Solving Lateral Beam Buckling Problems by Means of Solid Finite Elements and Nonlinear Computational Methods

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Abstract: The present paper deals with the comparison of values of resistance of a hot-rolled steel beam with initial imperfections. Very detailed 3D calculation models of beams were created in the computer programme Ansys applying the finite elements SOLID185. Eight nodes having three degrees of freedom at each node define the element SOLID185. Resistances were computed by application of geometrically and material nonlinear methods to the selected degree of non-dimensional slenderness depending on nominal length of the beam. Beams are subjected to bending, and their total and elastic resistance are compared with analytical, empirical and standard types of resistance.

Key-Words: Lateral Beam Buckling, Resistance, Imperfections, Hot-rolled, Beam, Ansys

1 Introduction

The stiffness of steel beams subjected to bending is substantially higher in the plane associated with bending about their major principal axis than in the plane associated with bending about their minor principal axis [1].

The static resistance is the crucial quantity influencing the safety and reliability of steel beams. Initial imperfections differ according to the manufacturing type of the beam [2]. Initial imperfections of steel plated girder resistance arise during the welding process in particular [3]. On the other side, initial imperfections of hot-rolled steel beams are often modelled so that they originate in the first eigenmode of lateral beam buckling [4].

The imperfections of a hot-rolled steel I-beam loaded by equal end bending moments of opposite sense M consist of initial axis curvature v_0 and initial cross-section rotation φ_0 [5]. The imperfection modelled according to the first eigenmode of lateral beam buckling assumes that v_0 and φ_0 are dependent functionally, and have the correlation 1 [6]. From the point of view of design reliability, it is conservative because it leads to the most rapid decrease of resistance.

In general, reliability analyses are important parts of multi-objective optimization of economic aspects of building engineering and mechanical one [7-9]. Large attention is to be paid to computational models of supporting elements in particular.

The presented paper is concentrated on finite element modelling and nonlinear analyses of resistance of a hot-rolled steel beam I200.

2 Computational Model

The computational model represents a hot-rolled beam I200 steel grade 235. The cross-section geometry was simplified according to Fig.1b) so that it could be possible to define it by four quantities, h, b, t_1 and t_2.

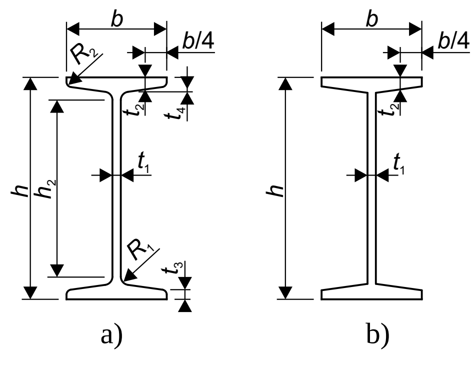


Fig. 1: Profile I200: a) real, b) idealized

The initial geometrical imperfection of beams is designed according to the first eigenmode of buckling at the stability loss by lateral beam buckling. It consists of initial displacement of axis v_0 and of initial rotation of cross-sections φ_0. These imperfections are considered to be affine to the final shape as the functions sinus, v_0 being the curvature of the beam axis in the direction of major axis, i.e., in plane xy, and φ_0, rotation of cross-sections along the beam length, see Fig.2.

$$v_0 = a_{v0} \sin\left(\frac{\pi x}{L}\right), \ \varphi_0 = a_{\varphi 0} \sin\left(\frac{\pi x}{L}\right) \tag{1}$$

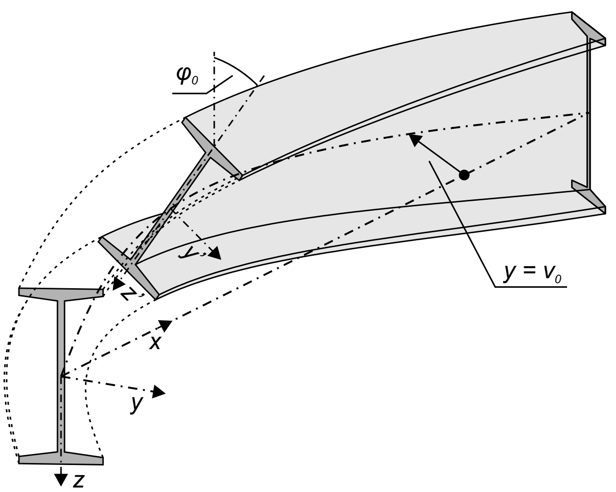


Fig. 2: Initial imperfections

If the beam is curved according to the first eigenmode, thus it holds for amplitudes a_{v0} and $a_{\varphi 0}$

$$a_{vo} = \frac{e_0}{1+\dfrac{h}{2}\dfrac{\pi^2 EI_z}{M_{cr}L^2}}, \ a_{\varphi 0} = a_{v0}\frac{\pi^2 EI_z}{M_{cr}L^2}, \tag{2}$$

where e_0 is the amplitude of one half-wave of the sine function relating to the upper edge of the cross-section, and is designed as $L/1000$ [10], h is the cross-section height, E is the Young's modulus of elasticity, I_z is the inertia moment to the axis z, L is the beam length, and M_{cr} is the elastic critical moment at lateral beam buckling. The elastic behaviour of beams can be analysed using two differential equations [11]:

$$EI_z\frac{\partial^2 v}{\partial x^2} + M(\varphi + \varphi_0) = 0, \tag{3}$$

$$EI_\omega \frac{\partial^3 \varphi}{\partial x^3} - GI_t\frac{\partial \varphi}{\partial x} + M\left(\frac{\partial v}{\partial x} + \frac{\partial v_0}{\partial x}\right) = 0, \tag{4}$$

where G is the shear modulus, I_ω is the warping constant, and I_t is the torsion constant.

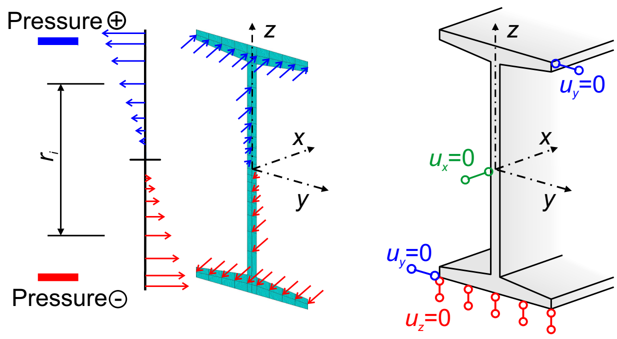
2.1 Loading and Boundary Conditions

The beam is considered as simply supported and loaded at both ends by equal bending moment of opposite sense. For such a loading case, the relation for M_{cr} can be derived according to [11], in the form

$$M_{\mathrm{cr}} = \frac{\pi}{L}\sqrt{EI_z GI_t}\sqrt{1+\frac{\pi^2 EI_\omega}{GI_t L^2}}. \tag{5}$$

The bending moment on the edge cross-section of 3D model was created as a pair of forces in its nodes. The forces act during loading perpendicularly to the end cross-section, and so, their arms r_i remain constant, see Fig.3a).

Boundary conditions are set so that the edge cross-sections can warp, see Fig.3b). The support in direction of axis x $u_x = 0$ is introduced at one end only.



a) b)

Fig. 3: a) Loading of edge cross-sections, b) boundary conditions

2.2 Finite Element Model

Computational models were created in the programme Ansys, using 3D elements SOLID185. SOLID185 is an 8-node element which can be used for 3D modelling of solid structures and has plasticity, hyperelasticity, stress stiffening, creep, large deflection, and large strain capabilities. It is defined by eight nodes having three degrees of freedom at each node: translations in the nodal x, y, and z directions. The element was set to be a homogeneous structural solid element. The enhanced strain formulation was considered. The enhanced strain formulation prevents shear locking in bending-dominated problems and volumetric locking in nearly incompressible cases. The formulation introduces certain number of internal (and inaccessible) degrees of freedom to overcome shear locking, and an additional internal degree of freedom for volumetric locking (except for the case of plane stress in 2-D elements). All internal degrees of freedom are introduced automatically at the element level and condensed out during the solution phase of the analysis [12].

An example of the computational model is presented in Fig.4 and Fig.5. Initial imperfections are illustrated here in magnified scale.

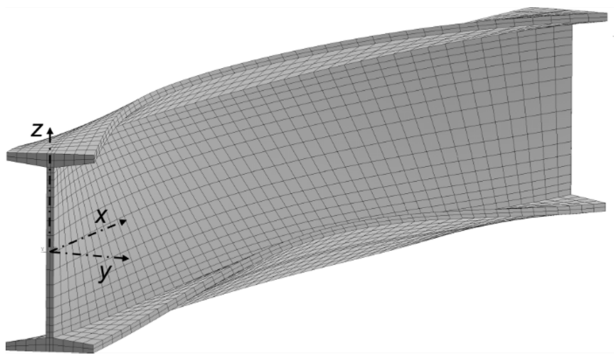


Fig. 4: Computational model in Ansys - axonometry

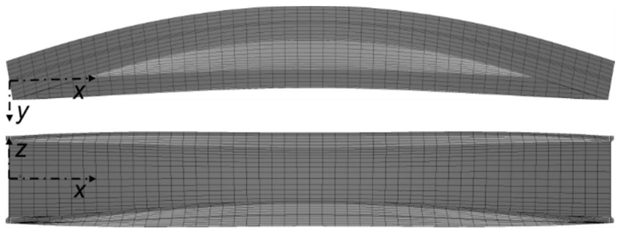


Fig. 5: Computational model in Ansys – views

2.2.1 Material Properties

An elastic-plastic stress-strain diagram without hardening according to the standard ENV 1993-1-1:1992 is used for the computation. The value of yield strength f_y is considered by nominal value 235 MPa.

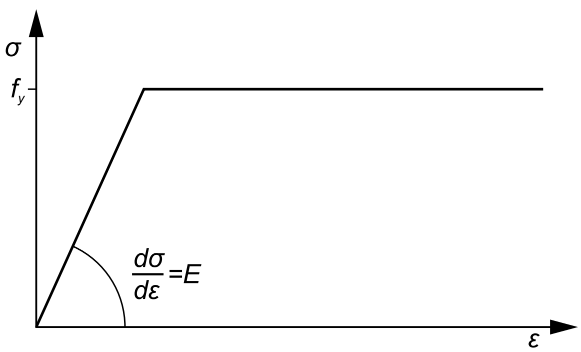


Fig. 6: Bilinear stress-strain diagram

3 Load-Carrying Capacity

The value of elastic resistance M_R of the beam can be derived on behalf of equations (3) and (4). M_R represents the bending moment at which the maximum value of the von Mises stress corresponds to yield strength f_y of the steel, and is given by the relation [4]:

$$M_R = \frac{2D_1 + D_4 + D_5}{4M_{cr}W_z} - \frac{\sqrt{4D_1^2 + \left(D_4 + D_5\right)^2 + 4D_1\left(D_4 - 2M_{cr}D_3\right)}}{4M_{cr}W_z} \tag{6}$$

where

$$D_1 = f_y M_{cr} W_y W_z,$$
$$D_2 = M_{cr} W_z + P_z \left|a_{v0}\right| W_y,$$
$$D_3 = M_{cr} W_z - P_z \left|a_{v0}\right| W_y,$$
$$D_4 = 2P_z^2 I_y \left|a_{v0}\right|,$$
$$D_5 = 2M_{cr} D_2,$$
$$P_z = \pi^2 \frac{EI_z}{L^2}.$$

W_y is the cross-section module to axis y, and W_z is the cross-section module to axis z.

Table 1: Cross-section characteristics

Characteristic	**Symbol**	**Value**
Cross-section height	h	0.200 m
Cross-section width	b	0.090 m
Web thickness	t_1	0.007 5 m
Flange thickness at quarter of the width	t_2	0.011 3 m
Second moment of area about axis y	I_y	21.235E-6 m^4
Second moment of area about axis z	I_z	1.188E-6 m^4
Torsion constant	I_t	1.187E-7 m^4
Warping constant	I_ω	1.017E-8 m^6
Section modulus about axis y	W_y	21.235E-5 m^3
Section modulus about axis z	W_z	2.639E-5 m^3
Plastic section modulus about axis y	$W_{pl,y}$	24.684E-5 m^3

To calculate the plastic resistance $M_{pl,R}$, it is possible to apply, by means of (6), the empirical relation according to [13]

$$M_{pl,R} = M_R \frac{W_{pl,y}}{W_y}\alpha + M_R\left(1-\alpha\right), \tag{7}$$

where

$$\alpha = \left(\frac{1}{1+\bar{\lambda}_{LT}^4}\right)^4. \tag{8}$$

$\bar{\lambda}_{LT}$ is the non-dimensional slenderness at lateral beam buckling according to the Eurocode 3

$$\bar{\lambda}_{LT} = \sqrt{\frac{W_{pl,y} f_y}{M_{cr}}}, \tag{9}$$

where $W_{pl,y}$ is the plastic cross-section module to axis y. Cross-section characteristics of the idealized profile I200 according to Fig.1b) are given in Table 1.

3.1 Resistance According to Eurocode 3

The design resistance moment of the beam at lateral beam buckling $M_{b,Rd}$ of a horizontally not supported beam is determined from the relation

$$M_{b,Rd} = \chi_{LT} W_y \frac{f_y}{\gamma_{M1}} \tag{10}$$

The cross-section I200 is the cross-section class 1, and therefore the cross-section module W_y can be determined as $W_y = W_{pl,y}$. For the partial resistance factor of cross-section when evaluating the stability γ_{M1} holds γ_{M1} =1.0. Reduction factor for lateral-torsional buckling χ_{LT} for the appropriate slenderness $\bar{\lambda}_{LT}$ may be determined from

$$\chi_{LT} = \frac{1}{\Phi_{LT} + \sqrt{\Phi_{LT}^2 - \bar{\lambda}_{LT}^2}}, \tag{11}$$

in which

$$\Phi_{LT} = 0.5\left[1 + \alpha_{LT}\left(\bar{\lambda}_{LT} - 0.2\right) + \bar{\lambda}_{LT}^2\right]. \tag{12}$$

The curve of lateral beam buckling b can be used for the cross-section I200. The value of imperfection factor for lateral-torsional buckling is $\alpha_{LT} = 0.34$.

3.2 Resistance of the Computational Model

The computational model in the programme Ansys is loaded increasingly, and calculated in geometrically nonlinear way by the Newton-Raphson method. The plastic resistance $M_{pl,Ansys}$ is defined as the maximum value of bending moment M, when the determinant of the stiffness matrix is non-zero, and the calculation converges. The elastic resistance $M_{R,Ansys}$ is given by reaching of prescribed stress (of yield strength f_y) in any point of the beam. With regard to the plane symmetry of the beam along the plane passing through its centre, and in parallel with the yz, reaching the yield strength takes place in one of cross-section tops in the middle of span. The linear regression was carried out to accurately quantify the elastic resistance for a narrow set of data including the value of acting moment and corresponding value of the von Mises stress near the yield strength. As the basic linear regression model, the polynomial of the seventh degree was applied, where the absolute error was still negligible.

4 Comparison of Resistances

The values of analytically computed resistances according to (6), (7) and (10) are depicted by the curve in Fig.7. The diagram is completed by the Euler hyperbola according to (5), and the values of resistance M_d given by the relation

$$M_d = f_y W_{pl,y} \tag{13}$$

The range of non-dimensional slenderness $\bar{\lambda}_{LT}$ is from 0 to 2.1. According to (9) and (5), non-dimensional slenderness is, in the present problem, dependent only on the beam length L. This is limited to the length of 12 metres by the manufacturer of hot-rolled profiles. The relation for the dependence of length on non-dimensional slenderness can be thus derived in the form:

$$L = \frac{\bar{\lambda}_{LT} I_z^{0.25} \pi \left(2EI_\omega\right)^{0.5}}{\left(Q_1^{0.5} - \bar{\lambda}_{LT}^{\ 2} I_z^{0.5} G I_t\right)^{0.5}} \tag{14}$$

in which

$$Q_1 = \bar{\lambda}_{LT}^{\ 4} I_z G^2 I_t^2 + 4 f_{y,n}^{\ 2} W_{pl,y}^{\ 2} I_\omega .$$

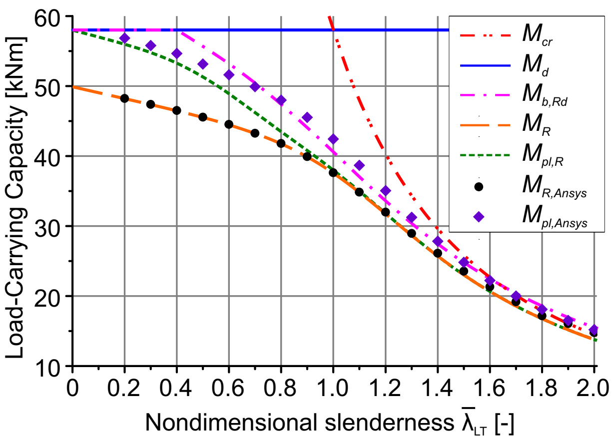


Fig. 7: Resistance vs. non-dimensional slenderness

Due to the non-linear relation between $\bar{\lambda}_{LT}$ and L, the resistance is laid out also to the beam length, see. Fig.8.

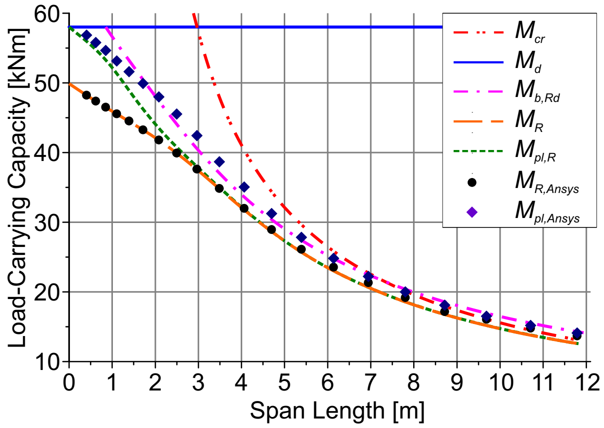


Fig. 8: Diagram resistance vs. span length

4.1 Stress of the Beam under Limit State

The elastic resistance is given by reaching the yield strength f_y in any point of the beam, without occurrence of plasticization of the cross-section. The course of stress σ_x in the span middle is illustrated in Fig.9a). It is evident from the diagram in Fig.7 that the absolute difference of resistance $M_{pl,Ansys}$ and $M_{R,Ansys}$ is increasing with decreasing value of slenderness. There takes place the use of plastic reserve of the cross-section. For the slenderness approximately higher that 1.4, this difference is negligible.

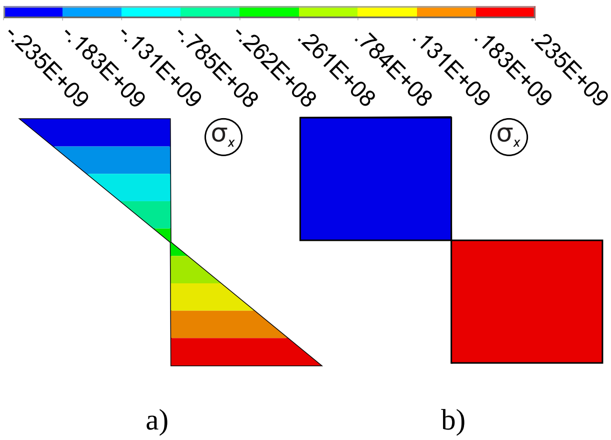


Fig. 9: Stress course in web σ_x at reaching : a) elastic resistance, b) plastic resistance

It can be noticed that the plastic resistance of the cross-section subjected to bending is an important part of numerous optimization analyses [14]. The cross-section can theoretically plasticize totally according to Fig.9b). However, the reality is so that even for the lowest considered values of slenderness, the cross-section of the beam need not plasticize fully at reaching the total resistance. Such a case of stress course σ_x is, observed, e.g., for the slenderness $\bar{\lambda}_{LT} = 0.6$, presented in Fig.10. The von Mises stress which decides on the resulting value of resistance, is depicted in Fig.11 and Fig.12.

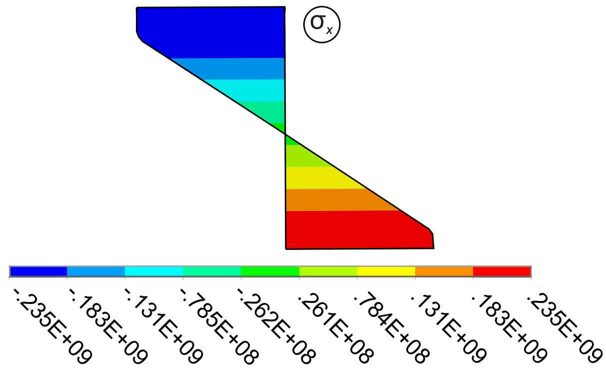


Fig. 10: Stress course in web σ_x for slenderness $\bar{\lambda}_{LT} = 0.6$ at reaching the total resistance

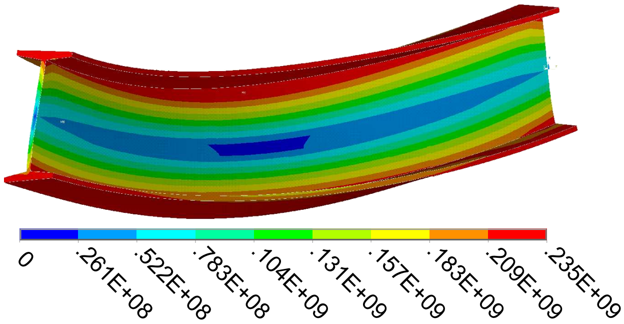


Fig. 11: Course of the von Mises stress σ_{vM} in the beam for $\bar{\lambda}_{LT} = 0.6$ at reaching the total resistance

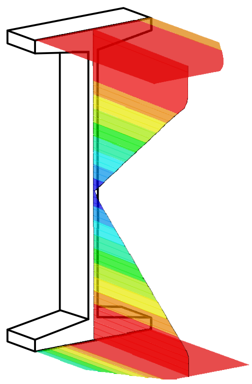


Fig. 12: Course of the von Mises σ_{vM} in the cross-section in the span middle for slenderness $\bar{\lambda}_{LT} = 0.6$ at reaching the total resistance

5 Conclusion

It is clear from Fig.7 that, with increasing slenderness, the values of analytically and by programme computed resistances approach the Euller hyperbola, and the problem becomes a stability problem. In case of elastic resistance, the values from computational model $M_{R,Ansys}$ fully agree with the analytical M_R. When comparing the values of standard resistance $M_{b,Rd}$ with total computed $M_{pl,Ansys}$, there is obtained good agreement approximately from $\bar{\lambda}_{LT} \geq 0.7$. For the lower non-dimensional slenderness, the standard resistance is by 2 – 6 % higher. The empirical total resistance $M_{pl,R}$ according to (7) gives the values lower than $M_{b,Rd}$ and $M_{pl,Ansys}$, and thus, it is rather safe assessment of total resistance based on the elastic resistance M_R.

Although it is well known that the influence of residual stress decreases the resistance of hot-rolled struts under compressions [15], it was not considered in the present problem. It remains, however, the topic of future studies.

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