoff functions; one per player.
- $u_i\,(s_1, \ldots, s_n) = r,\ where\ (s_1, \ldots, s_n) \in S_1 \times \ldots \times S_n, r \in \mathbb{R}$.

3.2 Nash equilibrium and Pareto efficiency

The Nash equilibrium [23] is a widely used mathematical concept, especially in the modeling of non-cooperative games. To identify the strategy profiles that fit the condition of Nash equilibrium, every strategy profile is evaluated with the payoff functions of the players, and the chosen profiles are those that for every player it is the options that produces less loss for each, individually, in a non-cooperative way. For mathematical definition, let ${s_1}^*, \ldots, {s_n}^*$ and ${s_i}^*$ the each non-cooperative player's strategies from i to the $n-1$ other players' strategies, so $({s_1}^*, \ldots, {s_i}^*, \ldots, {s_n}^*)$ fits the Nash equilibrium condition if and only if maximizes the payoff function in equation (2):

$$u_i(\,{s_1}^*, \ldots, \boldsymbol{s_i}^*, \ldots, {s_n}^*) \geq u_i(\,{s_1}^*, \ldots, \boldsymbol{s_i}, \ldots, {s_n}^*) \quad \forall i \in P, \boldsymbol{s_i} \in S_i \tag{2}$$

Every strategy profile is each payoff function valued and compared with all of the others to determine whether or not it is dominated. Given a strategy profile x_1 for each player i, the strategy profile is modified by altering the player's current strategy whilst keeping the strategies of the other $n-1$ players unchanged; if any deviation from x_1 evaluated by u_i dominates, that means, the player i's profit is higher by $u_i\,(x_2)$ then x_1 is a dominated by x_2 profile and x_1 is discarded. All the dominated profiles are discarded and the non-dominated profiles fit the Nash equilibrium. Any game in (finite) normal form has at least one strategy profile that fits the Nash equilibrium [23]. Observe that in NE every player is applying a non-cooperative perspective – less bad for him regarding the other players' strategies. The non-dominated strategy profiles, is got by the algorithm which rough coding is in **Table 3**.

Table 3. Nash equilibrium algorithm for selection of strategies in baseball gaming

Input each strategy profile and its payoff value
1:for all $x = (x_1, \ldots, x_n)$ strategy profiles
2: for all player $i = (1, \ldots, n)$
3: if x is labelled as non-dominated
4: Do the derivations in x for player i
5: if x is dominated by at least one derivation of i

6: labeled x as dominated, move to the next strategy profile
7: end if
8: end if
9: else move to the next strategy profile
10: end for
11: end for

In a broad perspective to deal with valuations on strategy profiles for multiple players, definition of Pareto dominance follows: a vector $\vec{v} = (v_1, \dots, v_k)$ is said to dominate $\bar{\bar{v}} = (\bar{v}_1, \dots, \bar{v}_k)$ if and only if $\vec{v}$ is at least partially better off than $\bar{\bar{v}}$, formally in (3) [24].

$$\forall j \in \{1, \dots, k\}, v_j \geq \bar{v}_j \ \wedge \exists i \in \{1, \dots, k\}: v_i > \bar{v}_i \quad (3)$$

Let $x = (s_1, \dots, s_n)$ be a strategy profile, and $\vec{u} = (u_1(x), \dots, u_n(x))$ be the vector with all of the valuations from payoff functions u_i, $i \in P$. Vector $\vec{u}$ is Pareto efficient if and only if there is not another vector $\bar{\bar{u}}$ which dominates $\vec{u}$. Thus, one strategy profile results in a Pareto efficient valuation if and only if it is not dominated. In other words, a strategy profile is Pareto efficient valued if there is no other strategy profile such that all players are better off and at least one player is strictly better off. Algorithm for PE is in **Table 4**.

Table 4. Pareto efficiency algorithm for selection of strategies in baseball gaming

Input each strategy profile and its payoff value
1: for each $x = (x_1, \dots, x_n)$ strategy profiles
2: Create the vectors $u(j) = ((u_1(x), \dots, u_n(x)), j = 1$ to the total number of profiles
3: end for
4: $PF = find - nondominate(v)$, PF contains profiles which are Pareto efficient
5: find in PF the profile(s) which is (are) cooperative (for all players as team)

Pareto efficiency (PE) or optimality is foundational for comparisons and discussions on social welfare and choice, as well as on the use of social welfare functions [25]. By applying Pareto efficiency for selection of strategy profiles in baseball, we select those where the profits are maximized as a group and not only individually. Each player uses the strategy such that all players, as a team, get maximum utility, so multi-player cooperation is achieved.

A set of payoff matrices comprises the quantitative analysis for a whole baseball match, by considering: 1) if the match is at the first, middle or late innings; 2) the score conditions (up, down or tie); 3) the number of outs in the innings; and 4) the eight players' positions on the bases following the methodology in [5]. Payoff matrices comprise the payoff function valuations of the strategy profiles. Each matrix entry arranges each player's strategy profile valuation. The M payoff matrix for the n players is arranged from the set of M^i payoff matrix of every player i. The M entries are the strategy profiles joint to the profile payoff value r_z, hence $((s_1, \dots, s_i, \dots, s_n), r_z)$.The payoff matrices data can support the manager's decision-making in the course of a match. One matrix for each analysis, based on NE or PE, is constructed.

3.3 Payoff functions for batter and runners

In this section, payoff functions for the baseball runners and batters are defined. Parameter $\boldsymbol{\delta}$ indicates the score conditions: $\boldsymbol{\delta}$ = 1, $\boldsymbol{\delta}$ = 0.5 and $\boldsymbol{\delta}$ = 0.2 when team is down, tied and up on score. Parameter $\boldsymbol{\eta}$ gives information about innings: $\boldsymbol{\eta = 0.2}$, $\boldsymbol{\eta = 0.5}$ and $\boldsymbol{\eta = 1}$ when the match is in first (1-3), middle (4-6) and late (7-9) innings. Parameter $\boldsymbol{\beta}$ gives information about the number the outs in the inning: $\boldsymbol{\beta = 0}$ for two outs and $\boldsymbol{\beta = 1}$ for zero or one out. The batter is identified with $\boldsymbol{a}$ and the runners with $\boldsymbol{b, c, d}$. Let $\boldsymbol{\psi} \in [\boldsymbol{0, 1}]$ be a weighting factor used in the runners' payoff function to consider the batter's strategies; and let $\boldsymbol{\gamma, \mu} \in [\boldsymbol{0, 0.1}]$ be parameters to determine the playing style by regarding in turns parameters $\boldsymbol{\alpha}$ and $\boldsymbol{\eta}$. Let $\boldsymbol{\rho_{si}}$ be the statistical occurrence (SO) of the strategy $\boldsymbol{s}$ for player $\boldsymbol{i}$, and $\boldsymbol{v_i}$ is the preference value of the player $\boldsymbol{i}$ to the profile that is being analyzed. The payoff function is given by of $\boldsymbol{\rho_{si}}$ and the parameters described previously.

The strategy profile is $(\boldsymbol{s_1}, \dots, \boldsymbol{s}, \dots, \boldsymbol{s_n})$ highlighting in bold the focus player's strategy. We observe that baseball strategy profiles for the offensive team, with men in the bat position and runners are at most a 4-tuple; actually, 3 runners at most in the field, one per base, and the batter. Hence strategy profile analysis is restricted by this condition. To define payoff functions, we should regard combinations according to the next conditions:

Payoff function for runners

Let $u_b(s_1, \dots, s, \dots s_n)$ be the runner $b's$ payoff function, with $s \in \{r, wb\}$; r notation is for try to steal the forward base, and wb for wait the batter's action.

Case man in 1st or 2nd base. If runner's statistical occurrence to steal a base ρ_{rb} is such that $\rho_{rb} < 1 - \rho_{rb}$, payoff function is in equation (4),

$$\begin{gathered} u_b(s_1, \dots, \boldsymbol{r}, \dots, s_n) = 0 + v_b, \\ u_b(s_1, \dots, \boldsymbol{wb}, \dots, s_n) = 0.1 + v_b. \end{gathered} \tag{4}$$

Otherwise, if $\rho_{rb} > 1 - \rho_{rb}$, we need to consider if any statistical occurrence of batter' strategy is greater than ψ , and use equation (5),

$$\begin{gathered} u_b(s_1, \dots, \boldsymbol{r}, \dots, s_n) = \psi * \beta + v_b, \\ u_b(s_1, \dots, \boldsymbol{wb}, \dots, s_n) = \psi + v_b. \end{gathered} \tag{5}$$

Otherwise, use equation (6),

$$\begin{gathered} u_b(s_1, \dots, \boldsymbol{r}, \dots, s_n) = 0.1 + v_b, \\ u_b(s_1, \dots, \boldsymbol{wb}, \dots, s_n) = 0 + v_b. \end{gathered} \tag{6}$$

Case man in 2nd and 1st. Let runner b in 2nd base and runner c in 1st base. In this case the advance of c depends on the advance of b, which payoff function is obtained as in the previous case. If ρ_{rc} steal a base from c is such that $\rho_{rc} < 1 - \rho_{rc}$, use equation (7),

$$\begin{gathered} u_c(s_1, \dots, \boldsymbol{r}, \dots, s_n) = 0 + v_c, \\ u_c(s_1, \dots, \boldsymbol{wb}, \dots, s_n) = 0.1 + v_c. \end{gathered} \tag{7}$$

Otherwise, if $\rho_{rc} > 1 - \rho_{rc}$, consider if $u_b(s_1, \dots, \boldsymbol{r}, \dots, s_n) > u_b(s_1, \dots, \boldsymbol{wb}, \dots, s_n)$, so if runner b tries to steal base greater than wait for the batter's action, and use equation (8),

$$\begin{gathered} u_c(s_1, \dots, \boldsymbol{r}, \dots, s_n) = \rho_{rc} * \beta + v_c, \\ u_c(s_1, \dots, \boldsymbol{wb}, \dots, s_n) = \rho_{rc} + v_c. \end{gathered} \tag{8}$$

Otherwise, use equation (9),

$$\begin{gathered} u_c(s_1, \dots, \boldsymbol{r}, \dots, s_n) = 0 + v_c, \\ u_c(s_1, \dots, \boldsymbol{wb}, \dots, s_n) = 0.1 + v_c. \end{gathered} \tag{9}$$

Case man in 3rd, or 3rd and 2nd, or 3rd, 2nd and 1st. In this case, the payoff function may include when b is in 3rd base, c in 2nd base and d in 1st base. Base stealing is neutralized since it is highly unlikely that any runner tries base stealing in these positions, and use equations in (10).

$$\begin{gathered} u_b(s_1, \dots, \boldsymbol{r}, \dots, s_n) = 0 + v_b, \\ u_b(s_1, \dots, \boldsymbol{wb}, \dots, s_n) = 0.1 + v_b. \\ u_c(s_1, \dots, \boldsymbol{r}, \dots, s_n) = 0 + v_c, \\ u_c(s_1, \dots, \boldsymbol{wb}, \dots, s_n) = 0.1 + v_c. \\ u_d(s_1, \dots, \boldsymbol{r}, \dots, s_n) = 0 + v_d, \\ u_d(s_1, \dots, \boldsymbol{wb}, \dots, s_n) = 0.1 + v_d. \end{gathered} \tag{10}$$

Payoff function for batter

To explain how to define the payoff function for batters, we use the strategies, home run h, hit hi, sacrifice flies fs and sacrifice bunt tb, so, $s \in \{h, hi, fs, tb\}$, even other strategies may be also used. The γ, μ parameters indicate the playing style, aggressive or conservative, according to results of a set of experiments, and by regarding to the score condition α and the information on innings, η. To model an aggressive style use ($\delta * \eta$==1), that yield to $\mu = 0.03, \gamma = 0$. For a conservative style use ($\delta * \eta$==0.5) that yields to $\mu = 0, \gamma = 0.08$. Otherwise, $\mu = 0, \gamma = 0$ that means that these parameters do not affect the playing style, and playing is sole restricted to characteristic of players.

The values of μ and ã came from some experiments, and their assignation is open. They are independent and will affect to different baseball plays in order to induce the playing style. They may be asymmetrical or not. The value of μ will weight to home run and hit plays, for playing aggressively, and the value of γ will weight to sacrifice fly, for playing conservatively.

Case no-runner. With no runners on bases, we only consider the playing style and the statistical occurrence of batter' strategy to define the payoff function in (11):

$$\begin{gathered} u_a(\boldsymbol{fs}) = 0. \\ u_a(\boldsymbol{tb}) = p_{tba} * \beta + \gamma. \\ u_a(\boldsymbol{h}) = \rho_{ha} + \mu. \\ u_a(\boldsymbol{hi}) = \rho_{hia} + \mu. \end{gathered} \tag{11}$$

Case one man on base. Runner b is the man in base. In this case consider the statistical occurrence of batter a strategies, the preference value v_a of the batter a on strategy profile (s, s_1), $s_1 \in \{r, wb\}$, the statistical occurrence $\rho_{s_1 b}$ from runner b on strategy s_1, and the playing style. The payoff function follows, (12).

$$\begin{gathered} u_a(\boldsymbol{fs}, s_1) = \big((\rho_{fsa} + v_a) - \rho_{s_1 b}\big) * \beta + \gamma. \\ u_a(\boldsymbol{tb}, s_1) = ((\rho_{tba} + v_a) - \rho_{s_1 b}) * \beta + \gamma. \\ u_a(\boldsymbol{h}, s_1) = ((\rho_{ha} + v_a) - \rho_{s_1 b}) + \mu. \\ u_a(\boldsymbol{hi}, s_1) = ((\rho_{hia} + v_a) - \rho_{s_1 b}) + \mu. \end{gathered} \tag{12}$$

Case: two men on base. b is the more advanced runner in base and c is the other runner. In this case, we consider the statistical occurrence of each batter

a strategies, the preference value v_a of the batter a on strategy profile (s, s_1, s_2), $s_1, s_2 \in \{r, wb\}$ for the runners b and c, the statistical occurrence of strategies s_1 and s_2 from b and c for the, respectively, and the playing style. The payoff function follows, (13).

$$u_a(\boldsymbol{fs}, s_1, s_2) = \big((\rho_{fsa} + v_a) - (\rho_{s_1b} + \rho_{s_2c})\big) * \beta + \gamma.$$
$$u_a(\boldsymbol{tb}, s_1, s_2) = \big((\rho_{tba} + v_a) - (\rho_{s_1b} + \rho_{s_2c})\big) * \beta + \gamma.$$
$$u_a(\boldsymbol{h}, s_1, s_2) = \big((\rho_{ha} + v_a) - (\rho_{s_1b} + \rho_{s_2c})\big) + \mu. \quad (13)$$
$$u_a(\boldsymbol{hi}, s_1, s_2) = \big((\rho_{hia} + v_a) - (\rho_{s_1b} + \rho_{s_2c})\big) + \mu.$$

Case: there men on base. Runner b is in 3rd base, c on 2nd base and d on 1st base. In this case, consider the statistical occurrence of each batter a strategies, the preference value v_a of the batter a on strategy profile (s, s_1, s_2, s_3), $s_1, s_2, s_3 \in \{r, w\}$ for the runners b, c and d, the statistical occurrence of the strategies s_1, s_2 and s_3 from b, c and d, respectively, and the playing style. The payoff function follows, (14).

$$u_a(\boldsymbol{fs}, s_1, s_2, s_3) = \big((\rho_{fsi} + v_a) - (\rho_{s_1b} + \rho_{s_2c} + \rho_{s_3d})\big) * \beta + \gamma.$$
$$u_a(\boldsymbol{tb}, s_1, s_2, s_3) = \big((\rho_{tba} + v_a) - (\rho_{s_1b} + \rho_{s_2c} + \rho_{s_3d})\big) * \beta + \gamma.$$
$$u_a(\boldsymbol{h}, s_1, s_2, s_3) = \big((\rho_{ha} + v_a) - (\rho_{s_1b} + \rho_{s_2c} + \rho_{s_3d})\big) + \mu. \quad (14)$$
$$u_a(\boldsymbol{hi}, s_1, s_2, s_3) = \big((\rho_{hia} + v_a) - (\rho_{s_1b} + \rho_{s_2c} + \rho_{s_3d})\big) + \mu.$$

3.4 Examples

Consider the followings circumstances in a baseball match: last innings with the match score tied, one out in the inning and runner b on 3rd base. The b options are, base stealing r, or wait (wb) for the batter's action. For batter a options are, *homerun h*, or a *sacrifice hit* (*fs*). Let $\beta = 1$, $\delta = 1$ and $\eta = 0.5$, hence $\mu = 0$, and $\gamma = 0.08$, so playing style is conservative. Using payoff functions the strategy profiles values are calculated to identify the ones that fit NE or PE.

For runner b, let his statistical occurrence of $\rho_{rb} = 0.2$. Using payoff function (4):

- $u_b(h, \boldsymbol{r}) = 0 + v_b = 0 + 0.5 = 0.5$. $u_b(h, \boldsymbol{wb}) = 0.1 + v_b = 0.1 + 0.4 = 0.5$.
- $u_b(fs, \boldsymbol{r}) = 0 + v_b = 0 + 0.3 = 0.3$. $u_b(fs, \boldsymbol{wb}) = 0.1 + v_b = 0.1 + 0.3 = 0.4$.

For batter a, using payoff function (12):

- $u_a(\boldsymbol{h}, r) = ((\rho_{ha} + v_a) - \rho_{rb}) + \mu = ((0.3 + 0.4) - 0.2) + 0 = 0.5$.
- $u_a(\boldsymbol{h}, wb) = ((\rho_{ha} + v_a) - \rho_{wbb}) + \mu = ((0.3 + 0.4) - 0.8) + 0 = -0.1$.
- $u_a(\boldsymbol{fs}, r) = ((\rho_{fsa} + v_a) - \rho_{rb}) * \beta + \delta = ((0.62 + 0.0) - 0.2) * 1 + 0.08 = 0.5$.
- $u_a(\boldsymbol{fs}, wb) = ((\rho_{fsa} + v_a) - \rho_{wbb}) * \beta + \delta = ((0.62 + 0.4) - 0.8) * 1 + 0.08 = 0.3$.

The strategy profiles and the utility value assigned by the payoff function of each player to each profile are shown in **Fig. 2**.

	$x = (s_1, \ldots, s_n)$	$u_a(x)$
1	(*h*, *r*)	0.5
2	(*h*, *wb*)	-0.1
3	(*fs*, *r*)	0.5
4	(*fs*, *wb*)	0.3

$x = (s_1, \ldots, s_n)$	$u_b(x)$
(*h*, *r*)	0.5
(*h*, *wb*)	0.5
(*fs*, *r*)	0.3
(*fs*, *wb*)	0.4

Fig. 2. Entries for the payoff matrices of players a and b

In **Fig. 3** the deviations in the strategy profiles is illustrated, such that in the analysis, depending on the values assigned by the payoff function, those strategy profiles being not dominated are identified. The example illustrates the steps to be applied to find the profiles that satisfy NE condition. For a player, x_1/x_2 means that profile x_1 dominates profile x_2, so for player 1 we have 2/4**;** for player b domination is by 3/4. Therefore, the non-dominated profiles for all players are the profiles 1, (*h*, *r*) and 4, (*fs*, *wb*), and both satisfy the NE condition. The only profile that satisfies PE condition is (*h*, *r*) because, in this profile, both players get the maximum profit as a team.

	$x = (s_1, \ldots, s_n)$	$u_a(x)$
	(*h*, *r*)	0.5
2	(*h*, *wb*)	-0.1
	(*fs*, *r*)	0.5
4	(*fs*, *wb*)	0.3

	$x = (s_1, \ldots, s_n)$	$u_b(x)$
	(*h*, *r*)	0.5
	(*h*, *wb*)	0.5
3	(*fs*, *r*)	0.3
4	(*fs*, *wb*)	0.4

Fig. 3. Deviations in the strategy profiles

Table 5 summarizes: the strategy profiles, the payoff values assigned to profiles by each player, the statistical occurrence of strategies, and the profiles which are NE or Pareto efficient or none, following the example above. Usually, the strategies in NE profiles are statistically more frequent of occurrence than strategies in Pareto efficient profiles. Particularly, sacrifice hit (fs) strategy is in NE profile and home run (h) is in Pareto efficient profile; statistically, fs is more frequent to occur than h, although h is more profitable than hi. The Pareto efficient profiles are the theoretical most profitable, but their occurrence in practice is too low.

Table 5. Summary of the analysis of strategy profiles

	Strategy profiles	Payoff value by player	Statistical occurrence (average)	NE	PE
	(h, r)	0.5, 0.5	0.3, 0.2	✓	✓
	(h, wb)	-0.1, 0.5	0.3, 0.8		
	(fs, r)	0.5, 0.3	0.7, 0.2		
	(fs, wb)	0.3, 0.4	0.7, 0.8	✓	

Profiles (h, r) and (fs, wb) fit the NE condition because hold equation (2)

- $u_a(h, r) \geq u_a(fs, r)$
- $u_b(h, r) \geq u_b(h, wb)$

and

- $u_a(fs, wb) \geq u_a(h, wb)$
- $u_b(fs, wb) \geq u_b(fs, r)$

Profile (h, r) is Pareto efficient because $\vec{u} = ((u_a(h, r), u_b(h, r))$ given $s_1 \in \{hi, h\}, s_2 \in \{r, wb\}$, there is not a vector $\bar{\bar{u}} = (u_a(s_1, s_2), u_b(s_1, s_2))$ which dominates $\vec{u}$, see equation (3).

Next, the analytical comparison of the use of HM for player-positions assignment combined with NE or PE for selection of strategies follows. The way to assign player-positions is of great importance and a team perspective analysis is needed. In addition, the selection of strategies is essential for a good team performance.

4 Team performance comparisons

The best team performance is obtained by jointly applying the HM for team formation with a method for the selection of strategies. Actually, the HM+NE combination allows the best team performance during matches gaming. The use of HM produces the right player-positions assignment regarding the best team performance that should be applied based on statistical information from the MLB results. Data on two MLB teams and each player's statistics are input to a baseball simulator. The statistical measures to be considered in this analysis, for batting, pitching and fielding, are shown in **Table 6**. A total of 44 players from the 2012 MLB season, 22 from the Boston Red Sox and 22 from the New York Yankees are considered. The different combinations gamed by the baseball teams are practiced by a baseball simulator.

Table 6. Statistical measure to be considered to quantify the baseball players

Batting		**Pitching**		**Fielding**	
R	Runs	**W-L%**	Percentage of games won	**E**	Errors
SB	Stealing bases	**ERA**	Runs allowed per game	**DP**	Double plays
OPS	OBP + SLG	**WHIP**	(BB + Hits) / Innings	**RF/9**	(PO + A) / Innings
GDP	Double plays	**H/9**	Hits per game	**SB**	Stealing bases
SH	Sacrifice hits	**HR/9**	Home runs per game		
SF	Sacrifice flys	**BB/9**	Based on balls per game		
IBB	BB intentional	**SO/9**	Strikeouts per game		

In **Table 6**, OBP is the percentage of times an offensive player reaches a base, SLG is the total bases reached by the total number at-bat, BB is for the bases on balls allowed the player, and PO the outs achieved by the defensive team. From the MLB players, we selected 12 pitchers and 30 field players, 6 and 15 from each team, respectively. The statistical data is normalized [0, 1] for an easy analysis. Applying HM on MLB players statistics, we formed a team of 10 players selected from the 42 possible ones. In this process, a weight matrix W that indicates the relevance of each measure to each position is required, and its values, given by baseball experts, are shown in **Table 7**.

Table 7. The *W* matrix of weight

Position/ Measure	**C**	**1B**	**2B**	**SS**	**3B**	**LF**	**CF**	**RF**	**D**	**P**
R	0.3	0.25	0.25	0.15	0.2	0.3	0.3	0.35	0.45	0
SB	0	0.1	0.1	0.05	0.1	0	0	0	0	0
OPS	0.5	0.45	0.5	0.5	0.55	0.75	0.7	0.75	0.85	0
GDP	-0.05	-0.1	-0.1	0	-0.1	-0.25	-0.25	-0.3	-0.3	0
SH	0	0.1	0.1	0.1	0.15	0	0	0	0	0
SF	0.1	0	0.05	0	0.05	0.1	0.15	0.1	0	0
IBB	0.05	0	0	0	0	0.05	0.1	0.05	0	0
W-L%	0	0	0	0	0	0	0	0	0	0.6
ERA	0	0	0	0	0	0	0	0	0	-0.15
WHIP	0	0	0	0	0	0	0	0	0	-0.05
H/9	0	0	0	0	0	0	0	0	0	-0.05
HR/9	0	0	0	0	0	0	0	0	0	-0.1
BB/9	0	0	0	0	0	0	0	0	0	-0.05
SO/9	0	0	0	0	0	0	0	0	0	0.8
E	-0.2	-0.2	-0.2	-0.35	-0.15	-0.05	-0.1	-0.05	0	-0.05
DP	0.05	0.2	0.3	0.25	0.15	0	0	0	0	0.1
RF/9	0.45	0.4	0.3	0.3	0.2	0.1	0.1	0.1	0	0.15
SB	-0.2	-0.2	-0.3	0	-0.15	0	0	0	0	-0.2

The selected MLB players using HM are listed in **Table 8,** and the probability matrix for offensive actions is presented in **Table 9**, where each abbreviation denotes: H-hits, HR-home run, 2B-doubles, SF-sacrifice fly, SH-sacrifice bunt, SB-stealing bases, GDP-double plays, SO-strikeouts, BB-bases on balls.

Table 8. Players selected by applying the Hungarian method

Player	**Original position**	**Team**	**Assigned position**	**Benefit**
Mark Teixeira	1B	Yankees	C	1.1187
Adrián González	1B	Red Sox	1B	0.9915
Dustin Pedroia	2B	Red Sox	2B	1.0352
Robinson Cano	2B	Yankees	SS	0.8308
Derek Jeter	SS	Yankees	3B	0.7256
Cody Ross	RF	Red Sox	LF	0.7446
David Ortiz	DH	Red Sox	CF	1.0607
Curtis Granderson	CF	Yankees	RF	0.9595
Nick Swisher	RF	Yankees	DH	0.9023
CC Sabathia	SP	Yankees	SP	1.1444
TOTAL				**9.5134**

Table 9. Probability of offensive actions per player

	H	**HR**	**2B**	**SF**	**SH**	**SB**	**GDP**	**SO**	**BB**
1	0.2996	0.0310	0.0764	0.0145	0.0000	0.0000	0.0186	0.1674	0.0641
2	0.2895	0.0266	0.0693	0.0107	0.0018	***0.0355***	0.0160	***0.1066***	0.0853
3	0.2668	0.0462	0.0714	0.0126	0.0021	0.0042	0.0231	0.2710	0.0882
4	***0.3179***	***0.0710***	***0.0802***	0.0093	0.0000	0.0000	0.0185	0.1574	***0.1728***
5	0.2156	0.0458	0.0515	***0.0229***	0.0000	0.0038	0.0210	0.1584	0.1031
6	0.2812	0.0473	0.0689	0.0029	0.0000	0.0043	0.0316	0.1377	0.0875
7	0.2919	0.0203	0.0432	0.0014	***0.0081***	0.0122	0.0324	0.1216	0.0608
8	0.2018	0.0629	0.0263	0.0102	0.0015	0.0146	***0.0073***	0.2851	0.1096
9	0.2340	0.0385	0.0577	0.0080	0.0016	0.0032	0.0144	0.2260	0.1234
Avg	**0.2665**	**0.0433**	**0.0606**	**0.0103**	**0.0017**	**0.0086**	**0.0203**	**0.1812**	**0.0994**

4.1 Results

In modeling and simulation of multi-player games, the verification and validation are essential prerequisites to the credible and reliable use of a model and its results. A total of six hundred computer simulations of baseball matches were performed for the next items, **one hundred (100) simulations** each.

– Team 1 (T_1) uses HM and NE; Team 2 (T_2) uses NE.
– T_1 uses HM and NE; T_2 uses PE.
– T_1 uses HM and PE; T_2 uses NE.
– T_1 uses HM and PE; T_2 uses PE.
– T_1 uses HM and NE; T_2 uses HM and PE.
– T_1 uses HM and PE; T_2 does not use any method.

The results in **Fig. 4** correspond when T_1 uses HM to assign player-positions and NE for strategic analysis while T_2 uses only NE; T_1 achieved more victories 55/45 and in a total of 5 baseball matches reached *extra innings*. **Fig. 5** shows the results when T_1 uses HM selection and NE while T_2 only uses PE; T_1 won more victories 60/40 and 5 baseball matches reached *extra innings*. **Fig. 6** shows the results when T_1 uses HM selection and PE while T_2 only uses NE; T_1 won more victories 53/47 and 4 baseball matches reached *extra innings*. **Fig. 7** shows the results when T_1 uses HM selection and PE while T_2 only uses PE; T_1 scored more victories 59/41 and 8 baseball matches reached *extra innings*. **Fig. 8** shows the results when T_1 uses HM selection and NE while T_2 uses HM selection and PE; T_1 won 54/46 and 4 baseball matches extended to *extra innings*. **Fig. 9** shows the results when T_1 uses HM selection while T_2 only uses its statistics; T_1 won 61/39 victories and only 6 baseball matches reached *extra innings*.

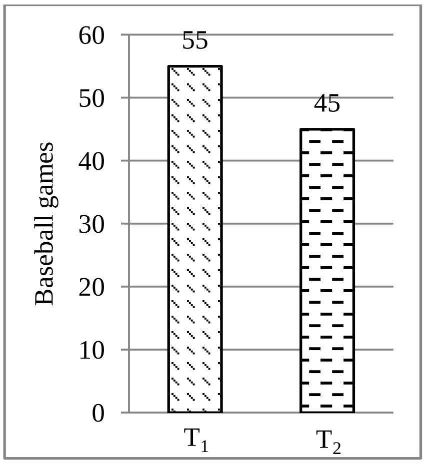

Fig. 4. HM+NE *versus* NE

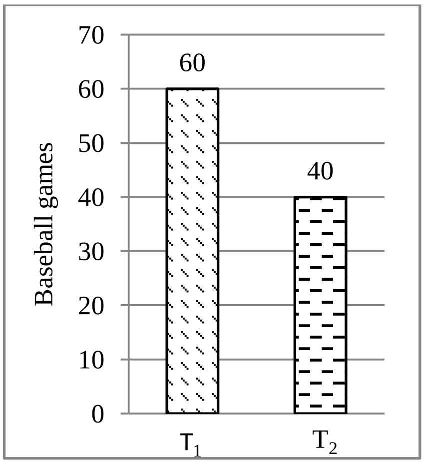

Fig. 5. HM+NE *versus* PE

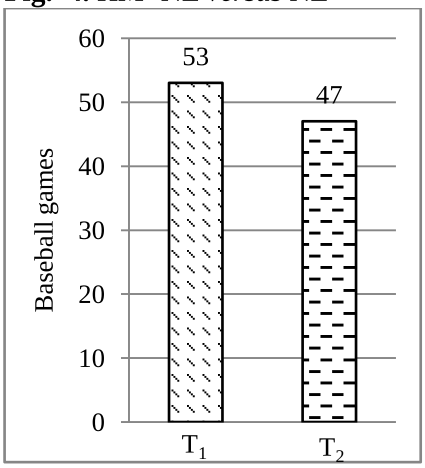

Fig. 6. HM+PE *versus* NE

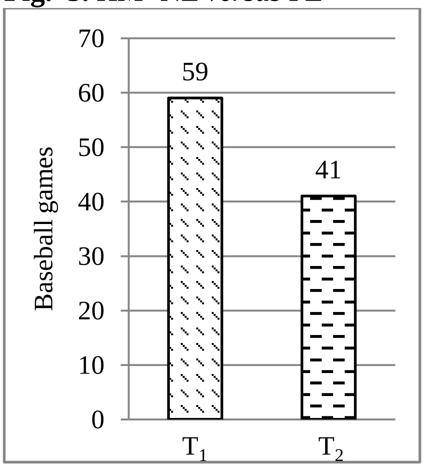

Fig. 7. HM+PE *versus* PE

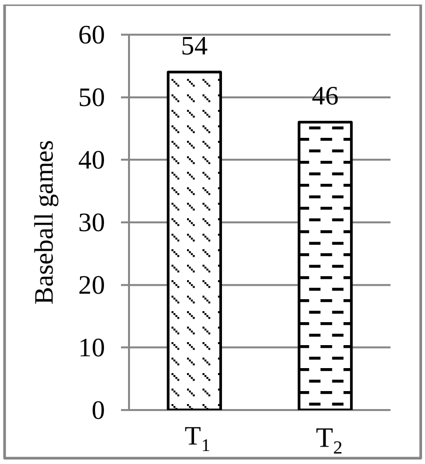

Fig. 8. HM+NE *versus* HM+PE

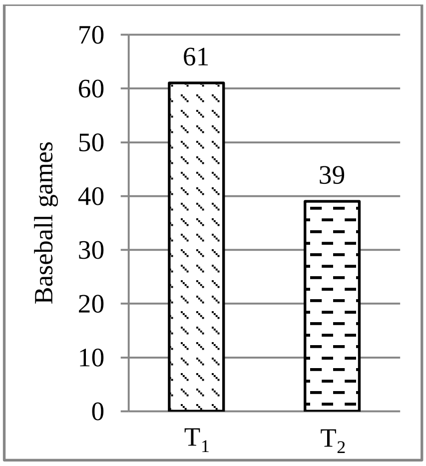

Fig. 9. HM *versus* No-Method

4.2 Comparative analysis

In **Fig.10** illustrates the behavior when T_1 only uses HM with NE while T_2 uses PE, NE, HM with PE and HM with NE. The T_1 performance is superior to all other techniques used by T_2 and the T_2 performance improves by changing to a better technique.

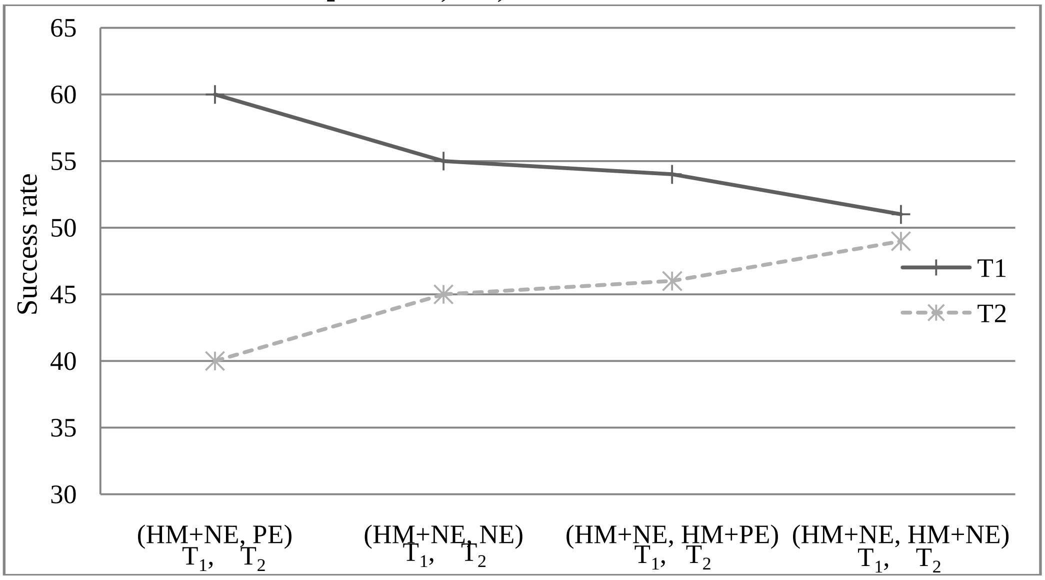

Fig. 10. T_1 only uses (HM+NE) and T_2 uses (PE, NE, HM+PE, HM+NE).

Fig. 11 describes the use of different techniques by T_2 while T_1 only uses HM+PE. The T_1 performance decays while the T_2 performance increases and even exceeds the T_1 performance when T_2 uses HM+NE.

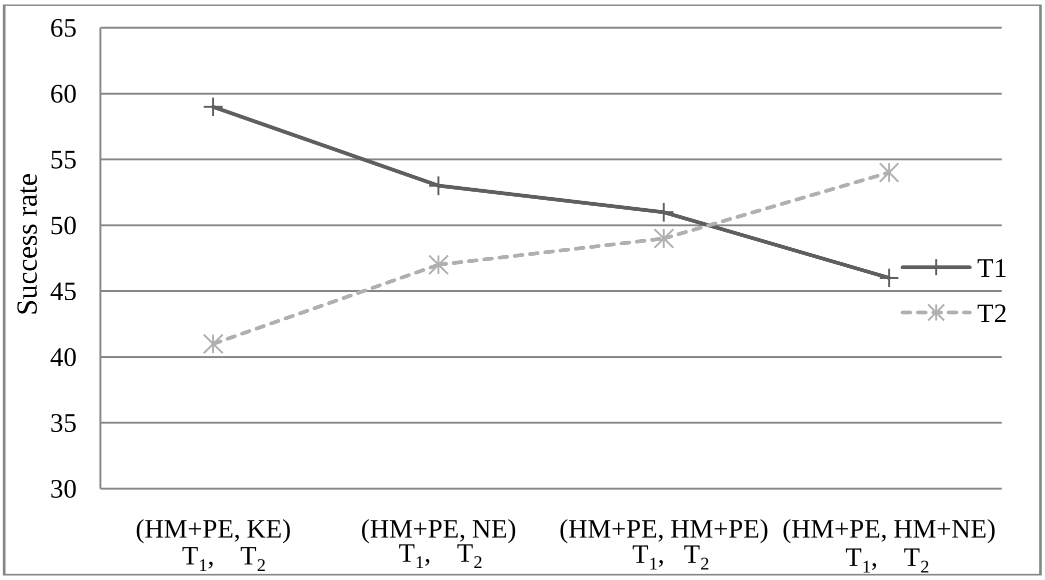

Fig. 11. T_1 only uses (HM+PE) and T_2 uses (PE, NE, HM+PE, HM+NE)

In summary, the team selected by HM and using NE for selection of strategies showed a better performance than any other team which used any other combination of methods. In addition, a comparison of when T_1 used HM for player-positions assignment when T_2 did not use HM or perform any selection of strategies shows that the T_1 performance is better.

5 Discussions

The task assignment approaches that use HM [23, 24], no longer require a strategic analysis after the assignment is made. However, in baseball, besides the player-positions assignment challenge, there are factors during a match that require a meticulous strategic analysis to identify suitable strategies to guide the team to victory. Nash equilibrium and Pareto efficiency for selection of strategies in baseball gaming, jointly used with the Hungarian method to choose a baseball team, fit the aim of improving the team performance. Actually, according to our set of computer simulation tests the combination HM+NE methods produces the best baseball team performance during match playing. HE+PE, theoretical best, frequently cannot be practiced by empirical reasons ever present in real matches. The home run is one of the best baseball offensive options and under some circumstances, it belongs to Pareto efficient profiles. If it happened is the best option for the team, but has a low of statistical occurrence in practice. Whenever the risk of trying to perform a home run is less than the benefit that the team could get then, it is a good option to try to perform it; otherwise it is better to use another option. We emphasize, on these kinds of games not always the best theoretical options can be practiced.

The classic NE need be complemented to deal with interactions in social or enterprise organizations so the hybrid approach fine model these complexities. The problem of coordination when a game has more than one NE, is taken by analyzing Pareto and risk dominations as criteria to select one of them [26]. In a two-player coordination game, the effectiveness of communication is sensitive to the structure of payoffs and the communication does not necessarily lead to the Pareto-dominant equilibrium [27]. The reengineering strategy in supply chain should deal with the conflicts and benefits of reengineering for multiples entities, the variety of reengineering modes and reengineering selection for cross-organizational [28], that cannot be fully allowed by lone NE analysis. The lazy bureaucrat scheduling problem through a game-theoretic issue [29], define the potential functions to prove the existence of a Nash equilibrium solution, and present the pseudo polynomial time algorithm to find such a solution.

As we discuss previously, the theoretical Pareto-efficient allows an optimal design of strategies. However, in real human baseball matches, the theoretical design cannot occur by the presence of uncertain factors –beyond the team’s control. Whereby, the use of theoretical Pareto efficient strategies under some circumstances in baseball game is low feasible. In a baseball match there are many uncertain factors such as, human ways of pitching, running and batting, or natural factors like wind speed or the height of the place, which affect the playing performance. The stochastic nature of the baseball game is well-modeled by our approach, and the convergence to some identified equilibrium points observes the statistics from real matches.

In Kantian equilibrium (KE) [30] all players have a common strategy space S, so the normal form game is $G = (S; u_1, \dots, u_n)$; a strategy profile $(s_1, \dots, s_n)$ fits Kantian equilibrium condition if equation (18) holds:

$$u_i\ (s_1, \dots, s_n) \geq u_i\ (\alpha(s_1, \dots, s_n))\ \ \forall\ i \in P, \alpha \in \mathbb{R}_+ \quad (18)$$

All of the player's action value is weighted by the same factor α. Kantian equilibrium models community cooperation in equal conditions and no one player takes improves doing worse any other. By KE usage every player is applying the Pareto efficient best own strategy from a cooperative perspective, and there is at least one strategy profile for a game in normal form that fits Kantian equilibrium, as for NE. For KE, all players get the maximum profit, in fact the player changes his strategy if and only if each player changes its strategy by the same multiplicative factor α, we interpret α as a change in the strategy profile for all players, and perhaps, we do not fit strictly with real definition on (18) but we assure at least that the profiles be Pareto efficient. Nevertheless, due to the lack of interpretation of KE in this kind of games, we can use KE as previously discussed.

Couple and team formation

In game theory (GT), a problem of finding a stable matching between two sets of elements is known as the stable marriage problem (SMP) [31]. In this problem, we have a set of *n* men and *n* women where each person has his/her own preference list of the persons that he/she wants to marry. The goal is to have a set of stables marriages, such that, there are no two persons of opposite set who would prefer other person than his/her current partner. Gale and Shapley in [31] proved that there is a stable set of marriages. In the case of baseball, the solution of SMP does not satisfy the player-positions assignment because SMP emphases finding solutions by couple rather than by group, i.e., the couple (*player*, *baseball-position*) is attended individually without regard the others couples. On the opposite, HM allows to find out the best couples (*player*, *baseball-position*) thinking of forming the best team. The best baseball-positions couples are not the assigned sole regarding the best statistics-player, but such that all baseball-positions-player are attended in a way that emerging team guarantees the best playing performance.

On the convergence to equilibrium points

In the examples in Section 3.4, different circumstances of a baseball match are analyzed using Nash equilibrium and Pareto efficiency for making strategic choices. Particularly, one strategy profile of sacrifice plays was finding by using Nash equilibrium. Qualitative analysis [32] and statistical studies [33] about the pertinence of sacrifice plays in a baseball match explain the best moment to apply them. The equilibrium analysis on the strategy profiles of sacrifice plays in a baseball match being supported by computer simulations [5], found that these profiles fit the Nash profiles when circumstances of baseball match are, the last innings, the match score tied, one player on third base and one/none out(s) in the inning. For these circumstances sacrifice plays are applied opportunely to reach the best result. The convergence to these profiles is by means of increase the probability of occurrence of these plays; in practice, the manager should indicate his players try to perform these plays, so the probability to these plays is increased. Moreover, according to our experimental results, the probability of convergence to the strategy profiles of sacrifice plays is over 60 percent, at the last innings and tied score. We claim a probabilistic convergence to desired profiles because the stochastic nature of the baseball game, many uncertain factors –beyond the team's control.

A convergence method proposed by Clempner and Poznyak [12], finds an equilibrium strategy profile using a vector Lyapunov-like function in strictly dominated games, where strategy profiles with dominated strategies are deleted. A Lyapunov strategy profile (point) is a Nash equilibrium point. The convergence method is applied to the prisoner's dilemma and battle of the sexes, two players and two strategies games. For the future, it would be interesting analyze the Lyapunov-like function for convergence to Lyapunov strategy profiles on multi-players games, like baseball that have many more than two players and strategies.

Conclusion

Results from computer simulations of baseball matches show that, although the usual *non-cooperative* qualification to NE, it is a relative adjective, up to the real circumstance. In the context of a baseball match, with several parameters out of the players' and manager's control, NE allows identify strategy profiles for effective cooperation in real circumstances of baseball gaming. The use of

NE prevents to try plays or strategies with low statistical occurrence, so to avoid the risk to lose score points. It means that NE strategy profiles frequently include plays and strategies with higher statistical occurrence, so they are more feasible in real circumstances of matches. Furthermore, NE, by avoiding the risk to lose score points induces an effective cooperation for a team. On the other hand, PE formal account, it induces to choose the theoretically optimum strategy profiles. We observe that the best plays and strategies have low statistical occurrence, so few time to be practiced in real baseball gaming circumstances. The Pareto efficient strategy profiles are less likely to occur than the Nash ones. Strategies in Pareto efficient profiles may be the most profitable but their probability of occurrence is low, and it moderates the use of Pareto efficiency to identify circumstances of cooperation in real circumstances of baseball gaming. Combined application of both, Nash and Pareto efficiency for strategic choices on multi-player baseball game is relevant. By applying the hybrid methods for guiding team actions in order to increase the probability of victory, is a major advantage on circumstances of complex social interactions.

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