- *If two or more groups have the same $x_c \geq 2.5$, then the group with the greater y_c performs better.*
- *If two or more groups have the same $x_c < 2.5$, then the group with the smaller y_c performs better.*

As it becomes evident from the above criterion, a group's performance depends mainly on the value of the x-coordinate of the COG of the corresponding level's area, which is calculated by the first of formulas (3). In this formula, greater coefficients (weights) are assigned to the higher grades. Therefore, the COG method focuses, similarly to the GPA index, on the group's ***quality performance.***

In case of the ***ideal*** performance ($y_5 = 1$ and $y_i = 0$ for $i \neq 5$) the first of formulas (3) gives that $x_c = \frac{9}{2}$. Therefore, values of x_c greater than $\frac{9}{4} = 2.25$ could be considered as demonstrating a more than satisfactory performance.

4. The Variations GRFAM, TFAM and TpFAM of the RFAM

A group's performance is frequently represented by numerical scores in a climax from 0-100. These scores can be assigned to the linguistic labels of *U* as follows: A (85-100), B(75-84), C (60-74), D(50-59) and F (0-49) [1].

Nevertheless, ambiguous cases appear frequently in practice, being at the boundaries between two successive assessment grades; e.g. something like 84-85%, being at the boundaries between A and B. In an effort to treat better such kind of cases, Subbotin [8] "moved" the rectangles of Figure 1 to the left, so that to share common parts (see Figure 2). In this way, the ambiguous cases, being at the common rectangle parts, belong to both of the successive grades, which means that these parts must be considered ***twice*** in the corresponding calculations.

The graph of the resulting fuzzy set is now the bold line of Figure 2. However, the method mentioned in Section II for calculating the coordinates of the COG of the area contained between the graph and the X-axis is not the proper one here, because in this way the common rectangle parts are calculated only once. The right method for calculating the coordinates of the COG in this case was fully developed by Subbotin & Voskoglou [9] and the resulting framework was called the ***Generalized Rectangular Fuzzy Assessment Model (GRFAM).*** The development of GRFAM involves the following steps:

1. Let y_1, y_2 , y_3, y_4 , y_3 be the ***frequencies*** of a group's members who obtained the grades F, D, C, B, A respectively. Then $\sum_{i=1}^{5} y_i = 1$ (100%).

2. We take the heights of the rectangles in Figure 2 to have lengths equal to the values of corresponding frequencies. Also, without loss of generality we allow the sides of the adjacent rectangles lying on the OX axis to share common parts with length equal to the 30% of their lengths, i.e. 0.3 units.[2]

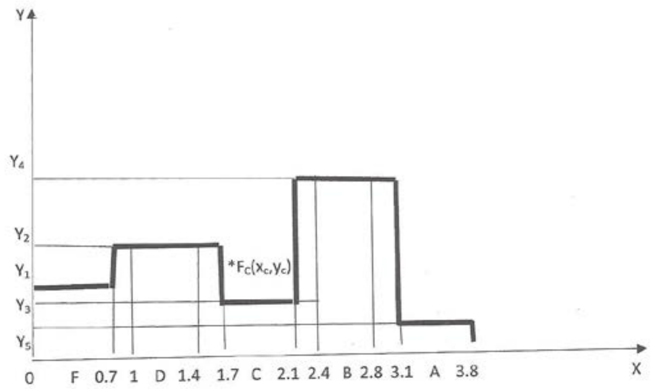

Figure 2: Graphical representation of the GRFAM

3. We calculate the coordinates (x_{c_i}, y_{c_i}) of the COG, say F_i , of each rectangle, i = 1, 2, 3, 4, 5 as follows: Since the COG of a rectangle is the point of the intersection of its diagonals, we have that $y_{c_i} = \frac{1}{2} y_i$. Also, since the x-coordinate of each COG F_i is equal to the x- coordinate of the middle of the side of the corresponding rectangle lying on the OX axis, from Figure 2 it is easy to observe that

[1] This way of assignment, although it satisfies the common sense, it is not unique; in a more strict assessment, for example, one could take A(90-100), B(80-89), C(70-79), D(60-69) and F (0-59), etc.

[2] Since the ambiguous assessment cases are situated at the boundaries between the adjacent grades, it is logical to accept a percentage for the common lengths of less than 50%.