

# Instability of a Compressible Plasma

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*Abstract:-* The aim of the present research was to study the thermosolutal instability of a compressible plasma due to the effects of the ion Larmor radius. Following the linearized stability theory, Boussinesq approximation and normal mode analysis, the dispersion relation is obtained. For stationary convection, the compressibility, stable solute gradient and finite Larmor radius stabilize the system. The system is found to be stable for  $(C_p/g)\beta < 1$ . The finite Larmor radius and the compressibility introduce oscillatory modes in the system for  $(C_p/g)\beta > 1$ . A condition for the system to be stable is obtained by using the Rayleigh-Ritz inequality. The sufficient conditions for the non-existence of overstability are also derived.

*Key-words:-* Thermosolutal convection, compressibility, Finite Larmor Radius

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## 1 Introduction

The theoretical and experimental results of the onset of thermal convection (Be'nard convection) in a fluid layer under varying assumptions of hydrodynamics and hydromagnetics has been summarized in the celebrated monograph by Chandrasekhar [1]. The properties of ionized space and laboratory magnetic fluids (plasmas) have been intensively investigated theoretically and experimentally in the past sixty years. One of the key aspects studied in this context is the stability of plasma structures. Usually, instabilities can be divided into two categories: macro- and micro-instabilities. Macro-instabilities occur with low frequencies compared to the plasma and cyclotron frequency and they are studied within the framework of magnetohydrodynamics (MHD). Physicists have understood the behaviour of macro-instabilities and they showed how to avoid the most destructive of them, but small-scale gradient driven micro-instabilities are still a serious obstacle for having a stable plasma for a large range of parameters. Micro-instabilities are described by models which include, e.g. finite Larmor radius (FLR) and collision less dissipative effects in plasmas. Time and length scales of micro-instabilities are comparable to the turbulent length scales and the length scales of transport coefficients. In general, the FLR effect is neglected. However, when the

Larmor radius becomes comparable to the hydromagnetic length of the problem (e.g. wavelength) or the gyration frequency of ions in the magnetic field is of the same order as the wave frequency, finiteness of the Larmor radius must be taken into account. Strictly speaking, the space and time scale for the breakdown of hydromagnetics are on the respective scales of ion gyration about the field, and the ion Larmor frequency. Finite Larmor radius effect on plasma instabilities has been the subject of many investigations. In many astrophysical plasma situations such as in solar corona, interstellar and interplanetary plasmas the assumption of zero Larmor radius is not valid. The effects of finiteness of the ion Larmor radius, showing up in the form of a magnetic viscosity in the fluid equations, have been studied by Rosenbluth et al. [2], Roberts and Taylor [3], Vandakurov [4] and Jukes [5]. Melchior and Popowich [6] have considered the finite Larmor radius (FLR) effect on the Kelvin-Helmholtz instability of a fully ionized plasma, while the effect on the Rayleigh-Taylor instability has been studied by Singh and Hans [7]. Sharma [8] has studied the effect of a finite Larmor radius on the thermal instability of a plasma. Hernegger [9] investigated the stabilizing effect of FLR on thermal instability and showed that thermal criterion is changed by FLR for wave propagation perpendicular to the magnetic field. Sharma [10] investigated the stabilizing effect of FLR on thermal instability of

rotating plasma. Ariel [11] discussed the stabilizing effect of FLR on thermal instability of conducting plasma layer of finite thickness surrounded by a non-conducting matter. Vaghela and Chhajlani [12] studied the stabilizing effect of FLR on magneto-thermal stability of resistive plasma through a porous medium with thermal conduction. Bhatia and Chhonkar [13] investigated the stabilizing effect of FLR on the instability of a rotating layer of self-gravitating plasma incorporating the effects of viscosity and Hall current. Vyas and Chhajlani [14] pointed out the stabilizing effect of FLR on the thermal instability of magnetized rotating plasma incorporating the effects of viscosity, finite electrical conductivity, porosity and thermal conductivity. Kaothekar and Chhajlani [15] investigated the problem of Jeans instability of self-gravitating rotating radiative plasma with finite Larmor radius corrections. The frictional effect of collisions of ionized with neutral atoms on Rayleigh-Taylor instability of a composite plasma in porous medium has been considered by Kumar and Mohan [16]. Thus FLR effect is an important factor in the discussion of thermal convection and other hydromagnetic instabilities.

The investigation of double-diffusive convection is motivated by its interesting complexities as a double-diffusion phenomena as well as its direct relevance to geophysics and astrophysics. The conditions under which convective motion in double-diffusive convection are important (e.g. in lower parts of the Earth's atmosphere, astrophysics, and several geophysical situations) are usually far removed from the consideration of a single component fluid and rigid boundaries and therefore it is desirable to consider a fluid acted on by a solute gradient and free boundaries. The problem of thermohaline convection in a layer of fluid heated from below and subjected to a stable salinity gradient has been considered by Veronis [17]. The physics is quite similar in the stellar case in that helium acts like salt in raising the density and in diffusing more slowly than heat. The problem is of great importance because of its application to atmospheric physics and astrophysics, especially in the case of the ionosphere and the outer layer of the atmosphere. The thermosolutal convection problems also arise in oceanography, limnology and engineering. For thermal and thermohaline convection problems, the Boussinesq approximation has been used, which is well justified in the case of incompressible fluids.

When the fluids are compressible, the equations governing the system become quite complicated. Spiegel and Venonis [18] have simplified the set of equations governing the flow of compressible fluids under the assumptions that (a) the depth of the fluid layer is much less than the scale height, as defined by them, and (b) the fluctuations in temperature, density, and pressure, introduced due to motion; do not exceed their total static variations.

Under the above approximations, the flow equations are the same as those for incompressible fluids, except that the static temperature gradient is replaced by its excess over the adiabatic one and  $C_v$  is replaced by  $C_p$ . Using these approximations, Sharma [19] has studied the thermal instability in compressible fluids in the presence of rotation and a magnetic field. In another study, Sharma and Sharma [20] have considered the thermosolutal instability of a plasma including the finite Larmor radius effect.

In the stellar case, the physics is quite similar to Veronis [17] thermohaline configuration, in that helium acts like salt in raising the density and in diffusing more slowly than heat. The problem of the onset of thermal instability in the presence of a solute gradient is of great importance because of its application to atmospheric physics and astrophysics, especially in the case of the ionosphere and the outer layer of the solar atmosphere (Spiegel [21]). The finite Larmor radius and compressibility effects are likely to be important in these regions.

Keeping in mind the importance of various parameters like compressibility, finite Larmor radius; in geophysics (e.g. Earth's molten core), soil sciences, atmospheric physics, astrophysics, and various applications mentioned above, our interest, in the present paper, is to bring out the effects of compressibility and finite Larmor radius on the thermosolutal instability of a plasma.

## 2 Description of the Problem and Perturbation Equations

Consider an infinite, horizontal, compressible, viscous, and conducting plasma layer of depth  $d$ , heated and soluted from below so that the temperature, density and solute concentrations at

the bottom surface  $z = 0$  are  $T_0, \rho_0$ , and  $C_0$  and at the upper surface  $z = d$  are  $T_d, \rho_d$  and  $C_d$ , respectively, and that a uniform temperature gradient  $\beta (= |dT/dz|)$  and uniform solute gradient  $\beta' (= |dC/dz|)$  are maintained. The gravity force  $\vec{g}(0,0,-g)$  and uniform magnetic field  $\vec{H}(0,0,H)$  pervade the system.

Speigel and Veronis' [18] defined  $f$  as any of the state variables (pressure ( $p$ ), density ( $\rho$ ) or temperature ( $T$ )) and expressed these in the form

$$f(x, y, z, t) = f_m + f_0(z) + f'(x, y, z, t), \tag{1}$$

where  $f_m$  is the constant space average of  $f$ ,  $f_0$  is the variation in the absence of motion and  $f'$  is the fluctuation from motion.

The initial state is therefore a state in which the density, pressure, temperature, solute concentration and velocity at any point in the plasma are given by

$$\rho = \rho(z), p = p(z), T = T(z), C = C(z), v = 0, \tag{2}$$

respectively, where

$$\begin{aligned} T(z) &= T_0 - \beta z, & C(z) &= C_0 - \beta' z, \\ p(z) &= p_m - g \int_0^z (\rho_m + \rho_0) dz, \\ \rho(z) &= \rho_m [1 - \alpha_m(T - T_m) + \alpha'_m(C - C_m) + K_m(p - p_m)], \\ \alpha_m &= -\left(\frac{1}{\rho} \frac{\partial \rho}{\partial T}\right)_m, & \alpha'_m &= -\left(\frac{1}{\rho} \frac{\partial \rho}{\partial C}\right)_m, & K_m &= \left(\frac{1}{\rho} \frac{\partial \rho}{\partial p}\right)_m. \end{aligned} \tag{3}$$

The linearized hydromagnetic perturbation equations appropriate to the problem are

$$\begin{aligned} \frac{\partial \vec{v}}{\partial t} &= -\left(\frac{1}{\rho_m}\right) \nabla \delta p - \left(\frac{1}{\rho_m}\right) \nabla \vec{P} + \nu \nabla^2 \vec{v} \\ &+ \frac{\mu_e}{4\pi \rho_m} (\nabla \times \vec{h}) \times \vec{H} \\ &+ \vec{g} \left(\frac{\delta \rho}{\rho_m}\right), \end{aligned} \tag{4}$$

$$\nabla \cdot \vec{v} = 0, \tag{5}$$

$$\frac{\partial \theta}{\partial t} = \left(\beta - \frac{g}{C_p}\right) \omega + \kappa \nabla^2 \theta, \tag{6}$$

$$\frac{\partial \gamma}{\partial t} = \beta' w + \kappa' \nabla^2 \gamma, \tag{7}$$

$$\frac{\partial \vec{h}}{\partial t} = (\vec{H} \cdot \nabla) \vec{v} + \eta \nabla^2 \vec{h}, \tag{8}$$

$$\nabla \cdot \vec{h} = 0, \tag{9}$$

where  $\delta \rho, \delta p, \vec{v}(u, v, w), \vec{h}(h_x, h_y, h_z), \theta$  and  $\gamma$  denote, respectively, the perturbations in  $\rho$  and  $p$ , the velocity, and the perturbations in the magnetic field  $\vec{H}, T$  and  $C, \mu, \nu (= \mu/\rho_m), \mu_e, \kappa, \kappa', g/C_p, \eta$  and  $\vec{P}$  stand for viscosity, kinematic viscosity, magnetic permeability, thermal diffusivity, solute diffusivity, adiabatic gradient, resistivity and stress tensor taking into account the finite Larmor radius effects, respectively.

The equation of state

$$\rho = \rho_m [1 - \alpha(T - T_m) + \alpha'(C - C_m)], \tag{10}$$

contains the thermal coefficient of expansion  $\alpha$  and an analogous solute coefficient  $\alpha'$ . The change in density is caused mainly by the temperature and solute concentration, and the suffix  $m$  refers to values at the reference level  $z = 0$ . The change in density  $\delta \rho$ , caused by the perturbations  $\theta$  and  $\gamma$  is given by

$$\delta \rho = -\rho_m (\alpha \theta - \alpha' \gamma). \tag{11}$$

For the magnetic field along the  $z$ -axis the stress tensor  $\vec{P}$ , taking into account the finite ion gyration radius (Vandakurov [4]), has the components

$$\begin{aligned} P_{xx} &= -\rho_m \nu_0 \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right), \\ P_{xy} &= P_{yx} = \rho_m \nu_0 \left(\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}\right), \\ P_{xz} &= P_{zx} = -2\rho_m \nu_0 \left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right), \\ P_{yy} &= \rho_m \nu_0 \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right), \\ P_{yz} &= P_{zy} = 2\rho_m \nu_0 \left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right), & P_{zz} &= 0, \end{aligned} \tag{12}$$

where  $\rho_m \nu_0 = NT/4\omega_H$ ,  $\omega_H$  being the ion gyration frequency, while  $N$  and  $T$  are the number density and temperature of the ions, respectively.

### 3 The Dispersion Relation

Analyzing the disturbances into normal modes, we assume that the perturbation quantities are of the form

$$[w, h_z, \zeta, \xi, \theta, \gamma] = [W(z), K(z), Z(z), X(z), \Theta(z), \Gamma(z)] \exp(ik_x x + ik_y y + nt), \tag{13}$$

where  $k_x$  and  $k_y$  are the wave numbers in the  $x$  and  $y$  directions, respectively,  $k = (k_x^2 + k_y^2)^{1/2}$  is the resultant wave number, and  $n$  is the growth rate, which is, in general, a complex constant.

$$\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \text{ and } \xi = \frac{\partial h_y}{\partial x} - \frac{\partial h_x}{\partial y},$$

stand for the  $z$ -components of vorticity and current density, respectively.

Expressing the coordinates  $x, y, z$  in the new unit of length  $d$  and letting  $a = kd, \sigma = nd^2/\nu, p_1 = \nu/\kappa, p_2 = \nu/\eta, q = \nu/\kappa', G = C_p \beta/g$  and  $D = d/dz$ , equations (4)-(9), with the help of equations (11)-(13) in non-dimensional form, become

$$\begin{aligned} (D^2 - a^2)(D^2 - a^2 - \sigma)W & - \left(\frac{gd^2}{\nu}\right)a^2(\alpha\Theta - \alpha'\Gamma) \\ & + \frac{\mu_e Hd}{4\pi\rho_m\nu}(D^2 - a^2)DK \\ & - \left(\frac{\nu_0 d}{\nu}\right)(2D^2 + a^2)DZ \\ & = 0, \end{aligned} \tag{14}$$

$$\begin{aligned} (D^2 - a^2 - \sigma)Z & = -\frac{\mu_e Hd}{4\pi\rho_m\nu}DX \\ & - \left(\frac{\nu_0}{\nu d}\right)(2D^2 + a^2)DW, \end{aligned} \tag{15}$$

$$\begin{aligned} (D^2 - a^2 - p_1\sigma)\Theta & = -\frac{d^2}{\kappa}\left(\beta - \frac{g}{C_p}\right)W, \end{aligned} \tag{16}$$

$$(D^2 - a^2 - q\sigma)\Gamma = -\frac{\beta'd^2}{\kappa'}W, \tag{17}$$

$$(D^2 - a^2 - p_2\sigma)K = -\left(\frac{Hd}{\eta}\right)DW, \tag{18}$$

$$(D^2 - a^2 - p_2\sigma)X = -\left(\frac{Hd}{\eta}\right)DZ. \tag{19}$$

Consider the case where both boundaries are free as well as perfect conductors of both heat and solute, while the adjoining medium is electrically nonconducting. The boundary conditions for this case, using (13), are

$$W = D^2W = 0, DZ = X = \Theta = \Gamma = 0 \text{ and } \vec{h} \text{ are continuous at } z = 0 \text{ and } 1. \tag{20}$$

Eliminating  $Z, K, X, \Theta$  and  $\Gamma$  between equations (14)-(19) and substituting the proper solution  $W = W_0 \sin \pi z$ ,  $W_0$  being a constant, in the resultant equation, we obtain the dispersion relation

$$\begin{aligned} R_1 x & = \left(\frac{G}{G-1}\right) \left[ (1+x)(1+x+i\sigma_1)(1+x \right. \\ & + ip_1\sigma_1) + S_1 x \frac{(1+x+ip_1\sigma_1)}{(1+x+iq\sigma_1)} \\ & + \frac{Q_1(1+x)(1+x+ip_1\sigma_1)}{(1+x+ip_2\sigma_1)} \\ & \left. + \frac{U(2-x)^2(1+x+ip_1\sigma_1)(1+x+ip_2\sigma_1)}{(1+x+i\sigma_1)(1+x+ip_2\sigma_1)+Q_1} \right], \end{aligned} \tag{21}$$

where

$$\begin{aligned} R_1 & = \frac{g\alpha\beta d^4}{\nu\kappa\pi^4}, S_1 = \frac{g\alpha'\beta'd^4}{\nu\kappa'\pi^4}, Q_1 \\ & = \frac{\mu_e H^2 d^2}{4\pi\rho_m\nu\eta\pi^2} \text{ and } U = \frac{\nu_0^2}{\nu^2}. \end{aligned}$$

### 4 The Stationary Convection

For the stationary convection one has  $\sigma = 0$  and equation (21) reduces to

$$\begin{aligned} R_1 & = \left(\frac{G}{G-1}\right) \left[ \left(\frac{1+x}{x}\right) \{(1+x)^2 + Q_1\} \right. \\ & + \frac{U(2-x)^2(1+x)^2}{x\{(1+x)^2 + Q_1\}} \\ & \left. + S_1 \right], \end{aligned} \tag{22}$$

which expresses the modified Rayleigh number  $R_1$  as a function of the dimensionless wave number  $x$  and the parameters  $S_1, Q_1, U$  and  $G$ . For fixed values of  $Q_1, S_1$  and  $U$ , let the non-dimensional number  $G$  accounting for the

compressibility effects be also kept as fixed. Then we find that

$$\bar{R}_c = \left(\frac{G}{G-1}\right) R_c, \tag{23}$$

where  $R_c$  and  $\bar{R}_c$  denote the critical Rayleigh numbers in the absence and presence of compressibility. The effect of compressibility is, thus, to postpone the onset of thermal instability. The case  $G > 1$  is relevant here as  $G = 1$  and  $G < 1$  corresponds to infinite and negative Rayleigh numbers. Hence we obtain a stabilizing effect of compressibility. It is evident from equation (22) that

$$\frac{dR_1}{dU} = \left(\frac{G}{G-1}\right) \frac{(2-x)^2(1+x)^2}{x[(1+x)^2 + Q_1]}, \tag{24}$$

and

$$\frac{dR_1}{dS_1} = \left(\frac{G}{G-1}\right), \tag{25}$$

which are positive. The stable solute gradient and finite Larmor radius, therefore, stabilize thermosolutal instability of a plasma.

### 5 Some Important Theorems

**Theorem 1:** The system is stable for  $G < 1$ .

**Proof:** Multiplying equation (14) by  $W^*$ , the complex conjugate of  $W$ , and using equations (15)-(19) together with the boundary conditions (20), we obtain

$$\begin{aligned} & (I_1 + \sigma I_2) + \frac{g\alpha'\kappa'a^2}{v\beta'} (I_5 + q\sigma^* I_6) \\ & + d^2(I_7 + \sigma^* I_8) + \frac{\mu_e\eta d^2}{4\pi\rho_m\nu} (I_9 + p_2\sigma I_{10}) \\ & + \frac{\mu_e\eta}{4\pi\rho_m\nu} (I_{11} + p_2\sigma^* I_{12}) \\ & = \frac{C_p\alpha\kappa a^2}{v(G-1)} (I_3 \\ & + p_1\sigma^* I_4), \end{aligned} \tag{26}$$

where

$$\begin{aligned} I_1 &= \int_0^1 (|D^2W|^2 + 2a^2|DW|^2 + a^4|W|^2) dz, \\ I_2 &= \int_0^1 (|DW|^2 + a^2|W|^2) dz, \\ I_3 &= \int_0^1 (|D\Theta|^2 + a^2|\Theta|^2) dz, \quad I_4 = \int_0^1 |\Theta|^2 dz, \\ I_5 &= \int_0^1 (|D\Gamma|^2 + a^2|\Gamma|^2) dz, \quad I_6 = \int_0^1 |\Gamma|^2 dz, \\ I_7 &= \int_0^1 (|DZ|^2 + a^2|Z|^2) dz, \quad I_8 = \int_0^1 |Z|^2 dz, \\ I_9 &= \int_0^1 (|DX|^2 + a^2|X|^2) dz, \quad I_{10} = \int_0^1 |X|^2 dz, \\ I_{11} &= \int_0^1 (|D^2K|^2 + 2a^2|DK|^2 + a^4|K|^2) dz, \\ I_{12} &= \int_0^1 (|DK|^2 + a^2|K|^2) dz. \end{aligned} \tag{27}$$

The integrals  $I_1 - I_{12}$  are all positive definite. Putting  $\sigma = \sigma_r + i\sigma_i$  and equating the real and imaginary parts of equation (26), we obtain

$$\begin{aligned} & \left[ I_2 + \frac{g\alpha'\kappa'a^2}{v\beta'} qI_6 + \frac{\mu_e\eta}{4\pi\rho_m\nu} p_2I_{12} \right. \\ & \quad + \frac{\mu_e\eta d^2}{4\pi\rho_m\nu} p_2I_{10} + d^2I_8 \\ & \quad \left. + \frac{C_p\alpha\kappa a^2}{v(1-G)} p_1I_4 \right] \sigma_r \\ & = - \left[ I_1 + \frac{g\alpha'\kappa'a^2}{v\beta'} I_5 \right. \\ & \quad + \frac{\mu_e\eta}{4\pi\rho_m\nu} I_{11} + \frac{\mu_e\eta d^2}{4\pi\rho_m\nu} I_9 \\ & \quad \left. + d^2I_7 + \frac{C_p\alpha\kappa a^2}{v(1-G)} I_3 \right], \end{aligned} \tag{28}$$

and

$$\left[ I_2 - \frac{g\alpha'\kappa'a^2}{\nu\beta'} qI_6 - \frac{\mu_e\eta}{4\pi\rho_m\nu} p_2I_{12} + \frac{\mu_e\eta d^2}{4\pi\rho_m\nu} p_2I_{10} - d^2I_8 + \frac{C_p\alpha\kappa a^2}{\nu(G-1)} p_1I_4 \right] \sigma_i = 0. \quad (29)$$

It is evident from equation (28) that  $\sigma_r$  is negative if  $G < 1$ . The system is therefore stable for  $G < 1$ .

**Theorem 2:** The modes may be oscillatory or non-oscillatory in contrast to the case of no magnetic field and in the absence of finite Larmor radius and stable solute gradient where modes are non-oscillatory, for  $G > 1$ .

**Proof:** It is clear from equation (29) that, for  $G > 1$ ,  $\sigma_i$  may be zero or nonzero, meaning that the modes may be non-oscillatory or oscillatory. The oscillatory modes are introduced due to the presence of a magnetic field, a finite Larmor radius and a solute gradient.

In the absence of a magnetic field and solute gradient, equation (29) gives

$$\left[ I_2 + \frac{C_p\alpha\kappa a^2}{\nu(G-1)} p_1I_4 \right] \sigma_i = 0, \quad (30)$$

and the terms in brackets are positive when  $G > 1$ . Thus  $\sigma_i = 0$ , which means that oscillatory modes are not allowed and the principle of exchange of stabilities is satisfied, but in the presence of solute gradient, magnetic field and finite Larmor radius effects, the oscillatory modes come into play.

**Theorem 3:** The system is stable for  $\frac{1}{G-1} \frac{C_p\alpha\kappa}{\nu} \leq \frac{27\pi^4}{4}$  and under the condition  $\frac{1}{G-1} \frac{C_p\alpha\kappa}{\nu} > \frac{27\pi^4}{4}$ , the system becomes unstable.

**Proof:** From equation (29) it is clear that  $\sigma_i$  is zero when the quantity multiplying it is not zero and arbitrary when this quantity is zero.

If  $\sigma_i \neq 0$ , equation (28) upon utilizing (29) and the Rayleigh-Ritz inequality gives

$$\left[ \frac{27\pi^4}{4} - \frac{1}{G-1} \frac{C_p\alpha\kappa}{\nu} \right] \int_0^1 |W|^2 dz + \frac{\pi^2 + a^2}{a^2} \left\{ \frac{\mu_e\eta}{4\pi\rho_m\nu} I_{11} + \frac{\mu_e\eta d^2}{2\pi\rho_m\nu} p_2I_{10}\sigma_r + \frac{\mu_e\eta d^2}{4\pi\rho_m\nu} I_9 + d^2I_7 + \frac{g\alpha'\kappa'a^2}{\nu\beta'} I_5 \right\} \leq 0, \quad (31)$$

since the minimum value of  $\frac{(\pi^2+a^2)^3}{a^2}$  with respect to  $a^2$  is  $\frac{27\pi^4}{4}$ .

Now, let  $\sigma_r \geq 0$ , we necessarily have from inequality (31) that

$$\frac{1}{G-1} \frac{C_p\alpha\kappa}{\nu} > \frac{27\pi^4}{4}. \quad (32)$$

Hence, if

$$\frac{1}{G-1} \frac{C_p\alpha\kappa}{\nu} \leq \frac{27\pi^4}{4}, \quad (33)$$

then  $\sigma_r < 0$ . Therefore, the system is stable.

Thus, under the condition (33), the system is stable and under condition (32) the system becomes unstable.

**Theorem 4:**  $\kappa < \eta, \kappa < \kappa'$  and  $\kappa < \nu$  are sufficient conditions for the non-existence of overstability.

**Proof:** For overstability, we wish to determine the critical Rayleigh number for the onset of instability via a state of pure oscillations, it is suffice to find conditions for which equation (21) will admit of solutions with  $\sigma_1$  real. Equating the real and imaginary parts of equation (21) and eliminating  $R_1$  between them, we obtain

$$A_4c^4 + A_3c^3 + A_2c^2 + A_1c + A_0 = 0, \quad (34)$$

where we have written  $c = \sigma_1^2, b = 1 + x$  and

$$A_4 = p_2^4 q^2 (1 + p_1) b, \quad (35)$$

$$A_0 = b^2 (b^2 + Q_1)^2 [(1 + p_1) b^3 + S_1 (p_1 - q) (b - 1) + Q_1 (p_1 - p_2) b] + U b^4 (3 - b)^2 [(p_1 - 1) b^2 + Q_1 (p_1 + p_2)]. \quad (36)$$

The four values of  $c, \sigma_1$  being real, are positive. The product of the roots is  $A_0/A_4$ , which is

positive if  $A_0 > 0$  (since from equation (35),  $A_4 > 0$ ). It is clear from equation (36) that  $A_0$  is always positive if

$$p_1 > p_2, p_1 > q, p_1 > 1. \quad (37)$$

This means that

$$\kappa < \eta, \quad \kappa < \kappa', \quad \text{and} \quad \kappa < \nu. \quad (38)$$

Thus  $\kappa < \eta$ ,  $\kappa < \kappa'$  and  $\kappa < \nu$  are the sufficient conditions for the non-existence of overstability, the violation of which does not necessarily imply the occurrence of overstability.

## 6 Conclusions

Formation of stars in the interstellar medium is one of the most fascinating and important process in plasma astrophysics. The birth of stars is a vast field of research in modern astrophysics and cosmology. The thermosolutal instability of a compressible plasma due to the effects of the ion Larmor radius is considered in the present paper. The investigation is motivated by its interesting complexities as a double diffusion phenomena as well as its direct relevance to astrophysics and geophysics. Thermosolutal convection problems arise in oceanography, limnology and engineering. Ponds built to trap solar heat and some Antarctic lakes provide examples of particular interest. The main conclusions from the analysis of this paper are as follows:

- For the case of stationary convection, the stable solute gradient and finite Larmor radius are found to have stabilizing effects on the system.
- The system is found to be stable for  $(C_p/g)\beta < 1$ .
- The finite Larmor radius and the compressibility introduce oscillatory modes in the system for  $(C_p/g)\beta > 1$ .
- It is observed that the system is stable for  $\frac{1}{G-1} \frac{C_p \alpha \kappa}{\nu} \leq \frac{27\pi^4}{4}$  and under the condition  $\frac{1}{G-1} \frac{C_p \alpha \kappa}{\nu} > \frac{27\pi^4}{4}$ , the system becomes unstable.
- The case of overstability is also considered. The conditions  $\kappa < \eta$ ,  $\kappa < \kappa'$  and  $\kappa < \nu$  are the sufficient

conditions for the non-existence of overstability, the violation of which does not necessarily imply the occurrence of overstability.

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