Electromagnetic Analysis of a Dielectric Rod loaded by Metasurface

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Abstract: - This paper introduces an analytical solution for a dielectric rod leaky wave antenna loaded by a metasurface. The advantage of adding metasurface on the dielectric rod is that it introduces an additional degree of freedom in the design of this antenna by controlling the outer boundary condition. Modified Generalized Sheet Transition Condition (GSTC) on a cylindrical geometry is used to model the metasurface on the dielectric rod. The analysis is based on developing the characteristic equation of the proposed structure. This characteristic equation is used to obtain the complex propagation constant of this structure and the corresponding field distribution inside and outside the rod including the resulting radiated fields. Parameters of the metasurface are extracted from the reflection and transmission properties of its elements in infinite periodic structure. An example of a dielectric rod loaded by a metasurface is presented analytically based on the introduced analysis and it also simulated numerically for comparison.

Key-Words: - Metasurface; Leaky wave antenna; Dielectric rod antenna; Generalized sheet transition condition.

1 Introduction

Dielectric rod waveguide is an important category of wave guiding structures, especially in mm-wave and higher frequency applications due to the low loss compared with conducting guiding structures in these bands. Dielectric rod waveguides are extensively studied by using different techniques [1-3]. The design of dielectric rod waveguide for a specific application is based on controlling the diameter and the electrical properties of the dielectric rod. Controlling the diameter of the rod includes also tapering and shaping of the rod.

In addition to be used as guiding structures, dielectric rod waveguides can also be used as radiating elements. Leaky wave dielectric antenna is an excellent example of using dielectric rod as an antenna in mm-wave range [4]. The key point in dielectric rod antenna is that it can be represented as an open-boundary waveguide structure which can be adjusted with appropriate design to leak electromagnetic wave continuously through its boundary to introduce a narrow beam radiation pattern. The resulting pattern is usually highly directive omni-directional pattern with a tilted conical beam with respect to the axis of the dielectric rod. The properties of this beam make it suitable for wireless communication and satellite communication systems [5].

On the other hand, metasurfaces are artificial surfaces composed of periodic elements with periodicity much smaller than the operating wavelength. By controlling the shape and the electrical properties of these elements it would be possible to control the resulting electric and magnetic susceptibilities of the corresponding surface. This property can be used to tailor the required boundary conditions. Metasurfaces can be considered as a two-dimensional form of metamaterials. Adding metasurface to the boundary of the dielectric rod waveguide structure introduces an additional degree of freedom in the design of this structure by controlling the outer boundary conditions. Metasurfaces can be designed as partially reflecting surface which can be used to enhance the leaky wave performance of leaky wave dielectric rod antenna. In a previous study [6] it was shown that a hollow cylindrical shell of metamaterial is suitable to design cylindrical leaky wave antennas. The disadvantage of this method is that it requires using bulk curved metamaterial which makes the fabrication of this structure may be quite complicated. Thus a metasurface printed on a cylindrical dielectric rod can be considered as a good alternative for the previously studied metamaterial cylindrical shell [6]. This configuration has also been studied from another point of view as a cloaking surface for cylindrical shapes [7]. However, in this case the metasurface was represented as two-dimensional surface impedance.

The formulation of the present problem is based on obtaining the corresponding boundary conditions of the metasurface and how these boundary conditions can be controlled by adjusting the shapes and the electrical properties of the elements of this metasurface. Previous studies of metasurfaces are based on Generalized Sheet Transition Conditions (GSTC) which is used to represent the boundary conditions of metasurfaces. This GSTC is based on infinite metasurface. For the case of a metasurface on a dielectric rod, this condition may be satisfied on the longitudinal direction of the rod. However, it can be approximately satisfied in the azimuth direction of the rod if the circumference of the rod is much greater than the dimensions of the elements of metasurface and the corresponding periodicity.

The problem of the cylindrical dielectric rod loaded with a metasurface is formulated by using modal analysis as an infinite set of orthogonal modes inside and outside the dielectric rod. By applying the GSTC of the metasurface at the boundary between the inside and outside parts of the dielectric rod it would be possible to obtain an eigenvalue problem where the eigenvalues correspond to the propagation constants and the eigen vectors corresponds to the amplitudes of the modes. By controlling the dielectric constant of the rod, the diameter of the rod and the GSTC of the metasurface one can control the complex propagation constant of the modes of this dielectric rod structure. Based on the signs of the real and imaginary parts of this complex propagation constant it would be determined the guiding and radiating properties of this rod structure. Positive real propagation constant with slightly negative imaginary attenuating coefficient would corresponds to an attenuating guided wave which continuously leaks from the dielectric rod to for a leaky wave antenna. The direction of the radiated beam depends on the real propagation constant while the corresponding beamwidth depends on the imaginary attenuation coefficient. Thus, appropriate usage of the previously mentioned design parameters it would be possible to design a cylindrical leaky wave antenna with specific beam tilt and specific beamwidth.

In the following section the basic theory of GSTC on cylindrical structure is investigated. This modified GSTC is used to formulate the boundary value problem of a dielectric rod loaded by a metasurface to obtain the corresponding eigenvalue problem. This eigenvalue problem is formulated as a characteristic equation. In Sec. III, this characteristic equation is solved numerically by using Davidenko's method [8] to obtain the corresponding complex propagation constant.

2 Characteristic Equation of Dielectric Rod Loaded By Metasurface

Figure 1 shows the geometry of the proposed structure. It consists of a dielectric rod loaded by a metasurface boundary. Assuming large circumference of the rod compared with the periodic cells of the metasurface, the boundary condition at the metasurface can be represented as:

$$\mathbf{a}_\rho \times \mathbf{H}\big|_{\rho=a^-}^{a^+} = j\omega\varepsilon_0 \bar{\bar{\alpha}}_{ES}\cdot \mathbf{E}_{t,av}\big|_{\rho=a} - \frac{1}{\mu_0}\mathbf{a}_\rho \times \nabla_t\left[\alpha_{MS}^{\rho\rho} B_{\rho,av}\right]_{\rho=a}, \quad \text{(1-a)}$$

$$\mathbf{E}\big|_{\rho=a^-}^{a^+} \times \mathbf{a}_\rho = j\omega\mu_0 \bar{\bar{\alpha}}_{MS}\cdot \mathbf{H}_{t,av}\big|_{\rho=a} - \frac{1}{\varepsilon_0}\nabla_t\left[\alpha_{ES}^{\rho\rho} D_{\rho,av}\right]_{\rho=a} \times \mathbf{a}_\rho, \quad \text{(1-b)}$$

where $\bar{\bar{\alpha}}_{ES}$ and $\bar{\bar{\alpha}}_{MS}$ are the diagonal electric and magnetic susceptibilities

$$\bar{\bar{\alpha}}_{ES} = \alpha_{ES}^{\rho\rho}\mathbf{a}_\rho\mathbf{a}_\rho + \alpha_{ES}^{\phi\phi}\mathbf{a}_\phi\mathbf{a}_\phi + \alpha_{ES}^{zz}\mathbf{a}_z\mathbf{a}_z, \quad \text{(1-c)}$$

$$\bar{\bar{\alpha}}_{MS} = \alpha_{MS}^{\rho\rho}\mathbf{a}_\rho\mathbf{a}_\rho + \alpha_{MS}^{\phi\phi}\mathbf{a}_\phi\mathbf{a}_\phi + \alpha_{MS}^{zz}\mathbf{a}_z\mathbf{a}_z. \quad \text{(1-d)}$$

The components of electric and magnetic susceptibilities in cylindrical coordinates can be represented approximately in terms of the corresponding susceptibilities of a unit cell of the metasurface in Cartesian coordinates as follows [9-11]:

$$\alpha_{ES}^{\rho\rho} = \frac{N\langle\alpha_{E,ww}\rangle}{1+\frac{N\,\varepsilon_\perp}{4R\varepsilon_0}\langle\alpha_{E,ww}\rangle}, \quad (2\text{-a})$$

$$\alpha_{ES}^{\phi\phi} = \frac{N\langle\alpha_{E,uu}\rangle}{1-\frac{N\,\varepsilon_0}{4R\varepsilon_\parallel}\langle\alpha_{E,uu}\rangle}, \quad (2\text{-b})$$

$$\alpha_{ES}^{zz} = \frac{N\langle\alpha_{E,vv}\rangle}{1-\frac{N\,\varepsilon_0}{4R\varepsilon_\parallel}\langle\alpha_{E,vv}\rangle}, \quad (2\text{-c})$$

$$\alpha_{MS}^{\rho\rho} = \frac{N\langle\alpha_{M,ww}\rangle}{1-\frac{N\langle\alpha_{M,ww}\rangle}{4R}}, \quad (2\text{-d})$$

$$\alpha_{MS}^{\phi\phi} = \frac{N\langle\alpha_{M,uu}\rangle}{1+\frac{N\langle\alpha_{M,uu}\rangle}{4R}}, \quad (2\text{-e})$$

$$\alpha_{MS}^{zz} = \frac{N\langle\alpha_{M,vv}\rangle}{1+\frac{N\langle\alpha_{M,vv}\rangle}{4R}}, \quad (2\text{-f})$$

The u, v and w directions are assumed to be local Cartesian coordinates which are coincident with the ϕ, z and ρ directions in cylindrical coordinates respectively. Assuming that the dielectric constant of the rod is $\varepsilon_1 = \varepsilon_{r1}\varepsilon_0$, the corresponding normal and parallel permittivity in (2) are given by:

$$\varepsilon_\perp = \frac{2\varepsilon_1\varepsilon_0}{\varepsilon_1+\varepsilon_0}, \quad (3\text{-a})$$

$$\varepsilon_\parallel = \frac{\varepsilon_1+\varepsilon_0}{2}, \quad (3\text{-b})$$

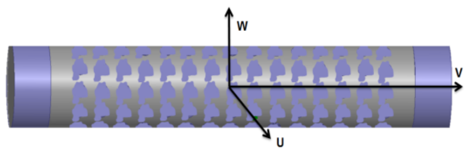


Fig. 1 Cylindrical metasurface structure.

The analysis of the problem for the hybrid mode of circular dielectric rod surrounded by a metasurface can be formulated by using modal analysis in terms of a combination of TE and TM modes. In this case the fields inside the dielectric rod are formulated in terms of Bessel function while the fields outside the dielectric rod are formulated in terms of Hankel functions of second kind to ensure the boundary condition at infinitely. Thus the fields inside the dielectric rod can presented as:

$$H_z = A_m J_m(\beta_\rho^d \rho) e^{-jm\phi} e^{-j\beta_z z}, \quad (4\text{-a})$$

$$E_z = B_m J_m(\beta_\rho^d \rho) e^{-jm\phi} e^{-j\beta_z z}, \quad (4\text{-b})$$

$$E_\rho = \left[-A_m \frac{m\omega\mu_0}{(\beta_\rho^d)^2\rho} J_m(\beta_\rho^d\rho) - B_m \frac{j\beta_z}{\beta_\rho^d} J_m'(\beta_\rho^d\rho)\right] e^{-jm\phi} e^{-j\beta_z z}, \quad (4\text{-c})$$

$$E_\phi = \left[A_m \frac{j\omega\mu_0}{\beta_\rho^d} J_m'(\beta_\rho^d\rho) - B_m \frac{m\beta_z}{(\beta_\rho^d)^2\rho} J_m(\beta_\rho^d\rho)\right] e^{-jm\phi} e^{-j\beta_z z}, \quad (4\text{-d})$$

$$H_\rho = \left[-A_m \frac{j\beta_z}{\beta_\rho^d} J_m'(\beta_\rho^d\rho) + B_m \frac{m\omega\varepsilon_1}{(\beta_\rho^d)^2\rho} J_m(\beta_\rho^d\rho)\right] e^{-jm\phi} e^{-j\beta_z z}, \quad (4\text{-e})$$

$$H_\phi = \left[-A_m \frac{m\beta_z}{(\beta_\rho^d)^2\rho} J_m(\beta_\rho^d\rho) - B_m \frac{j\omega\varepsilon_1}{\beta_\rho^d} J_m'(\beta_\rho^d\rho)\right] e^{-jm\phi} e^{-j\beta_z z}, \quad (4\text{-f})$$

where $(\beta_\rho^d)^2 + (\beta_z)^2 = \varepsilon_{r1}k_0^2$ and $\mathrm{Im}(\beta_z) \le 0$.

On the other hand, the fields outside the dielectric rod can be represented as:

$$H_z = C_m H_m^{(2)}(\beta_\rho^0\rho) e^{-jm\phi} e^{-j\beta_z z}, \quad (5\text{-a})$$

$$E_z = D_m H_m^{(2)}(\beta_\rho^0\rho) e^{-jm\phi} e^{-j\beta_z z}, \quad (5\text{-b})$$

$$E_\rho = \left[-C_m \frac{m\omega\mu_0}{(\beta_\rho^0)^2\rho} H_m^{(2)}(\beta_\rho^0\rho) - D_m \frac{j\beta_z}{\beta_\rho^0} H_m^{'(2)}(\beta_\rho^0\rho)\right] e^{-jm\phi} e^{-j\beta_z z}, \quad (5\text{-c})$$

$$E_\phi = \left[C_m \frac{j\omega\mu_0}{\beta_\rho^0} H_m^{'(2)}(\beta_\rho^0\rho) - D_m \frac{m\beta_z}{(\beta_\rho^0)^2\rho} H_m^{(2)}(\beta_\rho^0\rho)\right] e^{-jm\phi} e^{-j\beta_z z}, \quad (5\text{-d})$$

$$H_\rho = \left[-C_m \frac{j\beta_z}{\beta_\rho^0} H_m^{'(2)}(\beta_\rho^0\rho) + D_m \frac{m\omega\varepsilon_0}{(\beta_\rho^0)^2\rho} H_m^{(2)}(\beta_\rho^0\rho)\right] e^{-jm\phi} e^{-j\beta_z z}, \quad (5\text{-e})$$

$$H_\phi = \left[-C_m \frac{m\beta_z}{(\beta_\rho^0)^2\rho} H_m^{(2)}\left(\beta_\rho^0\rho\right) - D_m \frac{j\omega\varepsilon_0}{\beta_\rho^0} H_m^{'(2)}\left(\beta_\rho^0\rho\right)\right] e^{-jm\phi} e^{-j\beta_z z}, \qquad \text{(5-f)}$$

where $\left(\beta_\rho^0\right)^2 + (\beta_z)^2 = k_0^2$.

By substituting (4) and (5) and applying the GSTC boundary condition (1) one can obtain the hybrid modes of circular dielectric rod covered by a metasurface.
For a special case of TM modes where $H_z = 0$, the characteristic equation can be simplified as:

$$-\alpha_{ES}^{zz} + \left[\frac{1}{\beta_\rho^d} - \frac{(k_0)^2\alpha_{ES}^{zz}}{4\beta_\rho^d}\left(\alpha_{MS}^{\phi\phi} + \alpha_{ES}^{\rho\rho}\right)\right]\frac{\varepsilon_{r1}J_1(\beta_\rho^d a)}{J_0(\beta_\rho^d a)} - \left[\frac{1}{\beta_\rho^0} - \frac{(k_0)^2\alpha_{ES}^{zz}}{4\beta_\rho^0}\left(\alpha_{MS}^{\phi\phi} + \alpha_{ES}^{\rho\rho}\right)\right]\frac{H_1^{(2)}(\beta_\rho^0 a)}{H_0^{(2)}(\beta_\rho^0 a)} - \left[\frac{(k_0)^2\varepsilon_{r1}}{\beta_\rho^0\beta_\rho^d}\left(\alpha_{MS}^{\phi\phi} + \alpha_{ES}^{\rho\rho}\right)\right]\frac{J_1(\beta_\rho^d a)H_1^{(2)}(\beta_\rho^0 a)}{J_0(\beta_\rho^d a)H_0^{(2)}(\beta_\rho^0 a)} = 0. \qquad (6)$$

Similarly for the case of TE modes where $E_z = 0$, the characteristic equation can be simplified as:

$$-\alpha_{MS}^{zz}\frac{J_0(\beta_\rho^d a)H_0^{(2)}(\beta_\rho^0 a)}{J_1(\beta_\rho^d a)H_1^{(2)}(\beta_\rho^0 a)} - \left[\frac{1}{\beta_\rho^0} - \frac{(k_0)^2\alpha_{MS}^{zz}}{4\beta_\rho^0}\left(\alpha_{MS}^{\rho\rho} + \alpha_{ES}^{\phi\phi}\right)\right]\frac{J_0(\beta_\rho^d a)}{J_1(\beta_\rho^d a)} + \left[-\frac{(k_0)^2\alpha_{MS}^{zz}}{4\beta_\rho^d}\left(\alpha_{MS}^{\rho\rho} + \alpha_{ES}^{\phi\phi}\right) + \frac{1}{\beta_\rho^d}\right]\frac{H_0^{(2)}(\beta_\rho^0 a)}{H_1^{(2)}(\beta_\rho^0 a)} - \frac{(k_0)^2}{2\beta_\rho^d\beta_\rho^0}\left(\alpha_{ES}^{\phi\phi} + \alpha_{MS}^{\rho\rho}\right) = 0. \qquad (7)$$

It can be noted that if the electric and magnetic susceptibilities of the metasurface are assumed to vanishing, the above characteristic equations in (6) and (7) would be exactly the same of dielectric rod waveguide for both TM and TE modes respectively [1].

3 Results and Discussions

In this section a dielectric rod loaded by a metasurface is studied as shown in Fig. 2. The radius of the dielectric rod is 5mm and its dielectric constant is assumed to be $\varepsilon_{r1} = 5$ as in the case of [4]. The properties of the metasurface depend on the unit cell and the shape of its element. For simplicity, rectangular perfect conducting patches in rectangular cells are introduced as a metasurface. The size of the patches are $a = 1.84$ mm and $b = 1.5$ mm in z and ϕ directions respectively. The spacing between the patches are $d = 0.06$ mm and $c = 0.07$ mm in z and ϕ directions. It should be noted that the dimension of the patch and the spacing in the ϕ direction are chosen such that an integer number of patches of the same size and the same periodical spacing would cover the circumference of the dielectric rod.

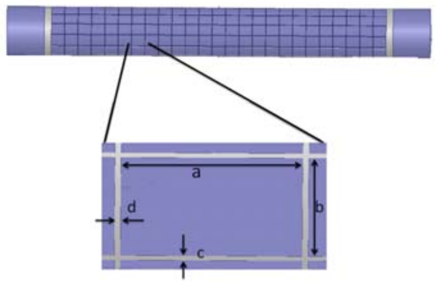
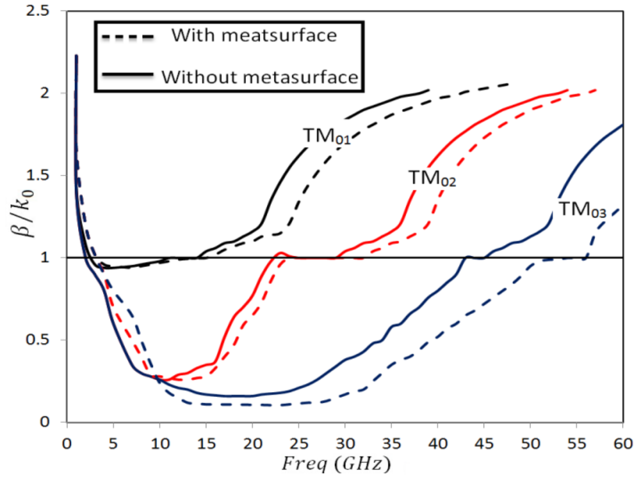


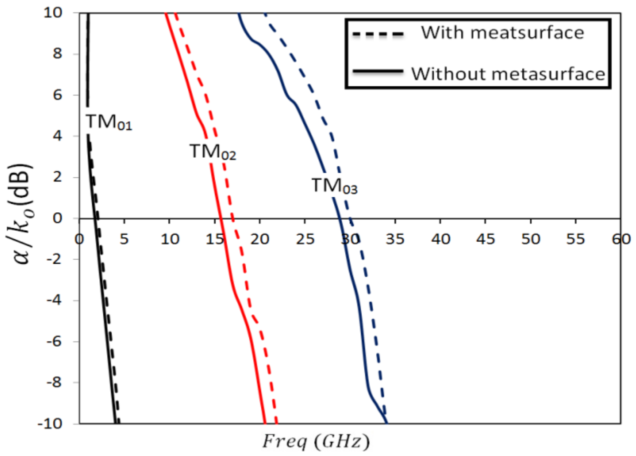
Fig. 2. Geometry of the dielectric rod loaded by a metasurface.

The equivalent susceptibilities of the proposed metasurface are obtained by using the reflection and transmission coefficients of this metasurface [10],[11]. These reflection and transmission coefficients for infinite planar metasurface can be obtained by solving for a single element with periodic boundary conditions. The approximation of using planar metasurface here would be appropriate since the periodic spacing in the ϕ direction is much smaller than the circumference of the dielectric rod, thus the element of the metasurface is assumed to be locally planar at the dielectric rod. The reflection and transmission coefficients of a single element with periodic boundary conditions are obtained numerically by using CST. Then these coefficients are used to obtain the equivalent susceptibilities.
The present analysis is mainly focused on obtaining omni-directional leaky wave antenna based on TM_{0n} leaky wave modes. These modes would require obtaining $\alpha_{MS}^{\Phi\Phi}$, α_{ES}^{zz} and $\alpha_{ES}^{\rho\rho}$ as discussed in (8). Based on the obtained results of the reflection and transmission coefficients for the above mentioned dimensions of the unit cell of the metasurface, the values of the corresponding susceptibilities in this case are $\alpha_{ES}^{zz} = .0024$ and $\alpha_{ES}^{\rho\rho} = \alpha_{MS}^{\Phi\Phi} = 0$. These values are used in (8). Then the resulting characteristic equation is solved numerically by using Davidenko's method [8] to obtain the complex propagation constant β_z along the loaded dielectric rod structure as a function of the operating frequency.
Figure 3 shows a comparison between the real and imaginary parts of the normalized complex propagation constant α/k_0 id dB scale and β/k_0 in linear scale of the TM_{0n} modes for the above mentioned dielectric rod with and without metasurface loading. It can be noted that the

dielectric rod with the metasurface has leaky wave propagation modes of $\beta/k_0 < 1$ and $\alpha/k_0 \ll 1$ at wider frequency ranges compared with the dielectric rod without metasurface. This property is found to be common in all TM_{0n} modes. This result shows that by controlling the shapes and the dimensions of the loading metasurface it would be possible to control the wave propagation on dielectric waveguides.



(a) Normalized propagation wave number



(b) Normalized attenuation coefficient

Fig. 3. Normalized complex wave number of a cylindrical dielectric waveguide; with and without a metasurface.

4 Conclusion

Modified Generalized Sheet Transition Condition (GSTC) for cylindrical configuration is introduced. This modified GSTC is used to develop the characteristic equation of a cylindrical dielectric rod loaded by a metasurface. This configuration is found to be suitable as a leaky wave antenna. The advantage of using metasurface is that it can be used to control the operating modes of the dielectric rod. The obtained results show that the loading metasurface can control the center frequency and the operating bandwidth of the leaky wave modes of this structure.

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