

In this paper, we demonstrated in a practical example that invariant system behavior can be guaranteed in accordance with theory in the case when different subsets of state variables must satisfy different consistency conditions. It is important to emphasize that although we examined a specific case involving a mix of unit-consistent and rotation-consistent state variables, the general approach is agnostic to the particular consistency conditions that are enforced, i.e., we could have chosen two different conditions or included different conditions for additional subsets of state variables and the same invariant behavior should be expected.

We also provided a formal proof that the unit-consistency properties of the UC inverse are preserved under applications of the Kronecker product. With this we believe the theory of consistent inverses is now largely complete.

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