

Solution evolutionary method of compressible boundary layers stability problems

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Abstract: In the paper the solution evolutionary method of hydrodynamic stability problems is offered. The essence of this method consists that an arbitrary initial disturbance is described by one wave with the greatest increment on large times, which varies according to the law $\exp(-i\omega\tau)$. In order to verify the new method and to work out the numerical scheme the stability calculations were carried out also on the base of the classical theory. The evolutionary method is used to study the effects of the gas injection direction through a porous surface on stability of a supersonic boundary layer at the Mach number $M=2$. It was established that with reduction of a slope angle of a gas injection to the flat plate the stability of a boundary layer increases, and the tangential blowing influence on a boundary layer stability of is poorly

Key-Words: evolutionary method, compressible boundary-layer, hydrodynamic stability, gas injection, numerical scheme

1 Introduction

Porous cooling is an effective method for a thermal protection of heat-stressed elements of technical apparatus [1-2]. The basic mechanism of a porous cooling consists in absorption of the thermal energy of the hot gas by a cold gas which is injected through a permeable surface. While the direction of a blowing cold gas relatively streamlined surfaces may be different (from normal to tangential). In addition to the heat protection, there is another important problem. It is connected with the control of the laminar-turbulent transition. It is known that with an increase of a gas density near the wall the boundary layer stability increases. To raise density near the walls it is possible to blow heavy gas through a porous wall. For a case of a subsonic boundary layer the possibility of its stabilization on the basis of a heavy gas injection was confirmed in [3]. Normal inflow relatively a streamlined surface promotes to an appearance of an inflection point in a velocity profile that leads to the destabilization of flow [4-6]. For reduction of this effect it is possible to blow a gas under some angle to the main flow direction. The tangential inflow is the limiting case the vectored injection. The boundary layer stability with the gas injection under angles not equal to $\pi/2$ to the surface has not been investigated so far.

It served as motivation of the real paper in which the case of the uniform gas injection is considered.

As for methods of the stability characteristics calculation, it is necessary to notice that, as a rule, authors use the standard method of elementary waves leading to the solution of the eigenvalue

problem of the homogeneous system of ordinary differential equations with homogeneous boundary conditions [4.7]. The lack of this method is in the difficulty to find the waves with the highest increment. Its search comes to the end successfully under a condition if its approximate value is known. Therefore, growing in time waves given the front direction and wave number, depending on the incoming values of main flow, such as Mach number, Reynolds number and others, are calculated on the base of small changes of determinative parameters. However, the wave with the maximum increment for some basic terms will not be the determinative one (with a maximum growth factor), for the other flow parameters. Therefore, it is desirable to have such a calculation method which would guarantee uniquely obtaining of the wave with the highest increment. For linear problems, this can be achieved by an evolutionary method by the integration over time of partial differential equations. Because any disturbance, satisfying uniform boundary conditions can be decomposed into the sum of the waves with different increments, the wave with the largest increment will dominate at large times. This method can be called by the usual term - the establishing method. In contrast to the generally accepted method of establishing when the solution goes to the constant, in our case the solution goes to the exponential dependence on time. In the hydrodynamic stability theory there are the temporary instability (wave number on uniform spatial coordinates are real) and the spatial instability (when perturbations with real frequencies growth in the space). At low

amplification rate in the space and time, which is characteristic for the boundary layers, temporal and spatial increments are associated with the simple approximate relation: the amplification rate in space equals the negative temporary divided by the wave group velocity [4.8]. If necessary, the more precise value of the spatial amplification rate can be obtained by the classical method.

In this paper we investigate theoretically influence of the gas blowing direction through the porous surface the supersonic boundary layer stability using the classical method of elementary waves and evolutionary method.

2 Basic equations

The initial equations of the disturbances evolution in a supersonic boundary layer are well known Navier - Stokes, continuity, energy and state [9]:

$$\begin{aligned} \rho^* \frac{d\mathbf{v}^*}{dt} &= -\text{grad}(p^*) - \frac{2}{3} \text{grad}(\mu^* \text{div}(\mathbf{v}^*)) + 2\text{Div}(\mu^* \dot{S}), \\ \frac{d\rho^*}{dt} + \rho^* \text{div}(\mathbf{v}^*) &= 0, \\ c_p \rho^* \frac{dT^*}{dt} - \frac{dp^*}{dt} &= 2\mu^* \dot{S}^2 - \frac{2}{3} \mu^* (\text{div}(\mathbf{v}^*))^2 + \\ &+ \text{div} \left(\mu^* \text{grad} \left(\frac{c_p T^*}{Pr} \right) \right), \\ p^* &= \rho^* R T^*. \end{aligned}$$

Here \mathbf{v}^* – velocity with components (u^*, v^*, w^*) in x, y, z -directions, p^*, ρ^*, T^* – pressure, density and temperature, c_p – specific heat at constant pressure, R – gas constant, \dot{S} – velocity tensor, $Pr = c_p \mu^* / \lambda^*$, λ^* – thermal conductivity, μ^* – dynamic viscosity.

In this paper the disturbance in supersonic boundary layers on a flat plate at high Reynolds numbers $Re_x = u_e^* x \rho_e^* / \mu_e^*$ is explored, where u_e^*, ρ_e^*, μ_e^* – velocity, density and dynamic viscosity on external border of boundary layer, x – distance from the front edge of the plate. In this case the main flow is independent on the transverse z coordinate, weakly depends on x - coordinate and velocity in y - direction is low. Therefore, the main (stationary) flow can be considered as a plane-parallel. All its parameters depend on the one coordinate y , only velocity in the x -direction $u^*(y)$ is unequal to zero. We have introduced the dimension-

less: coordinates, time, and flow parameters in the form: $dX = dx / \delta$, $dY = dy / \delta$, $dZ = dz / \delta$, $d\tau = u_e^* dt / \delta$, $v = v^* / u_e^*$, $p = p^* / p_e^*$, $T = T^* / T_e^*$, $\rho = \rho^* / \rho_e^*$, where $\delta = \sqrt{x \mu_e^* / \rho_e^* u_e^*}$ – the boundary layer thickness, index (e) indicates that the value is taken at the outer edge of the boundary layer.

Velocity, density, pressure and temperature of the compressible gas in the boundary layer can be represented in the form:

$$\begin{aligned} u &= U(Y) + \varepsilon u', v = \varepsilon v', w = \varepsilon w', p = P(Y) + \varepsilon p', \\ T &= T_0(Y) + \varepsilon \theta', \rho = 1 / T_0(Y) + \varepsilon \zeta', \end{aligned}$$

where $U(Y), P, T_0(Y)$ – velocity, pressure, and temperature in the unperturbed laminar boundary layer. The perturbed parameters are marked by the prime, which depend on X, Y, Z and τ . Equations for linear disturbances in the approximation of Dana-Lin, Alekseev [7, 10] for the two-dimensional boundary layer have the form [10]:

$$\begin{aligned} \frac{1}{T_0} \left(\frac{\partial u'}{\partial \tau} + U \frac{\partial u'}{\partial X} + \frac{\partial U}{\partial Y} v' \right) &= -\frac{1}{\gamma M^2} \frac{\partial p'}{\partial X} + \frac{\mu}{Re} \frac{\partial^2 u'}{\partial Y^2}, \\ \frac{1}{T_0} \left(\frac{\partial v'}{\partial \tau} + U \frac{\partial v'}{\partial X} \right) &= -\frac{1}{\gamma M^2} \frac{\partial p'}{\partial Y}, \\ \frac{1}{T_0} \left(\frac{\partial w'}{\partial \tau} + U \frac{\partial w'}{\partial X} \right) &= -\frac{1}{\gamma M^2} \frac{\partial p'}{\partial Z} + \frac{\mu}{Re} \frac{\partial^2 w'}{\partial Y^2}, \\ \frac{\partial \zeta'}{\partial \tau} + U \frac{\partial \zeta'}{\partial X} + \frac{d}{dY} \left(\frac{1}{T_0} \right) v' + \frac{1}{T_0} \left(\frac{\partial u'}{\partial X} + \frac{\partial v'}{\partial Y} + \frac{\partial w'}{\partial Z} \right) &= 0, \\ \frac{1}{T_0} \left(\frac{\partial \theta'}{\partial \tau} + U \frac{\partial \theta'}{\partial X} + \frac{dT_0}{dY} v' \right) &= \\ = \frac{\gamma - 1}{\gamma} \left(\frac{\partial p'}{\partial \tau} + U \frac{\partial p'}{\partial X} \right) + \frac{\mu}{Pr Re} \frac{\partial^2 \theta'}{\partial Y^2}, \\ p' / P &= T_0 \zeta' + \theta' / T_0. \end{aligned} \quad (1)$$

The system (1) should be solved with boundary conditions [4]:

$$u' = v' = c\theta' + d \frac{d\theta'}{dY} = 0, \text{ at } Y = 0, \infty. \quad (2)$$

The classical theory of stability founded on the method of elementary waves $\mathbf{a}', p' = (\mathbf{a}(Y), \pi) \exp(i(\alpha X + \beta Z - \omega \tau))$. Here components the vector \mathbf{a} $f, \varphi, h, \theta, \zeta$ are amplitudes of perturbations $u', v', w', \theta', \zeta'$. Equations (1) are given to system of the linear ordinary differential equations:

$$\frac{1}{T_0} \left[i\alpha(U - c)f + \varphi \frac{dU}{dY} \right] + \frac{i\alpha\pi}{\gamma M^2} = \frac{\mu}{Re} \frac{d^2 f}{dY^2},$$

$$\begin{aligned} \frac{1}{T_0} i\alpha(U-c)h + \frac{i\beta\pi}{\gamma M^2} &= \frac{\mu}{Re} \frac{d^2 h}{dY^2}, \\ \frac{1}{T_0} i\alpha(U-c)\varphi + \frac{1}{\gamma M^2} \frac{d\pi}{dY} &= 0, \\ i\alpha(U-c)\zeta + \frac{d}{dY} \left(\frac{1}{T_0} \right) \varphi + \frac{1}{T_0} \left(i\alpha f + i\beta h + \frac{d\varphi}{dY} \right) &= 0, \\ \frac{1}{T_0} \left[i\alpha(U-c)\theta + \varphi \frac{dT}{dY} \right] + (\gamma-1) \left(i\alpha f + i\beta h + \frac{d\varphi}{dY} \right) &= \\ = \frac{\gamma\mu}{PrRe} \frac{d^2\theta}{dY^2}, \\ \pi/P &= T_0\zeta + \theta/T_0. \end{aligned} \tag{3}$$

From (2) it is possible to receive new boundary conditions:

$$f = \varphi = c\theta + d \frac{d\theta}{dY} = 0 \text{ при } Y = 0, \infty. \tag{4}$$

Wave numbers α and β are real at the temporary instability and frequency ω is complex-valued, which is a result of solving the eigenvalues problem of homogeneous equations with homogeneous boundary conditions. The flow in the boundary layer is unstable for positive values of the imaginary part of $\omega = \omega_r + i\omega_i$.

In general, the number of eigenvalues is infinite, or at least large. However, we are interested primarily in frequency with the highest values of the imaginary part. The search of such frequencies is a challenge.

The evolutionary method for finding of such frequencies is proposed and realized in this paper for the first time.

The essence of this method consists that an arbitrary initial disturbance is described by one wave with the greatest increment on large times, which varies according to the law $\exp(-i\omega\tau)$.

For disturbances $\mathbf{a}' = \tilde{\mathbf{a}}(\tau, X, Y) \exp(i\beta Z)$ equations (1) and boundary conditions (2) take the form:

$$\begin{aligned} \frac{\partial \tilde{f}}{\partial \tau} &= -U \frac{\partial \tilde{f}}{\partial X} - \tilde{\varphi} \frac{\partial U}{\partial Y} - \frac{T_0}{\gamma M^2} \frac{\partial \tilde{\pi}}{\partial X} + \frac{\mu T_0}{Re} \frac{\partial^2 \tilde{f}}{\partial Y^2}, \\ \frac{\partial \tilde{h}}{\partial \tau} &= -U \frac{\partial \tilde{h}}{\partial X} - \frac{T_0}{\gamma M^2} i\beta \tilde{\pi} + \frac{\mu T_0}{Re} \frac{\partial^2 \tilde{h}}{\partial Y^2}, \\ \frac{\partial \tilde{\varphi}}{\partial \tau} &= -U \frac{\partial \tilde{\varphi}}{\partial X} - \frac{T_0}{\gamma M^2} \frac{\partial \tilde{\pi}}{\partial Y}, \\ \frac{\partial \tilde{\zeta}}{\partial \tau} &= -U \frac{\partial \tilde{\zeta}}{\partial X} - \tilde{\varphi} \frac{\partial r}{\partial Y} - \frac{d}{dY} \left(\frac{1}{T_0} \right) \left(\frac{\partial \tilde{f}}{\partial X} + i\beta \tilde{h} + \frac{\partial \tilde{\varphi}}{\partial Y} \right), \end{aligned} \tag{5}$$

$$\begin{aligned} \frac{\partial \tilde{\theta}}{\partial \tau} &= - \left[U \frac{\partial \tilde{\theta}}{\partial X} + \tilde{\varphi} \frac{\partial T}{\partial Y} \right] - \\ &- (\gamma-1) T_0 \left(\frac{\partial \tilde{f}}{\partial X} + i\beta \tilde{h} + \frac{\partial \tilde{\varphi}}{\partial Y} \right) + \frac{\gamma\mu T_0}{PrRe} \frac{\partial^2 \tilde{\theta}}{\partial Y^2}, \\ \tilde{\pi} &= \tilde{r} / \rho + \tilde{\theta} / T, \\ \tilde{f}(0) = \tilde{\varphi}(0), \tilde{h}(0), \left(c\tilde{\theta} + d \frac{d\tilde{\theta}}{dY} \right) \Big|_{y=0} &= 0, \\ \tilde{f} \Big|_{y=\infty}, \tilde{\theta} \Big|_{y=\infty}, \tilde{\zeta} \Big|_{y=\infty}, \tilde{h} \Big|_{y=\infty} &= 0. \end{aligned}$$

3 The computational domain and the numerical scheme

The problem was solved for the periodic perturbation in the coordinate x , i.e. $\tilde{\mathbf{a}}(Y, X, \tau) = \tilde{\mathbf{a}}(Y, X + L, \tau)$ and monochromatic conditions on the lateral coordinate z . The region of an integration in the normal direction was enclosed in the interval $0 < Y < Y^*$. We took into account the conditions of equality to zero disturbances at Y^* . Value Y^* was accepted rather large that its additional increase did not lead to essential change of disturbances increments.

For the integration of the system (5) we used 2-step finite-difference scheme [11].

The first step:

$$\begin{aligned} \frac{\tilde{f}^{n+\frac{1}{2}} - \tilde{f}^n}{\Delta} &= -U' \tilde{\varphi}^n + \\ &+ \frac{\mu T_0}{Re} \frac{\tilde{f}_{i+1}^{n+\frac{1}{2}} - 2\tilde{f}_i^{n+\frac{1}{2}} + \tilde{f}_{i-1}^{n+\frac{1}{2}}}{h_y^2} - \frac{T_0}{\gamma M^2} \frac{(\tilde{\pi}_{j+1}^n - \tilde{\pi}_{j-1}^n)}{2h_x}, \\ \frac{\tilde{h}^{n+\frac{1}{2}} - \tilde{h}^n}{\Delta} &= -\frac{i\beta T_0 \tilde{\pi}^n}{\gamma M^2} + \frac{\mu T_0}{Re} \frac{\tilde{h}_{i+1}^{n+\frac{1}{2}} - 2\tilde{h}_i^{n+\frac{1}{2}} + \tilde{h}_{i-1}^{n+\frac{1}{2}}}{h_y^2}, \\ \frac{\tilde{\varphi}^{n+\frac{1}{2}} - \tilde{\varphi}^n}{\Delta} &= -\frac{T_0}{\gamma M^2} \frac{(\tilde{\pi}_{i+1}^n - \tilde{\pi}_{i-1}^n)}{2h_y}, \\ \frac{\tilde{\zeta}^{n+\frac{1}{2}} - \tilde{\zeta}^n}{\Delta} &= -\frac{d}{dY} \left(\frac{1}{T_0} \right) \tilde{\varphi}^n - \\ &- \frac{1}{T_0} \left(\frac{\tilde{f}_{j+1}^n - \tilde{f}_{j-1}^n}{2h_x} + i\beta \tilde{h}^n + \frac{(\tilde{\varphi}_{i+1}^n - \tilde{\varphi}_{i-1}^n)}{2h_y} \right), \\ \frac{\tilde{\pi}^{n+\frac{1}{2}}}{P} &= \frac{\tilde{r}^{n+\frac{1}{2}}}{\rho} + \frac{\tilde{\theta}^{n+\frac{1}{2}}}{T}, \end{aligned} \tag{6a}$$

$$\begin{aligned} \frac{\tilde{\zeta}^{n+\frac{1}{2}} - \tilde{\zeta}^n}{\Delta} &= -\frac{d}{dY} \left(\frac{1}{T_0} \right) \tilde{\varphi}^n - \\ &- \frac{1}{T_0} \left(\frac{\tilde{f}_{j+1}^n - \tilde{f}_{j-1}^n}{2h_x} + i\beta h^n + \frac{(\tilde{\varphi}_{i+1}^n - \tilde{\varphi}_{i-1}^n)}{2h_y} \right), \\ \frac{\tilde{\theta}^{n+\frac{1}{2}} - \tilde{\theta}^n}{\Delta} &= -T' \tilde{\varphi}^n - (\gamma - 1) T_0 \left(\frac{\tilde{f}_{j+1}^n - \tilde{f}_{j-1}^n}{2h_x} + i\beta \tilde{h}^n + \right. \\ &+ \left. \frac{\tilde{\varphi}_{i+1}^n - \tilde{\varphi}_{i-1}^n}{2h_y} \right) + \frac{\mu T_0}{Pr Re} \frac{\tilde{\theta}_{i+1}^{n+\frac{1}{2}} - 2\tilde{\theta}_i^{n+\frac{1}{2}} + \tilde{\theta}_{i-1}^{n+\frac{1}{2}}}{h_y^2}. \end{aligned}$$

The second step:

$$\begin{aligned} \frac{\tilde{a}^{n+1} - \tilde{a}^{n+\frac{1}{2}}}{\Delta} &= -U \frac{\tilde{a}_{j+1}^{n+1} - \tilde{a}_{j-1}^{n+1}}{2h_x}, \\ \frac{\tilde{\pi}^{n+1}}{P} &= \frac{\tilde{r}^{n+1}}{\rho} + \frac{\tilde{\theta}^{n+1}}{T}. \end{aligned} \tag{66}$$

The scheme is stable; the approximation order is $O(\tau, h_x^2, h_y^2)$. Values $\tilde{f}, \tilde{\varphi}, \tilde{r}, \tilde{\theta}, \tilde{\pi}, \tilde{h}$ on the $(n+1)$ layer were obtained from each equation in the appropriate order. Unknown values at the boundary were obtained by interpolating on three adjacent points.

The value ω was determined by the formula: $\omega = -(1/iN2\Delta) \ln(\pi^{n+N} / \pi^n)$. Calculations were performed until its value was constant with the acceptable accuracy. In this case the real or imaginary part of $q(Y_c, X, \tau_c)$ were changed according to the relation: $q_{r,i}(Y, X, \tau) = a_{r,i} \sin(\alpha X + \psi_{r,i})$. The value of $\alpha = 2\pi m / L$, where m - the number of periods stacked on the calculating range of L . We used a rectangular mesh with 240 points in the X - coordinate and 400 point in Y - coordinate with the time step $\Delta = 0.001$.

4 Boundary layer equations and their solution.

In self-similar variables boundary layer equations have the form [12]:

$$\begin{aligned} \frac{d}{dY} \left(\mu \frac{dU}{dY} \right) + g \frac{dU}{dY} &= 0, \\ \frac{d}{dY} \left(\frac{\mu}{Pr} \frac{dT}{dY} \right) + g \frac{dT}{dY} + (\gamma - 1) M_e^2 \mu \left(\frac{dU}{dY} \right)^2 &= 0, \\ \frac{dg}{dY} &= \frac{U}{2T}. \end{aligned}$$

Here $\gamma = C_p / C_v$ - ratio of specific heats,

$M_e = u_e^* / a_e$ - Mach number and a_e - sound velocity at the external border of boundary layer. At a uniform gas blowing through a wall at an angle λ to the main flow direction the velocity components on a wall are defined as follows: $V(0) = G \sin \lambda$, $U(0) = G \cos \lambda$. Due to the fact that $g(0) = -ReV(0) / T_w$ [3], it is possible to get $g(0) = -GRe \sin \lambda / T_w$. Let us the parameter $C_q = -ReG / T_w$ characterizes the intensity of the suction or blowing through the surface. In this case boundary conditions on thermally insulated surface can be written as:

$$\text{at } Y=0: \quad g = C_q \sin \lambda, \quad U = \frac{T_w}{Re} C_q \cos \lambda, \quad \frac{dT}{dY} = 0;$$

$$\text{at } Y=\infty: \quad T = U = 1.$$

Introducing the new variables:

$$z_1 = \mu \frac{dU}{dY}, \quad z_2 = g, \quad z_3 = U, \quad z_4 = \frac{\mu}{Pr} \frac{dT}{dY}, \quad z_5 = T;$$

boundary layer equations are written as a system of first order equations:

$$\begin{aligned} \frac{dz_1}{dY} &= g \frac{z_1}{\mu}, \quad \frac{dz_2}{dY} = \frac{U}{2T}, \quad \frac{dz_3}{dY} = \frac{z_1}{\mu}, \quad \frac{dz_5}{dY} = \frac{Pr z_4}{\mu}, \\ \frac{dz_4}{dY} &= -g \frac{Pr z_4}{\mu} - (\gamma - 1) M_e^2 \mu \left(\frac{F}{\mu} \right)^2. \end{aligned} \tag{7}$$

Boundary conditions are rewritten in the form:

$$z_2(0) = C_q \sin \lambda, \quad z_3(0) = \frac{T_w}{Re} C_q \cos \lambda, \quad z_4(0) = 0,$$

$$z_3(\infty) = z_5(\infty) = 1.$$

The system (7) is integrated by the Runge-Kutta method from wall to Y_m . Necessary values $a = z_1(0)$, $b = z_5(0)$ and Y_m are determined during the iterations, based on Newton's method, and a condition that $|z_3(Y_m) - 1|, |z_5(Y_m) - 1| < \varepsilon_1$. The dependence of μ on temperature was adopted in an accordance with the Sutherland's law which in dimensionless form can be written as follows:

$$\mu = (T_0)^{3/2} \frac{1 + T_s / T_e^*}{T_0 + T_s / T_e^*},$$

where $T_s = 110^\circ K$ - Sutherland's constant, T_e^* - temperature on the boundary layer edge. In wind tunnels without heating at a constant stagnation temperature T_0^* , $T_e^* = T_0^* / (1 + (\gamma - 1) M_e^2)$. It was accepted $T_0^* = 300^\circ K$ in this paper.

5 Results

The results of calculations of stationary values in the boundary layer at Mach number $M_e = 2$ are presented in figures 1-3. Distributions of the longitudinal velocity, temperature and dynamic viscosity are shown in Fig. 1 for the injection parameter $C_q=0$. It should be noted that all the stationary flow parameters come to unit approximately at $Y = 8$. The calculations results of longitudinal velocity profiles for different values of the parameter C_q are presented in Fig 2. Note, that approaching to value of velocity to unit is slowed with increasing injection rates. Thus one can clearly see that normal blowing leads to increasing of boundary layer thickens. Furthermore, an inflection point is appeared in the velocity profile which can contribute to destabilization of the boundary layer.

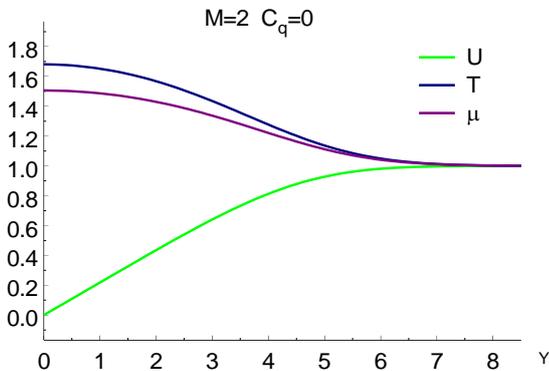


Fig. 1 Profiles of stationary flow parameters.

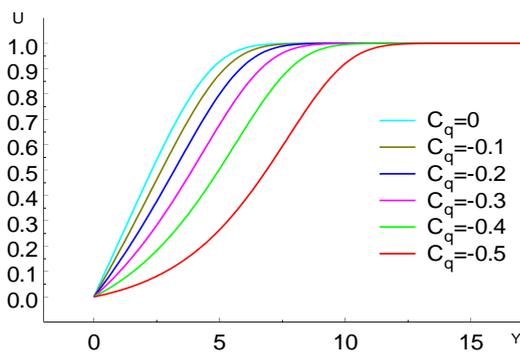


Fig. 2 Distribution of longitudinal velocities for various values of the parameter C_q

Influence of blowing direction in the distribution of longitudinal velocity is shown in Fig. 3. The velocity distribution without blowing is marked by symbols. From these data it follows that the stationary flow parameters are dependent on the tangential

injection weakly. The normal velocity component plays a decisive role in this respect.

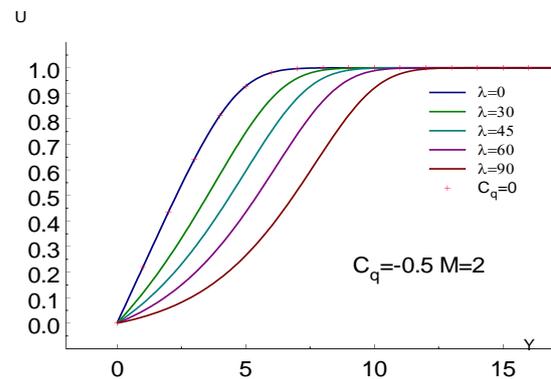


Fig. 3 Dependences of longitudinal velocities on the normal coordinate for the different λ

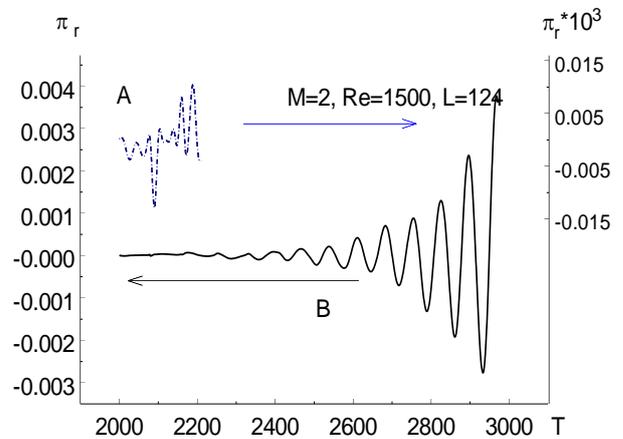


Fig. 4 The dependence of the real part of the pressure perturbation near the walls over time.

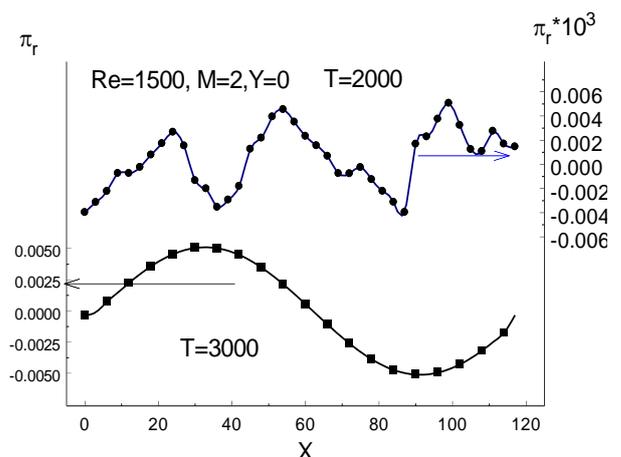


Fig. 5 The dependence of the real part of the pressure perturbation on the coordinate X .

Calculations results of perturbation parameters in the supersonic boundary layer are presented in Fig. 4-8, including the maximum degree of their

temporary growth. Main results were obtained on the basis of equations (3). Stability calculations were carried out by the classical theory for processing of the settlement scheme (4). As already mentioned, at large times the solution is described by an exponential dependence on the time, regardless of the initial data. Therefore, we will not dwell on the initial data which were set arbitrarily.

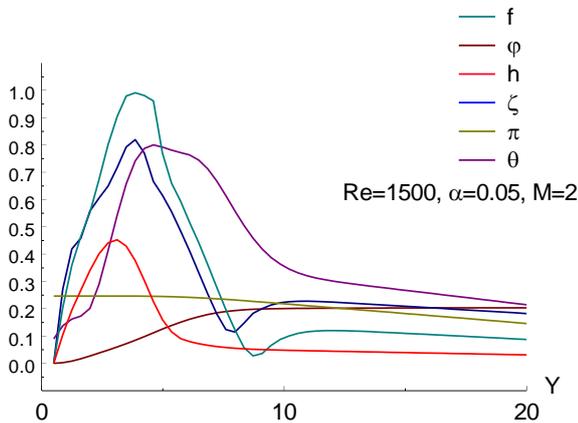


Fig. 6 Dependences of amplitudes of the velocity, density, temperature and pressure disturbances on the normal coordinate.

Fig. 4 shows the time variation of the real part of the pressure amplitude near the wall, $Y = 0$, when $Y^* = 40$. In the graph B the result is shown in the time interval $2000 < \tau < 3000$

Initial values of π_r of the graph B increased in one thousand times are shown on the top graph A, from which one can see that in the initial time moments there are several frequencies. However, over time the most growing frequency is allocated which changes under the law $\cos(\omega_r \tau + \psi_0) \exp \omega_i \tau$.

Fig. 5 shows the distribution of the real part of the pressure amplitude at the two times analogously to Fig. 4. It is seen that at large times the spatial dependence is described by a harmonic dependence with the wave number $\alpha = 2\pi/L$ rather well. The resulting increment ω_i is not different from the value of the classical theory in fact.

Profiles of the absolute values of the amplitudes of the perturbation are shown in Fig. 6. They correspond to the time when the solution came to an exponential dependence. Characteristically, that longitudinal velocity has the largest amplitude. Therefore, all amplitudes were normalized on the maximum longitudinal velocity value. It is also should be noted that the normal velocity amplitude perturbation reaches the maximum value at the outer edge.

The necessary computational domain is determined empirically by comparing of the growth rates for different values of Y^* with the data of the classical theory. From fig. 7 it is clearly visible that at the thickness $Y^* = 40$ results of numerical modeling differ from data of the classical theory a little.

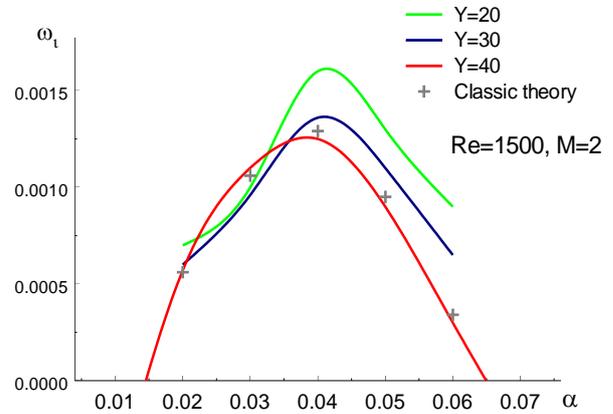


Fig. 7 Dependences of the increments on the wave number for different thicknesses.

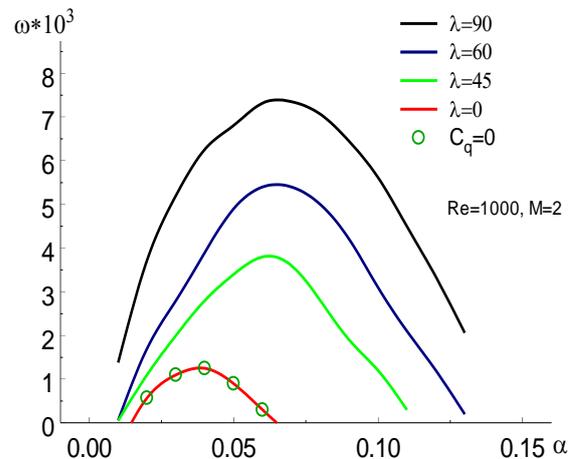


Fig. 8 Dependences of the grow rates on the parameter α for $C_q = -0.5$ and $C_q = 0$

Fig. 8 shows a change of grow rates depending on the wave number α for various injection directions of at $C_q = -0.5$. The red line corresponds to tangential blowing and green circles marks represent the results without blowing. It is seen that the boundary layer stability increases with decreasing of the angle λ , and tangential blowing ($\lambda = 0$) does not affect the boundary layer stability. At the same time normal blowing can increase the rate amplification in several times.

4 Conclusions

1. In the paper the new method of stability problem solving of the boundary layer is proposed, which is based on an evolutionary perturbations development in time.
2. Influence of the gas blowing direction through a porous surface on the supersonic boundary layer stability was studied for the first time. In the contrast to the strong influence of normal blowing on the boundary layer stability, tangential blowing has a little effect on it.
3. The developed method will be used in problems of the supersonic boundary layer stability with blowing of foreign gases, and the numerical scheme will be work for modeling of nonlinear problems of the laminar-turbulent transition.

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