Study the behaviors of the modified duffing equation by PPF control under the primary and internal resonance case

YASER AMER[1], HANAN ABD ELRAHMAN[2], MANSOUR ABD EL-SALAMR[3]
Department of Mathematics, Faculty of Science, Zagazig University, Zagazig, Egypt,, [2,3]Department of Basic Sciences, Higher Technological Institute, 10th of Ramadan City, EGYPT

Abstract: The nonlinear vibration control of a nonlinear dynamical system modeled as the well known Duffing oscillators is investigated within this article. The conventional positive position feedback (PPF) controller is proposed to mitigate system nonlinear vibrations. The whole system mathematical model is analyzed by applying the multiple time scales perturbation method. The slow-flow modulation equations that govern the oscillation amplitudes of both the main system and controller are derived. The stability analysis is investigated according to Routh–Hurwitz criterion. The obtained analytical and numerical results illustrated that the PPF controller can eliminate the main system nonlinear vibrations once the controller natural frequency is tuned to be the same value as the external excitation frequency, otherwise, the controller adds excessive vibrational energy to the main system rather than suppressing it. In addition, the PPF controller can destabilize the main system motion when excited by strong excitation force.

Key Words : Positive position feedback control; Modified Duffing equation; Stability analysis
Received: May 15, 2021. Revised: August 13, 2022. Accepted: September 11, 2022. Published: October 5, 2022.

1. Introduction

Vibrations are unwanted phenomenon, it damages a lot of dynamical systems. Therefore, much researches have been done to study how to control this phenomenon. So there are many types of control that are used for this reason. Recently, the vibrations of several vibration systems [1-6] have been suppressed using different types of control. Amer et al. [7] used the proportional derivative controller to suppress the vibrations of a Hybrid Rayleigh-Van der Pol- Duffing oscillator. They found that, the controller adds more damping to the vibrating system. Time delay strategy is one of the most important types of control used recently. Abdelhafez and Nassar [8], used the positive position feedback controllers in existence of two different time delays for suppressing the vibrations of a self-exited non-linear beam. They notified that, the time margin depends on the overall delays of the system $\tau_1 + \tau_2$. Liu et al. [9] investigated the influence of two different delays the first is displacement delay and the second is velocity delay in a cantilever beam. They used the method of multiple scales to determine all super-harmonic and sub-harmonic resonance cases. Effect of a pair of delay positive position feedback controller were used to control the vibrations of coupled Van der Pol harmonic oscillators by El-Sayed [10].

Ferrari and Amabili [11], offered an experimentally studying for the effectiveness of the PPF controllers on suspended the vibrations of sandwich plate. Niu et al [12] realize the fractional-order positive position feedback (FOPPF) controller. They found that, the FOPPF controller gives better results comparing with PPF controller. Omidi et al [13,14] presented three kinds of control to suppress the vibrations of vibrating systems such that, the Integral resonant controllers (IRC), PPF controllers and the non-linear

Integral Positive Position feedback (NIPPF). For the important of the positive position feedback controllers in suppressing the vibrations of many vibrating systems [15-17], it is a suitable for small natural frequencies as the bandwidth of the vibration reduction increases. Bauomey and El-Sayed [18], used a negative velocity feedback controllers to control the vibrations of the suspended cable. They investigated the suspended cable's stability near a sub-harmonic-combined simultaneous case. The controller succeeded in reducing the vibrations about to E_a (amplitude without control/amplitude with control)=2000 for x and E_a=800 for y.

In this article, we used PPF controller to suppress the vibrations of micro-electro-mechanical system. The multiple scale method is applied to deduce several resonance cases, the worst resonance case is simultaneous resonance case (one-to-one internal and primary) is studied to get the response of the non-linear system. The equations of frequency response are in use to investigate the stability of the obtained solution. The influence of some chosen coefficient is illustrated numerically and analytically. The rapprochement between numeric and analytic solution is offered.

2. Perturbation Analysis

Consider the model of micro-electro- mechanical system [19,20]

$$\begin{aligned} &u + 2\varepsilon\mu u + \omega_1^2 u + \varepsilon(\alpha_1 u^2 + \alpha_2 u^3) - \\ &\varepsilon\alpha(2u + 3u^2 + 4u^3) - \varepsilon(2u + 3u^2 + 4u^3) \\ &*(f_1 \cos(\Omega t) + f_2 \cos(2\Omega t)) - \\ &\varepsilon(\alpha + f_1 \cos(\Omega t) + f_2 \cos(2\Omega t)) = 0, \text{p} \ \ \varepsilon \text{ p } 1. \end{aligned} \quad (1)$$

This model represented the modified Duffing equation subjected to weakly non-linear parametric and external ex citations, and described

2

the main motions at time scales of the natural vibrations of the microstructure and fast dynamic at time scales of the high-frequency voltage, μ is the coefficient of viscous damping, ε is a small parameter, ω_1 is linear natural frequency , Ω is the frequency of the external excitation, α is the coefficient of linear term , α_1, α_2 are the coefficients of the nonlinear terms f_1, f_2 ~are the coefficient of linear and nonlinear parameters excitations.We present a positive position feedback (PPF) Controller(PPF), which designed to control the micro-electro- mechanical system. Then, the equation commanding the dynamics of the controller (PPF) is indicated as

$$y + 2\varepsilon\xi\omega_2 y + \omega_2^2 y = \varepsilon\gamma_2 F_f(t), \quad (2)$$

so the closed loop system equations are

$$\begin{aligned} &u + 2\varepsilon\mu u + \omega_1^2 u + \varepsilon(\alpha_1 u^2 + \alpha_2 u^3) - \\ &\varepsilon\alpha(2u + 3u^2 + 4u^3) - \varepsilon(2u + 3u^2 + 4u^3) \\ &*(f_1 \cos(\Omega t) + f_2 \cos(2\Omega t)) - \\ &\varepsilon(\alpha + f_1 \cos(\Omega t) + f_2 \cos(2\Omega t)) = \varepsilon\gamma_1 F_c(t) \end{aligned}$$

$$\ddot{y} + 2\varepsilon\xi\omega_2 \dot{y} + \omega_2^2 y = \varepsilon\gamma_2 F_f(t) \quad (3)$$

where γ_1, γ_2 are gains, ξ is damping coefficient of the (PPF) controller, ω_2 is the natural frequency of (PPF) controller we determine the control signal

$F_c = y$ and the feedback signal $F_f = u$.

2.1 Mathematical Treatment(MSPT)

The multiple scales method is applied to get the asymptotic first-order approximate solutions for the system (3) which we use the multiscale perturbedmethod

$$u(T_0,T_1,\varepsilon)=u_0(T_0,T_1)+\varepsilon u_1(T_0,T_1)+O(\varepsilon^2),$$
$$y(T_0,T_1,\varepsilon)=y_0(T_0,T_1)+\varepsilon y_1(T_0,T_1)+O(\varepsilon^2),$$
$$T_n=\varepsilon^n t. \tag{4}$$

where $T_0=t, T_1=\varepsilon t.$ are the fast and slow time scales, respectively. The time derivatives became

$$\frac{d}{dt}=D_0+\varepsilon D_1+...,$$
$$\frac{d^2}{dt^2}=D_0^2+2\varepsilon D_0 D_1+... \tag{5}$$

Substituting (4) and (5) into (3), and equating the coefficients of equal power of ε lead to:

$$O(\varepsilon^0):(D_0^2+\omega_1^2)u_0=0,$$
$$(D_0^2+\omega_2^2 y_0=0, \tag{6}$$

$$O(\varepsilon^1):(D_0^2+\omega_1^2)u_1=-2D_0D_1u_0-2\mu D_0u_0-\alpha_1u_0^2-\alpha_2u_0^3+\alpha(2u_0+3u_0^2+4u_0^3)+(2u_0+3u_0^2+4u_0^3)(f_1\cos(\Omega t)+f_2\cos(2\Omega t))+\gamma_1 y_0, \tag{7}$$
$$(D_0^2+\omega_2^2)y_1=-2D_0D_1y_0-2\xi\omega_2y_0+\gamma_2u_0.$$

The solution of system of equations(6) are

$$u_0(T_0,T_1)=A_1(T_1)e^{i\omega_1T_0}+c.c.,$$
$$y_0(T_0,T_1)=A_2(T_1)e^{i\omega_{21}T_0}+c.c. \tag{8}$$

Where A_1, A_2 are unknown complex function in T_1 and c.c.denotes the complex conjugate of the previous terms, insert eqs.(8) into eqs.(7) we get

$$(D_0^2+\omega_1^2)u_1=\alpha-(2i\omega_1D_1A_1+2i\mu\omega_1A_1-2\alpha A_1-12\alpha A_1^2\overline{A}_1+3\alpha_2A_1^2\overline{A}_1)e^{i\omega_1T_0}+A_1^2(3\alpha-\alpha_1)e^{2i\omega_1T_0}+\overline{A}_1^2(3\alpha-\alpha_1)e^{-2i\omega_1T_0}+A_1^3(4\alpha-\alpha_1)e^{3i\omega_1T_0}+\overline{A}_1^3(4\alpha-\alpha_1)e^{-3i\omega_1T_0}+f_1(0.5\ +3A_1\overline{A}_1)e^{i\Omega T_0}+f_2(0.5+3A_1\overline{A}_1)e^{2i\Omega T_0}+1.5A_1^2f_1e^{(i\Omega+2\omega_1)T_0}+$$

$$2\overline{A}_1^3f_2e^{i(2\Omega-3\omega_1)T_0}+1.5A_1^2f_2e^{i(2\Omega+2\omega_1)T_0}+2A_1^3f_2e^{i(2\Omega+3\omega_1)T_0}+6\alpha A_1\overline{A}_1-2\alpha_1A_1\overline{A}_1+f_1(A_1+6\overline{A}_1A_1^2)e^{i(\Omega+\omega_1)T_0}+\gamma_1A_2e^{i\omega_2T_0}+f_2(\overline{A}_1+6A_1\overline{A}_1^2)e^{i(2\Omega-\omega_1)T_0}+1.5\overline{A}_1^2f_1e^{i(\Omega-2\omega_1)T_0}+1.5\overline{A}_1^2f_2e^{i(2\Omega-2\omega_1)T_0}+2\overline{A}_1^3f_1e^{i(\Omega-3\omega_1)T_0}+f_2A_1(1+6\overline{A}_1)e^{i(2\Omega+\omega_1)T_0}+2A_1^3f_1e^{i(\Omega+3\omega_1)T_0}+f_1A_1(1+6\overline{A}_1^2)e^{i(\Omega-\omega_1)T_0}. \tag{9}$$

$$(D_0^2+\omega_2^2)y_1=A_1\gamma_1e^{i\omega_1T_0}-2i\omega_2(\xi A_2+D_1A_2)e^{i\omega_2T_0}. \tag{10}$$

the solutions of equations (9),(10) after eliminating the secular terms

$$u_1=\alpha+E_1e^{i(\Omega+\omega_1)T_0}+E_2e^{i(2\Omega+2\omega_1)T_0}+E_3e^{2i\omega_1T_0}+E_4e^{2i\Omega T_0}+E_5e^{i(\Omega+2\omega_1)T_0}+E_6e^{i(\Omega+3\omega_1)T_0}+E_7e^{i(2\Omega+2\omega_1)T_0}+E_8e^{i(2\Omega-2\omega_1)T_0}+E_9e^{i(\Omega-3\omega_1)T_0}+E_{10}e^{i(\Omega+3\omega_1)T_0}+E_{11}e^{3i\omega_1T_0}+E_{12}e^{i(2\Omega+3\omega_1)T_0}+E_{13}e^{i(\Omega-\omega_1)T_0}+E_{14}e^{-2i\omega_1T_0}+E_{15}e^{-3i\omega_1T_0}+E_{16}e^{i(2\Omega-\omega_1)T_0}+E_{17}e^{i\Omega T_0}+E_{18}e^{i(2\Omega-3\omega_1)T_0}+c.c. \tag{11}$$

$$y_1=E_{19}e^{i\omega_1T_0}+c.c. \tag{12}$$

where $E_i, \{1=1,2,...,19\}$ are presented at appendix.

3. Stability Analysis

In this paper, the case of the simultaneous primary and internal resonance $\Omega=\omega_1, \omega_2=\omega_1$ which is the worst resonance case, is considered to study the stability of the system of equations (3) . Introducing the detuning parameters σ_1,σ_2 according to:

$$\Omega=\omega_1+\varepsilon\sigma_1, \omega_2=\omega_1+\varepsilon\sigma_2, \quad (13)$$

and write

$$(\Omega-2\omega_1)T_0=(\varepsilon\sigma_1-\omega_1)T_0=-(\omega_1T_0-\sigma_1T_1),$$
$$(2\Omega-\omega_1)T_0=(2\varepsilon\sigma_1+\omega_1)T_0=(\omega_1T_0+2\sigma_1T_1),$$
$$(2\Omega-3\omega_1)T_0=(2\varepsilon\sigma_1-\omega_1)T_0=-(\omega_1T_0+2\sigma_1T_1). \quad (14)$$

Substituting equations (13) and (14) into equations (11) and (12) and eliminating the secular terms, leads to the solvability conditions for the first order approximation, hence the following differential equations are obtained:

$$2i\omega_1D_1A_1=-2i\mu\omega_1A_1+2\alpha A_1+12\alpha A_1^2\overline{A}_1- 3\alpha_2A_1^2\overline{A}_1+(0.5f_1+3A_1\overline{A}_1f_1)e^{i\sigma_1T_1}+A_2\gamma_1e^{i\sigma_2T_1}+ (f_2\overline{A}_1+6A_1\overline{A}_1^2f_2)e^{2i\sigma_1T_1}+1.5A_1^2f_1e^{-i\sigma_1T_1}+ 2A_1^3f_2e^{-2i\sigma_1T_1}, \quad (15)$$

$$2iD_1A_2=-2i\xi\omega_2A_2-\gamma_2A_1e^{i\sigma_2T_1}. \quad (16)$$

The solution of equations (15) and (16) can be analyzed by putting $A_1(T_1), A_2(T_1)$ in polar form,

$$A_1(T_1)=\frac{a_1(T_1)}{2}e^{i\phi_1(T_1)}, A_2(T_1)=\frac{a_2(T_1)}{2}e^{i\phi_2(T_1)}, \quad (17)$$

$$DA_1(T_1)=\frac{1}{2}(a_1(T_1)+ia_1\varphi_1)e^{i\varphi_1(T_1)},$$
$$DA_2(T_1)=\frac{1}{2}(a_2(T_1)+ia_2\varphi_2)e^{i\varphi_2(T_1)}, \quad (18)$$

where a_1, a_2 are the amplitudes of steady state, ϕ_1,ϕ_2 are the motions phases. By substituting equations (17),(18) into equations (15) ,(16), we get

$$(a_1+ia_1\varphi_1)=\frac{-i\alpha a_1}{\omega_1}-\mu a_1-\frac{3i\alpha a_1^3}{2\omega_1}+\frac{3i\alpha_2a_1^3}{8\omega_1}- \frac{i}{\omega_1}(\frac{1}{2}f_1+\frac{3}{4}a_1^2f_1)e^{i(\sigma_1T_1-\varphi_1)}-\frac{ia_2\gamma_1}{2\omega_1}e^{i(\sigma_2T_1-\varphi_1+\varphi_2)}- \frac{i}{2\omega_1}(\frac{1}{2}a_1f_2+\frac{3}{4}a_1^3f_2)e^{i(\sigma_1T_1-\varphi_1)}- \frac{3if_1a_1^2}{8\omega_1}e^{-i(\sigma_1T_1-\varphi_1)}-\frac{if_2a_1^3}{4\omega_1}e^{-2i(\sigma_1T_1-\varphi_1)}, \quad (19)$$

$$(\dot{a}_2+ia_2\dot{\phi}_2)=-\frac{\xi a_2}{2}+\frac{\gamma_2a_1}{4\omega_2}e^{-i(\sigma_2T_1-\phi_1+\phi_2)}. \quad (20)$$

compare the imaginary part and the real terms

$$a_1=-\mu a_1+\frac{1}{2\omega_1}(a_1f_2+\frac{3}{2}a_1^3f_2)\sin(2\theta_1)+ \frac{a_2\gamma_1}{2\omega_1}\sin(\theta_2)+\frac{1}{2\omega_1}(f_1+\frac{3}{2}a_1^2f_1)\sin(\theta_1)- -\frac{3f_1a_1^2}{8\omega_1}\sin(\theta_1)-\frac{f_2a_1^3}{4\omega_1}\sin(2\theta_1),$$

$$a_1\varphi_1=-\frac{\alpha a_1}{\omega_1}-\frac{3\alpha a_1^3}{2\omega_1}+\frac{3\alpha_2a_1^3}{8\omega_1}-\frac{3f_1a_1^2}{8\omega_1}\cos(\theta_1)- \frac{1}{2\omega_1}(a_1f_2+\frac{3}{2}a_1^3f_2)\cos(2\theta_1)-\frac{a_2\gamma_1}{2\omega_1}\cos(\theta_2)- \frac{1}{2\omega_1}(f_1+\frac{3}{2}a_1^2f_1)\cos(\theta_1)-\frac{f_2a_1^3}{4\omega_1}\cos(2\theta_1), \quad (21)$$

$$a_2=-\xi\omega_2a_2-\frac{\gamma_2a_1}{2\omega_2}\sin(\theta_2),$$

$$a_2\varphi_2=-\frac{\gamma_2a_1}{2\omega_2}\cos(\theta_2). \quad (22)$$

where $\theta_1=\sigma_1T_1-\phi_1, \theta_2=\sigma_2T_1-\phi_1+\phi_2$.

4. Fixed point solutions

The steady-state solution of our dynamical system corresponding to the fixed point of equations (21) , (22) is obtained when $a_m, \varphi_m, m=1,2,$

$$\mu a_1 = \frac{1}{2\omega_1}(a_1 f_2 + \frac{3}{2}a_1^3 f_2)\sin(2\theta_1) + \frac{a_2\gamma_1}{2\omega_1}\sin(\theta_2) + \frac{1}{2\omega_1}(f_1 + \frac{3}{2}a_1^2 f_1)\sin(\theta_1) - \frac{3 f_1 a_1^2}{8\omega_1}\sin(\theta_1) - \frac{f_2 a_1^3}{4\omega_1}\sin(2\theta_1), \quad (23)$$

$$a_1\sigma_1 = -\frac{\alpha a_1}{\omega_1} - \frac{3\alpha a_1^3}{2\omega_1} + \frac{3\alpha_2 a_1^3}{8\omega_1} - \frac{3 f_1 a_1^2}{8\omega_1}\cos(\theta_1) - \frac{1}{2\omega_1}(a_1 f_2 + \frac{3}{2}a_1^3 f_2)\cos(2\theta_1) - \frac{a_2\gamma_1}{2\omega_1}\cos(\theta_2) - \frac{1}{2\omega_1}(f_1 + \frac{3}{2}a_1^2 f_1)\cos(\theta_1) - \frac{f_2 a_1^3}{4\omega_1}\cos(2\theta_1), \quad (24)$$

$$\xi\omega_2 a_2 = \frac{-\gamma_2 a_1}{2\omega_2}\sin(\theta_2), \quad (25)$$

$$(\sigma_1 - \sigma_2)a_2 = -\frac{\gamma_2 a_1}{2\omega_2}\cos(\theta_2). \quad (26)$$

From equations (23) to (26) the amplitude and phase modulating equations take the form

$$a_1 = -\mu a_1 + \frac{1}{2\omega_1}(a_1 f_2 + \frac{3}{2}a_1^3 f_2)\sin(2\theta_1) + \frac{a_2\gamma_1}{2\omega_1}\sin(\theta_2) + \frac{1}{2\omega_1}(f_1 + \frac{3}{2}a_1^2 f_1)\sin(\theta_1) - \frac{3 f_1 a_1^2}{8\omega_1}\sin(\theta_1) - \frac{f_2 a_1^3}{4\omega_1}\sin(2\theta_1), \quad (27)$$

$$\theta_1 = \sigma_1 + \frac{\alpha}{\omega_1} + \frac{3\alpha a_1^2}{2\omega_1} - \frac{3\alpha_2 a_1^2}{8\omega_1} + \frac{3 f_1 a_1}{8\omega_1}\cos(\theta_1) + \frac{1}{2\omega_1}(f_2 + \frac{3}{2}a_1^2 f_2)\cos(2\theta_1) + \frac{a_2\gamma_1}{2\omega_1 a_1}\cos(\theta_2) + \frac{1}{2\omega_1}(\frac{f_1}{a_1} + \frac{3}{2}a_1 f_1)\cos(\theta_1) + \frac{f_2 a_1^2}{4\omega_1}\cos(2\theta_1), \quad (28)$$

$$a_2 = -\xi\omega_2 a_2 - \frac{\gamma_2 a_1}{2\omega_2}\sin(\theta_2), \quad (29)$$

$$\theta_2 = \sigma_2 + \frac{\alpha}{\omega_1} + \frac{3\alpha a_1^2}{2\omega_1} - \frac{3\alpha_2 a_1^2}{8\omega_1} + \frac{3 f_1 a_1}{8\omega_1}\cos(\theta_1) + \frac{1}{2\omega_1}(f_2 + \frac{3}{2}a_1^2 f_2)\cos(2\theta_1) + \frac{a_2\gamma_1}{2\omega_1 a_1}\cos(\theta_2) + \frac{1}{2\omega_1}(\frac{f_1}{a_1} + \frac{3}{2}a_1 f_1)\cos(\theta_1) + \frac{f_2 a_1^2}{4\omega_1}\cos(2\theta_1) - \frac{\gamma_2 a_1}{2a_2\omega_2}\cos(\theta_2), \quad (30)$$

where $\theta_1 = \sigma_{11} - \varphi_1, \theta_2 = \sigma_2 - \varphi_1 + \varphi_2.$

To determine the stability of the nonlinear solution,one lets

$$a_1 = a_{10} + a_{11}, a_2 = a_{20} + a_{21},$$
$$\theta_1 = \theta_{10} + \theta_{11}, \theta_2 = \theta_{20} + \theta_{21}. \quad (31)$$

where a_{m0}, θ_{m0} are the solutions of equations (27)-(30) and $a_m, \theta_m, m = 1,2,$ are perturbations which are assumed to be small compared the a_{m0}, θ_{m0} Substituting equation (31) into equations (27)-(30) and keeping only the linear terms in a_{m1}, θ_{m1} we obtain

$$a_{11} = \left\{-\mu + \frac{3 f_1 a_{10}}{4\omega_1}\sin(\theta_{10}) + \frac{1}{2\omega_1}(f_2 + 3a_{10}^2 f_2)\sin(2\theta_{10})\right\}a_{11} + \left\{\frac{1}{8\omega_1}(4 f_1 + 3a_{10}^2 f_1)\cos(\theta_{10}) + \frac{1}{4\omega_1}(4 f_2 a_{10} + 3a_{10}^3 f_2)\cos(2\theta_{10})\right\}\theta_{11} + \left\{\frac{a_{20}\gamma_1}{2\omega_1}\sin(\theta_{20})\right\}a_{21} + \left\{\frac{a_{20}\gamma_1}{2\omega_1}\cos(\theta_{20})\right\}\theta_{21}, \quad (32)$$

$$\theta_{11} = \left\{\frac{\sigma_1}{a_{10}} + \frac{\alpha}{a_{10}\omega_1} + \frac{3\alpha a_{19}}{\omega_1} - \frac{3\alpha_2 a_{10}}{4\omega_1} + \frac{9 f_1}{8\omega_1}\cos(\theta_{10}) + \frac{2 f_2 a_{10}}{\omega_1}\cos(2\theta_{10})\right\}a_{11} + \left\{\frac{-f_1}{2\omega_1 a_{10}} + \frac{9 f_1 a_{10}}{8\omega_1}\sin(\theta_{10}) - \frac{f_2(4 + 9a_{10}^2)}{4\omega_1}\sin(2\theta_{10})\right\}\theta_{11} + \left\{\frac{\gamma_1\cos(\theta_{20})}{2a_{10}\omega_1}\right\}a_{21} + \left\{\frac{\gamma_1 a_{20}}{2a_{10}\omega_1}\sin(\theta_{20})\right\}\theta_{21}, \quad (33)$$

$$\dot{a}_{21} = \left\{-\frac{\gamma_2}{2\omega_2}\sin(\theta_{20})\right\}a_{11} - \{\xi\omega_2\}a_{21} + \left\{\frac{\gamma_2}{2\omega_2}\cos(\theta_{20})\right\}\theta_{21}, \quad (34)$$

$$\theta_{21}=\begin{Bmatrix}\dfrac{-\gamma_2}{2a_{20}\omega_2}\cos(\theta_{20})+\dfrac{\sigma_1}{a_{10}}+\dfrac{\alpha}{a_{10}\omega_1}+\dfrac{3\alpha a_{10}}{\omega_1}\\ -\dfrac{3\alpha_2 a_{10}}{4\omega_1}+\dfrac{9f_1}{8\omega_1}\cos(\theta_{10})+\dfrac{2f_2a_{10}}{\omega_1}\cos(2\theta_{10})\end{Bmatrix}a_{11}+$$
$$\left\{\frac{-f_1}{2\omega_1 a_{10}}+\frac{9f_1a_{10}}{8\omega_1}\sin(\theta_{10})-\left(\frac{4f_2+9f_2a_{10}^2}{4\omega_1}\right)\sin(2\theta_{10})\right\}\theta_{11}+ \quad (35)$$
$$\left\{\frac{\sigma_2-\sigma_1}{a_{20}}+\frac{\gamma_1\cos(\theta_{20})}{2a_{10}\omega_1}\right\}a_{21}+\left\{\frac{\gamma_2 a_{10}}{2a_{20}\omega_2}\sin(\theta_{20})+\frac{\gamma_1 a_{20}}{2a_{10}\omega_1}\sin(\theta_{20})\right\}\theta_{21},$$

The following linear system is topologically equivalent to the nonlinear system given by Equations from(32) to (35) as long as the eigenvalues are hyperbolic

$$\begin{pmatrix}a_{11}\\ \theta_{11}\\ a_{21}\\ \theta_{21}\end{pmatrix}=\begin{pmatrix}r_{11}&r_{12}&r_{13}&r_{14}\\ r_{21}&r_{22}&r_{23}&r_{24}\\ r_{31}&r_{32}&r_{33}&r_{34}\\ r_{41}&r_{42}&r_{43}&r_{44}\end{pmatrix}\begin{pmatrix}a_{11}\\ \theta_{11}\\ a_{21}\\ \theta_{21}\end{pmatrix} \qquad (36)$$

The eigenvalues of the Jacobian matrix can be obtained by resolving the following determinant

$$\begin{pmatrix}\lambda-r_{11}&r_{12}&r_{13}&r_{14}\\ r_{21}&\lambda-r_{22}&r_{23}&r_{24}\\ r_{31}&r_{32}&\lambda-r_{33}&r_{34}\\ r_{41}&r_{42}&r_{43}&\lambda-r_{44}\end{pmatrix}=0, \qquad (37)$$

the values of eigenvalues are the roots of the following polynomial

$$\lambda^4+R_1\lambda^3+R_2\lambda^2+R_3\lambda+R_4=0, \qquad (38)$$

According to Routh–Hurwitz criterion, the necessary and sufficient conditions for the system stability are:

$$R_1\succ 0, R_1R_2-R_3\succ 0, R_3(R_1R_2-R_3)-R_1^2R_4\succ 0, R_4\succ 0.$$

5. Time history

we simulated numerically equation (1) which introduced the nonlinear dynamical model without and with involved PPF control to show the reduce of vibration after adding this control. After inserting the values of parameters as $\mu=0.1,\alpha=.01,\alpha_2=0.2,$ $\gamma_1=\gamma_2=3,\xi=0.01,\omega_1=\omega_2=4.$ the time history can be illustrated as in Fig.(1) a and b which represents the uncontrolled amplitude time history at primary resonance of the main model and the time histories of both controlled amplitude of the main model with PPF. It is worth to notice that from the Fig. (1) a, b that the steady-state amplitude of the micro-electro-mechanical system with PPF controller was reduced to about 99.9% from its value without PPF controller. This means that the effectiveness of the controller Ea (Ea= steady state amplitude of the micro-electro- mechanical system without controller steady state amplitude of the micro-electro- mechanical system with controller) is about 20 for the main system.

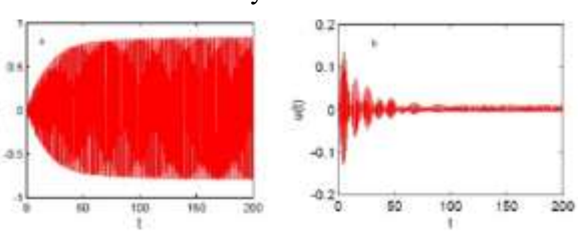

Fig. 1. The vibration amplitudes of main system : a without control and b with PPF control

We study the effects of different parameters by solving the frequency response equations (23) - (26). The results are illustrated graphically in Figs. (2 to 9). From the obtained figures, the steady state amplitudes a_1, a_2 and are presented against detuning parameters σ_1,σ_2 for the selected practical case $a_1\neq 0, a_2\neq 0$

The following curves represent the frequency response of the system with PPF control, where Fig. (a) shows the frequency response curves for the system) and Fig. (b) shows the frequency-response curves for PPF controller. At $\sigma_1=0$ the minimum steady-state amplitude $a_1=0$.Fig. (2), (3) shows that the steady state amplitudes for both the main system and the PPF controller are increased according to the increasing values of the excitation forces amplitudes f_1, f_2 Figs. (4), (5) shown the effect of the feedback signal gains γ_1,γ_2 the vibration reduction frequency bandwidth of the control for the amplitude of the main system a_1 is wider for increasing the values of γ_1, γ_2 and the controller amplitude a_2 decrease for increases σ_1, increase for increases γ_2.

Figure (6) shows that for increasing values of the damping coefficients μ both the main system and the controller are decreasing. Fig (7). show that the increase of linear natural frequency ω_1 makes an increases in the amplitude of the main system and the vibration reduction frequency bandwidth of the control for the amplitude of the main system a_1 is wider.The figure(8)shows that when taking different values of the internal detuning parameter σ_2 the shape of the frequency response curves for both the main system and the controller are affected by different values, for example when $\sigma_2 = 0.5$ the minimum steady state amplitude for the main system occurs when $\sigma_1 = 0.5$ for $\sigma_2 = 0$ the minimum steady state amplitude for the main system occurs when $\sigma_1 = 0$ and for $\sigma_2 = 0.5$ The steady-state widening of the main system of the small candle occurs when $\sigma_1 = 0.5$ So, the lower main system

Steady-state amplitude occurs when σ_1, σ_2 Fig.(9) represent the affect of the damping coefficient of the (PPF) controller for increasing ξ the amplitude of the main system and control are decreasing.

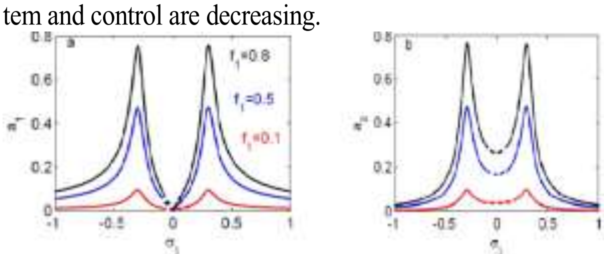

Fig. 2. Effect of the linear external excitation force f_1 on: a the main system a_1 and b the controller a_2

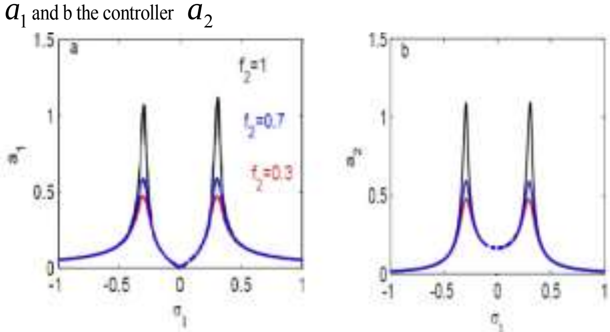

Fig.3 .Effect of the linear external excitation force f_2 on: a the main system a_1 and b the controller a_2

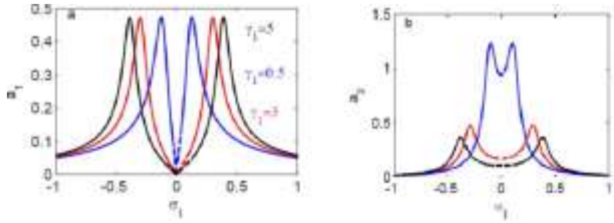

Fig. 4. The feedback gain γ_1 effectiveness on : a main system and b on the PPF controller

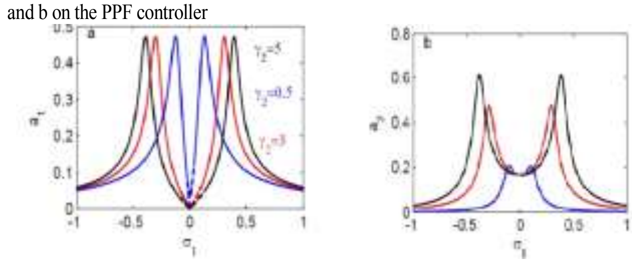

Fig.5. The feedback gain γ_2 effectiveness on: a main system and b on the PPF control

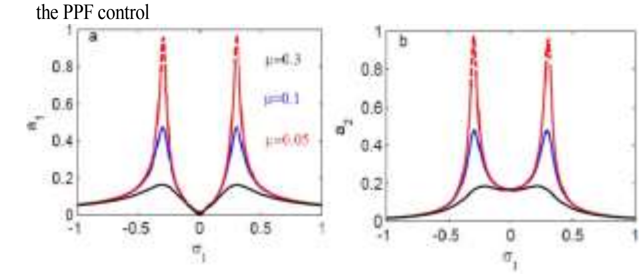

Fig.6. Effect of μ is the coefficient of viscous damping on the amplitudes of main system and PPF

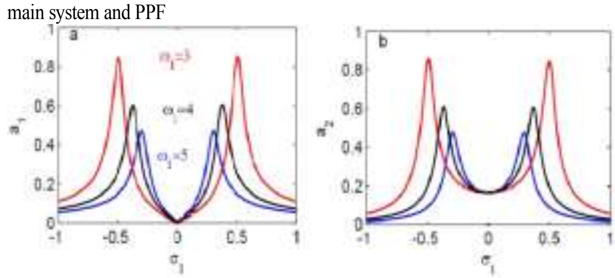

Fig. 7. Effect of linear natural frequency on the amplitudes of main system and PPF control

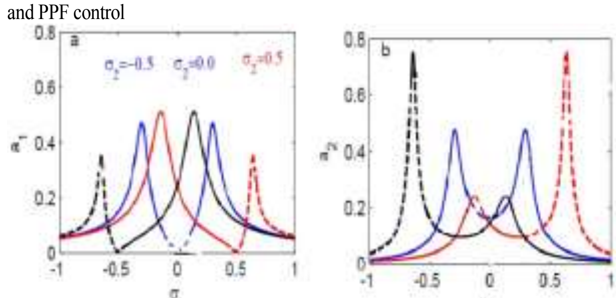

Fig. 8 The effect of damping parameter σ_2 on both the amplitudes of main system and PPF control

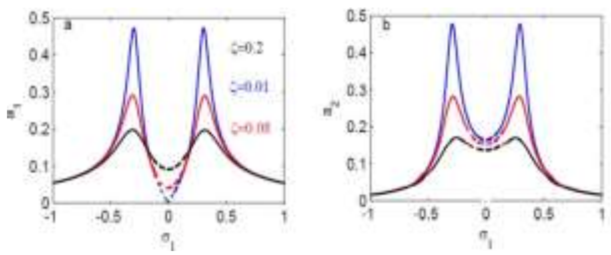

Fig. 9. The effect of damping coefficient of controller ξ on both the amplitudes of main system and PPF controller

6. Comparison between analysis and numerical solutions

Figure (10) represents the comparison between the numerical solution of equations (3) and the analytical solution The solution given by equations (28-31) for the modified Duffing equation with the PPF controller for chosen values of system parameters. The dashed lines show the analytical solution and represent the continuous lines numerical solution.

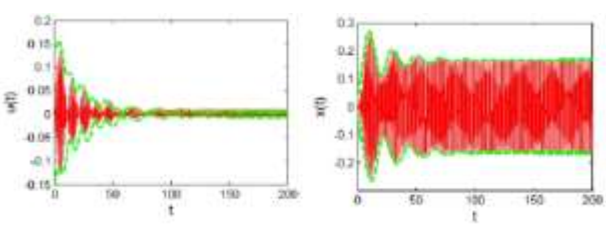

Fig.10. Comparison between the numerical solution and the perturbation analysis of closed loop

7. Conclusions

In this paper, the modified duffing equation is studied with PPF controller to reduce the vibration. We use the simultaneous primary and internal resonance case by the method of multiple scales. The stability of the system under the simultaneous resonances is studied to drive the frequency response equations. The effects of the different parameters of the system and the controller are studied numerically. The numerical results are focused on both the effects of different parameters and the response of the system.

Acknowledgments

The authors highly thank the referees for their observations and advice for advancing the quality of the paper.

References

[1] Kuang, J. H., Chen, C. J. (2004). Dynamic characteristics of shaped micro-actuators solved using the differential quadrature method. *J. Micromech. Microeng. 14*, 647-655.

[2] Abdel-Rahman, E. M., Younis, M. I., Nayfeh, A. H. (2002). Characterization of the mechanical behavior of an electrically actuated microbeam. *J. Micromech. Microeng. 12(6)*, 759-766.

[3] Younis, M. I., Nayfeh, A. H. (2003). A study of the nonlinear response of a resonant microbeam to an electric actuation. *Nonlinear Dynamics, 31(1*), 91-117.

[4] Amer, Y. A., EL-Sayed, A. T., EL-Salam, M. A . (2018). Non-linear Saturation Controller to Reduce the Vibrations of Vertical Conveyor Subjected to External Excitation. *Asian Research Journal of Mathematics, 11(2*), 1-26.

[5] Eftekhari, M., Ziaei-Rad, S., Mahzoon, M. (2013). Vibration suppression of a symmetrically cantilever composite beam using internal resonance under chordwise base excitation. *Int. J. Non-Linear Mech. 48*, 86-100.

[6] Nayfeh, A. H., Younis, M. I. (2005). Dynamics of MEMS resonators under superharmonic and subharmonic excitations. *J. Micromech. Microeng. 15(10)*, 1840-1847.

[7] Amer, Y. A., Abd EL-Salam, M. N., & EL-Sayed, M. A. (2022). Behavior of a Hybrid Rayleigh-Van der Pol-Duffing Oscillator with a PD Controller. *Journal of Applied Research and Technology*, *20*, 58-67.

[8] Abdelhafez, H., Nassar, M. (2016). Effects of time delay on an active vibration control of a forced and Self-excited nonlinear beam. *Nonlinear Dynamics, 86(1),* 137-151.

[9] Liu, C. X., Yan, Y., Wang, W. Q. (2019). Primary and secondary resonance analyses of a cantilever beam carrying an intermediate lumped mass with time-delay feedback. *Nonlinear Dynamics*, *97*(2), 1175-1195.

[10] EL-Sayed, A. T. (2021). Resonance behavior in coupled Van der Pol harmonic oscillators with controllers and delayed feedback. *Journal of Vibration and Control*, *27*(9-10), 1155-1170.

[11] Ferrari, G. Amabili, M. (2015). Active vibration control of a sandwich plate by non- collocated positive position feedback. *J. Sound Vib. 342*, 44-56.

[12] Niu, W., Li, B., Xin, T., Wang, W. (2018). Vibration active control of structure with parameter perturbation using fractional order positive position feedback controller. *J. Sound Vib. 430*, 101-114.

[13] Omidi, E., Mahmoodi, S. N. (2015). Sensitivity

analysis of the nonlinear integral positive position feedback and integral resonant controllers on vibration suppression of nonlinear oscillatory systems. *Commun. Nonlinear Sci. Numer. Simul. 22(1-3)*, 149-166.

[14] EL-Sayed, A. T., Bauomy, H. S. (2018). Outcomeof special vibration controller techniques linked to a cracked beam. *Appl. Math. Model. 63*, 266-287.

[15] El-Ganaini, W. A., Saeed, N. A., Eissa, M. (2013). Positive position feedback (PPF) controller for suppression of nonlinear system vibration. *Nonlinear Dynamics*, *72*(3), 517-537.

[16] El-Sayed, A. T., Bauomy, H. S. (2016). Nonlinear analysis of vertical conveyor with positive position feedback (PPF) controllers. *Nonlinear Dynamics, 83(1-2)*, 919-939.

[17] Amer, Y. A., EL-Sayed, A. T., N., M. (2022). A Suitable Active Control for Suppression the Vibrations of a Cantilever Beam. *Sound & Vibration, 56(2)*, 89–104.

[18]Bauomy H. S., El-Sayed A. T. and Metwaly T. M. N. (2016). Using negative velocity feedback controller to reduce the vibration of a suspended cable. *Journal of Vibroengineering*, *18*(2), (2016), 938-950

[19] A. M. Elnaggar, A. F. El-Bassiouny, K. M. Khalil1and A. M. Omran, Periodic Solutions of a Modified Duffing Equation Subjected to a Bi-Harmonic Parametric and External Excitations, British Journal of Mathematics & Computer Science 16(4) (2016)1-12.

[20] A. M. El-Naggar , K. M. Khalil and A. M. Omran, Subharmonic Solutions of Governed MEMS SystemSubjected to Parametric and External Excitations, Asian Research Journal of Mathematics,3(3) (2017)1-13.

Appendix

Coefficients of Eqs. (11) and (12)

$$E_1 = \frac{(f_1 A_1 + 6\bar{A}_1 A_1^2 f_1)}{(\omega_1^2 - (\Omega + \omega_1)^2)}, E_2 = \frac{(f_2 A_1 + 6\bar{A}_2 A_1^2 f_2)}{(\omega_1^2 - (2\Omega + \omega_1)^2)},$$

$$E_3 = \frac{(3\alpha A_1^2 - \alpha_1 A_1^2)}{-3\omega_1^2}, E_4 = \frac{(0.5 f_1 + 3A_1\bar{A}_1 f_1)}{(\omega_1^2 - 4\Omega^2)},$$

$$E_5 = \frac{1.5 A_1^2 f_1}{(\omega_1^2 - (\Omega + 2\omega_1)^2)}, E_6 = \frac{2\bar{A}_1^3 f_2}{(\omega_1^2 - (2\Omega - 3\omega_1)^2)},$$

$$E_7 = \frac{1.5\ A_1^2 f_2}{(\omega_1^2 - (2\Omega + 2\omega_1)^2)}, E_8 = \frac{1.5\bar{A}_1^2 f_2}{(\omega_1^2 - (2\Omega - 2\omega_1)^2)},$$

$$E_9 = \frac{2\bar{A}_1^3 f_1}{(\omega_1^2 - (\Omega - 3\omega_1)^2)}, E_{10} = \frac{2A_1^3 f_1}{(\omega_1^2 - (\Omega + 3\omega_1)^2)},$$

$$E_{11} = \frac{(4\alpha\bar{A}_1^3 - \alpha_1\bar{A}_1^3)}{-8\omega_1^2}, E_{12} = \frac{2A_1^3 f_2}{(\omega_1^2 - (2\Omega + 3\omega_1)^2)},$$

$$E_{13} = \frac{(f_1\bar{A}_1 + 6A_1\bar{A}_1^2 f_1)}{(\omega_1^2 - (\Omega - \omega_1)^2)}, E_{14} = \frac{(3\alpha\bar{A}_1^2 - \alpha_1\bar{A}_1^2)}{3\omega_1^2},$$

$$E_{15} = \frac{(4\alpha A_1^3 - \alpha_1 A_1^3)}{8\omega_1^2} + E_{16} = \frac{(f_2\bar{A}_1 + 6A_1\bar{A}_1^2 f_2)}{(\omega_1^2 - (2\Omega - \omega_1)^2)},$$

$$E_{17} = \frac{(0.5 f_1 + 3A_1\bar{A}_1 f_1)}{(\omega_1^2 - \Omega^2)}, E_{18} = \frac{2\bar{A}_1^3 f_2}{(\omega_1^2 - (2\Omega - 3\omega_1)^2)}$$

$$E_{19} = \frac{\gamma_2 A_1}{(\omega_2^2 - \omega_1^2)}.$$