On the Collision of Railcars as an Interaction of Nonlinear Soliton-Like Perturbations

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Abstract: - The article examines the collision of railcars based on the solution of hydrodynamic equations using shock waves during the transition from weak nonlinearity to perturbations of arbitrary amplitude. An approximate algorithm for analytical solution of the problem is proposed. This approach has not been previously considered and has a wide range of potential technical applications.

Key-Words: Hydrodynamics, railcar collision, analytical solution for slab collision

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1 Introduction

Tony Maxworthy was a prominent figure in the field of fluid mechanics. His work transformed the field and contributed enormously to the reputation of the USC Viterbi School of Engineering. His work was eclectic, elegant, simple without being simplistic, profound and artistic in many ways. He followed Einstein's call to "make everything as simple as possible, but not simpler".

The challenges associated with implementing the high-speed rail transport (see, for example, [1–4]) pose a large number of different physical and mathematical problems.

The use of a hydrodynamic approach to the collision of railcars as a collision of rods was proposed by the great Nikolay E. Zhukovsky, the father of Russian aviation, and was continued, for example, in [5]. In [6–8], for simplicity, we carried out this consideration in flat one-dimensional geometry, and the problem of collision of layers-slabs was reduced to a description of the interaction of Korteweg–de Vries solitons. Here we have obtained a description for the propagation of shock waves during the transition from soliton perturbations to perturbations of arbitrary amplitude.

A large body of literature on shock waves has been published, concerned with the analytical solution [9] and numerical problems [10–11].

An approximate solution of one-dimensional hydrodynamic equations using shock waves can be used in calculations of rail dampers (see, for example, [5, 12–15]) and construction equipment [16–20].

2 Formulation of the Model

Let us consider the propagation of perturbations of arbitrary amplitude using the equations of hydrodynamics [6–8]:

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho \upsilon)}{\partial x} = 0, \quad (1)$$

$$\frac{\partial(m\rho\upsilon)}{\partial t} + \frac{\partial(m\rho\upsilon^2 + P)}{\partial x} = 0. \quad (2)$$

For the energy density, we can use a simple expression

$$e = K(\rho - \rho_0)^2 , \quad (3)$$

where ρ_0 is the equilibrium density and $K = 9mc_{s0}^2$ is the compression modulus. The pressure is then

$$P = -\frac{\partial(e/\rho)}{\partial(1/\rho)} = K(\rho^2 - \rho_0^2) - \alpha\left(\frac{\partial\rho}{\partial x}\right)^2 . \quad (4)$$

Here we have added a dispersion term with the coefficient α, where $\frac{\alpha}{2mc_{so}^2}\rho_0 =$ (m)^2, and the speed of sound is $c_{s0} \approx 3$ 10^3 m/s. A collision between two railcars generates shock waves propagating at velocity D, which can be found from the hydrodynamic equations (1)–(2), assuming that $\frac{\partial}{\partial t} = -D\frac{\partial}{\partial x}$.

3 Problem Solution

Integrating these equations over the density jump we obtain

$$D = -\frac{\rho_0 \upsilon_0}{\rho - \rho_0} , \quad (5)$$

where υ_0 is the initial velocity of colliding layer-cars. Assuming velocity D to be equal to the speed of sound $c_s = \sqrt{\frac{\partial P}{m\partial\rho}}$, and taking into account the expression for pressure (4), we obtain the equation for density ρ:

$$K(\rho^2 - \rho_1^2) - \alpha\left(\frac{\partial\rho}{\partial x}\right)^2 = mc_s^2(\rho_1)(\rho - \rho_1) , \quad (6)$$

where ρ_1 is the maximum compression density on a shock wave,

$$c_s^2(\rho_1) = 2K\rho_1 / m = \frac{(\rho_0\upsilon_0)^2}{(\rho_1 - \rho_0)^2} . \quad (7)$$

From here

$$\frac{\partial\rho}{\partial x} = \pm\sqrt{\frac{K}{\alpha}}(\rho - \rho_1) , \quad (8)$$

and at $x. > 0$

$$\rho - \rho_1 = (\rho_1 - \rho_0)\left(\exp\left(-\sqrt{\frac{K}{\alpha}}x\right) - 1\right). \quad (9)$$

Isolating the main terms, since we are not currently interested in the details of the wave front structure, we can approximate solution (9) with the soliton solution

$$\rho = \rho_0 + 4\frac{(\rho_1 - \rho_0)}{(\exp(-\lambda x/2) + \exp(\lambda x/2))^2} , \quad (10)$$

where $\lambda = \sqrt{\frac{K}{\alpha}}$. Expression (10) describes the main features of solution (8)–(9). Also, as we did earlier with the Korteweg–de Vries solitons, we can integrate expression (10) over the length of the layer and consider the propagation of the shock wave front and its reflection from the boundaries. As a result of integration we obtain

$$\rho = \frac{1}{L}\int_{l_1}^{l_2} \rho' dx = \rho_0 + 4\frac{(\rho_1 - \rho_0)}{\lambda L} \times$$
$$\left[\frac{1}{1 + \exp(\lambda(x - l_2 - Dt))} - \frac{1}{1 + \exp(\lambda(x - l_1 - Dt))}\right], \quad (11)$$

where ρ' is formula (10), l_1 and l_2 are the boundaries of the railcar, and $L = l_2 - l_1$ is its size. Since at the maximum of the shock wave the velocity is $\upsilon = 0$, for the maximum, a wave equation is obtained from the equations of hydrodynamics that admits the d'Alembert solution.This is what we did. In this case, velocity can be found from the continuity equation, using expression (11) for density, taking into account possible reflections of shock waves from the boundaries of the system and the movement of the boundaries.

Figure 1 shows instantaneous photographs of the relative density (ρ/ρ_0) for the interaction of two rod cars in a system of equal speeds, when the train begins to move off at an initial velocity of $\upsilon_0 = 500$ m/s at times $t = 1;2;3;4;5$ ms. After the initial compression and formation of a hot spot, followed by expansion, at the expansion stage a rarefaction is observed in the center.

As for the size, the region of rarefaction turns out to be of the order of the railcar length (10 m) and, therefore, in accordance with the estimates of shock absorber parameters [5], does not result in destruction.

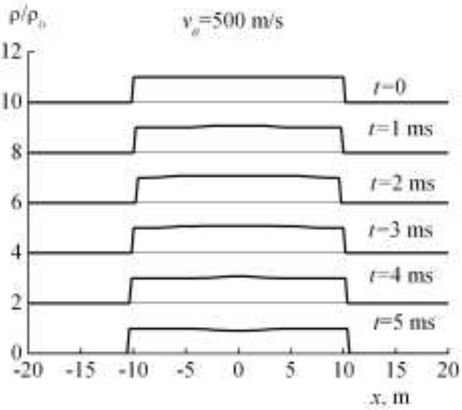

Fig. 1. Instantaneous photographs of the collision of rod cars at an initial velocity of $\upsilon_0 = 500$ m/s at times $t = 1; 2; 3; 4; 5$ ms

4 Conclusion

The approach itself based on hydrodynamic equations for perturbations of arbitrary amplitude is of independent fundamental interest for physics in general. It can be extended to a wide range of technical applications and used in the design of wheel dampers, pipes, transport structures, bridges and other transport and construction equipment (see, for example, [12–20]) in the light of high-speed transport problems.

To further develop the approach, it is necessary to take into account the dissipative terms in the hydrodynamic equations and the possible deviation from the one-dimensional problem.

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This approach can be extended to a wide range of technical applications and used in the design of wheel dampers, pipes, transport structures, bridges and other transport and construction equipment in the light of high-speed transport problems.

Conflict of Interest

The authors have no conflicts of interest to declare that are relevant to the content of this article.