Limits of Digital Holographic Interferometry used for Measurement of Temperature Fields

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Abstract: - Holographic interferometry (HI) is an optical measurement method that combines the principles of holography and classical interferometry. Compared to classical interferometry, two significant benefits are achieved. The first advantage is the ability to measure diffuse reflecting surfaces of objects, but this is not a matter of phase measurement. The second advantage is the fact that this is a differential technique. In principle, holographic interferometry, similar to classical interferometry, measures the change in the optical path of two waves. However, in classical interferometry, both optical paths of the interferometer must be optically equivalent, which means minimal difference in optical paths at each point. The resulting interferometric pattern is a combination of the measured deviation and variance introduced by the interferometer. Both deviations cannot be distinguished from one another. For this reason, the interferometer must be composed of highly accurate optical components and adjusted with high precision. In the case of HI, an initial reference state is recorded at first, than it is compared to the changed (in the optical point of view) state of the object. The optical inequality of both paths does not play a role and does not affect the measurement result. Digital holographic interferometry, compared with HI, uses numerical reconstruction of a hologram which is recorded on a digital camera.

The presented paper shows an overview of different modes and configurations of digital holographic interferometry (DHI) for precise measurement of temperature fields in fluids. It shows basic equations used for evaluation of achieved results and for the reconstruction of temperature fields. The paper shows basic configuration of interferometers (Mach-Zehnder and Twyman-Green) used for DHI. Paper also brings analysis of physical limits of the experimental method.

Key-Words: - Temperature measurement, digital holographic interferometry, limits

1 Introduction

Heat transfer is a complex issue that interferes with many scientific and industrial sectors. The problem can be encountered in a number of applications starting with the design and optimization of engineering systems such as heat exchangers, turbines, electronics cooling, hot water pipelines, food industry and ending with the development of new technologies such as sustainable energy, biotechnology, information technology, nanotechnology, etc.

It is very important to be able to accurately measure the related variable, such as temperature, to be able to fully understand and describe the studied phenomena and processes. The choice of the appropriate experimental method also depends on the measured magnitude and the investigated phenomenon.

Experimental methods used for the temperature distribution measurement can be divided into two main groups - contact and non-contact. Contact methods use a sensor located in the area of the experiment. The sensor is most often composed of thermocouples or various elements monitoring thermal expansion [1], capacity or a change of electrical resistance (for example CCA [2]). In most cases these are one-point measurement methods. The presence of the sensor placed inside the measured medium inevitably affects the measurement results, but also the phenomenon itself. The sensor must be able to withstand the temperatures or pressures within the measured area. Therefore, depending on the experiment, it is preferable to use non-contact, mainly optical methods. The use of optical measurement techniques has a very long history in the field of heat transfer. Although many techniques, as Schliere
method, methods of interferometry, infrared imaging, have been developed and described decades ago.

Optical methods have many advantages compared to classical, contact, methods. The most important and common feature of all optical methods is their non-invasiveness. The process being investigated is not, or very little, affected by the probe, which allows analyzing of very small changes and rapid processes. Optical methods used to measure temperature distributions include Schlieren method [1], absorption spectroscopy [3], Rayleigh and Raman scattering [4], methods based on thermal radiation [5], etc. An entirely unique method in terms of application width and overall versatility is holographic interferometry (HI), which is based on the principles of holography [6] and combines them with classical interferometry. While the above-mentioned methods are for example visual only, single-point or require presence of additive particles in the measured area, HI offers the possibility of wide-area measurements with a differential character with high spatial and temporal resolution. In addition, it is possible to visualize the process in the measured area in real time.

The high variability of the method allows its use in various areas from trivial applications to microscopy or supersonic holographic microscopy. Contactless character of this method allows measurements through the viewports in extreme environments (chambers with high pressure, temperature, vacuum, etc.). HI can be modified to measure different quantities not only as 2D but also due to tomographic reconstruction as well as 3D. Additionally, the continuing trend of rapid development in the field of optical detectors, semiconductor and computing, laser and fiber optics enables HI to grow and expand.

2 Digital holographic interferometry

Digital holographic interferometry (DHI) is non-contact, non-invasive and highly accurate experimental method for measuring quantities that affect the phase of passing or reflected light waves. For example, it can be used in mechanical strain and stress analysis, vibration analysis and measurement of refractive index distribution in transparent environment. This is also a case of measuring the temperature field distribution in fluids.

DHI records the intensity of the interference field of the object $U_o$ and the reference $U_r$ wave (digital hologram) from which it is possible to reconstruct the equiphase of recorded object waves. Fig. 1 shows the schematic arrangement for recording the hologram.

![Figure 1: Schematic arrangement for DHI measurement](image)

Within the experiment, at least two holograms $h_i$ (where $i = 1$ or $2$) must be recorded for comparison. Holograms correspond to the initial and final states of the object. Holograms are recorded on a digital sensor (e.g. a CCD camera), from where complex wave fields $U_i$ are reconstructed [7]:

$$U_i(x, y) = F^{-1} \left\{ h_i(\xi, \eta) r^* (\xi, \eta) e^{2\pi i [r^2 + y^2]} \right\}, \quad (1)$$

where $j = \sqrt{-1}$, $\lambda$ is the wavelength of the used light, $d$ is the distance of the object from the CCD camera, $r^*$ is the complex amplitude of the reference wave and the operator $F$ refers to the Fourier transformation. Coordinates in the hologram plane are marked $\xi, \eta$ and coordinates in the image plane are marked $x, y$.

It is necessary to calculate intensity $I_i(x, y)$ and phase $\varphi_i(x, y)$ of the reconstructed wave as:

$$I_i(x, y) = |U_i(x, y)|^2; \quad (2)$$
$$\varphi_i(x, y) = \arctg \left( \frac{\text{Im} [U_i(x, y)]}{\text{Re} [U_i(x, y)]} \right). \quad (3)$$

Value of interference phase modulo $2\pi$ (from equation (3)), which is affected by the change in temperature of the working fluid can be determined as (see [7]):

$$\Delta \varphi = \begin{cases} 
\varphi_1 - \varphi_2 & \text{if } \varphi_1 \geq \varphi_2 \\
\varphi_1 - \varphi_2 + 2\pi & \text{if } \varphi_1 < \varphi_2
\end{cases}. \quad (4)$$
The relationship between the interference phase and the change of the refractive index of the light is given by:

$$\Delta \varphi(x, y) = \frac{2\pi}{\lambda} \int_{0}^{L} \Delta n(x, y, z) dz,$$  \hspace{1cm} (5)

where $\Delta n(x, y, z) = n(x, y, z) - n_0$, where $n_0$ is the ambient light refractive index in measured fluid, $L = L_2 - L_1$ is the light beam length in measured environment.

### 3 Two- and three-dimensional temperature field measurement

Equation (5) defines the relationship between the wave transition phase and the refractive index of the environment. The solution of the integral (5) depends on the refractive index distribution, e.g. the temperature field in our case. There are three basic options how to evaluate $\Delta \varphi$. The simplest is a two-dimensional (2D) temperature field (e.g., temperature field in 2D boundary layer) when a constant temperature is assumed in the direction of the optical axis. Integration of equation (5) will be simplified to:

$$\Delta \varphi(x, y) = \frac{2\pi}{\lambda} \Delta n(x, y, z)L.$$  \hspace{1cm} (6)

Another type of temperature field is rotationally symmetrical array as e.g. jet flame or circular jet. The solution (5) leads to inverse Abel transformation. To measure general (unsymmetrical) temperature fields, a tomographic approach based on multi-directional measurement of several different projections is required.

The relationship between the refractive index and the gas temperature can be derived using the Gladstone-Dale equation:

$$n - 1 = K\rho,$$  \hspace{1cm} (7)

where $K$ is the Gladstone-Dale constant and $\rho$ is density of the measured fluid, and the state equation for the ideal gas at temperature $T$ and pressure $P$:

$$\rho = \frac{Mp}{RT},$$  \hspace{1cm} (8)

where $R = 8.314,3$ J·mol$^{-1}$ K$^{-1}$ is a universal gas constant.

By combining equations (7) and (8), a relation between the refractive index and the temperature is obtained as:

$$n - 1 = \frac{KMp}{RT}.$$  \hspace{1cm} (9)

If we consider air at normal pressure, room temperature, and wavelengths 532 nm and only minor changes in the phase, the sensitivity of the arrangement to the change of the refractive index is $\Delta n / \Delta T \sim 10^{-6}$.

The temperature field distribution can be measured with accuracy 10$^{-4}$. It means DHI method is among the most accurate method for measuring of temperatures.

Fig. 2 shows individual steps of processing a digital hologram on a temperature field. Two digital holograms (one in the initial and second in the final state after a change of temperature) are reconstructed using eq. (1) and the interference phase is calculated by eq. (4). Achieved result corresponds to the change of the object wave phase between the initial and final states of the object. This interference phase is then demodulated by adding of multiples of $2\pi$. In the considered case it is a 2D temperature field and therefore it is possible to apply eq. (6) to calculate the refractive index distribution. It is then recalculated to the temperature with using of eq. (9).

![Figure 2: Explanation of the individual steps for DHI temperature field measurement, [8]](image-url)
4 Types of interferometers used for DHI

Interferometers are devices for very accurate measurements using light interference. Two types of interferometers: Mach-Zehnder (MZ) and Twyman-Green (TG) are most often used for DHI experiments in the transmission samples.

Mach-Zehnder interferometer (Fig. 3) is used to determine the relative changes of the phase shift between the two collimated beams (1 and 2) resulting from the distribution of light by means of a polarizing unit BS1 from a single light source (laser). Beam 1 passes through the measured object and through the mirror M2, then is pointed to a recording camera. Beam 2 is the referential beam, which is also pointed to the camera. PC is used to process achieved holograms and to calculate the difference as mentioned above, it means to compare the phase shift between the reference object beam and the measured object beam.

The MZ interferometer is commonly used in aerodynamics, plasma physics and heat transfer for measurement of pressure, sensitivity and temperature changes in gases, [7]. As the phase change is relatively small, the interferometer sensitivity may not be sufficient for the measurement of the temperature field. That is the main reason why more sensitive, a Twyman-Green type interferometer, is used.

Twyman-Green interferometer is a Michelson variant of the interferometer, where the wave passes the measured area twice, thereby increasing the sensitivity of the measurement. The TG interferometer principle is explained in Fig. 4. The laser beam is expanded and collimated with a suitable optic. After the wave is split, the object wave passes through the measured area before and after the reflection from the mirror. The interference field is recorded by the imaging system on the detector after the recombination of the reference and object wave [7]. The double sensitivity is demonstrated in Fig. 5b, where a double phase change can be observed.

Figure 3: Schematic view of the Mach-Zehnder interferometer principle. BS1 - beam splitter, SF - space filter, CO - collimating lens, BS2 - non-polarizing splitter, O-focusing lens, M-mirror

Figure 4: Schematic view of the Twyman-Green interferometer principle. BS1 - beam splitter, SF - space filter, CO - collimating lens, BS2 - non-polarizing splitter, O-focusing lens, M-mirror
the mirror, passes through the measured object again and is pointed to the camera. During the PC processing, the object wave is numerically compared to the reference one. In the second case the wave is divided already in the splitter BS1, the reference wave is reflected from the mirror M2 and is pointed to the camera. The object wave passes through the measured object and is also pointed to the camera after being reflected from the M3 mirror and re-passing the measured object.

**5 Limits of the DHI method**

DHI is a very sensitive method for measuring the difference in the optical paths that are recorded in measured phase $\Delta \varphi$. The change of optical path is caused by the change of the refractive index $n$ according to equation (5) when measuring the temperature fields. With small temperature changes, the relationship between the temperature change and the refractive index can be approximated by the relationship $\frac{dn}{dT} = -1 \times 10^{-6}$. From this relationship and eq. (5), the uncertainty of the temperature field measurement can be estimated according to the uncertainties of the individual parameters.

In the case of asymmetric fields, total uncertainty of measurement is affected by three basic sources of error due to:
- Experiment optoelectronic properties (stability of the laser wave length, camera noise, coherent noise, etc.);
- Reconstruction of 3D temperature fields (limited number of projections, development of the environment, etc.)
- Refraction resulting from a refractive index change.

For the quantitative estimation of uncertainty given by the interferometer properties, 2D temperature fields can be considered:

$$\Delta T = -\frac{\lambda}{2\pi L} \Delta \varphi \times 10^6,$$

which can be derived from equation (6).

Uncertainty of wavelength $\varepsilon_\lambda$ and uncertainty in the phase $\varepsilon_{\Delta \varphi}$ (including the uncertainties arising in the optical and electronic part of the experiment) contribute to overall temperature field uncertainty as:

$$\varepsilon_{\Delta T} = \frac{1 \times 10^6}{2\pi L} \sqrt{(\Delta \varphi \varepsilon_\lambda)^2 + (\lambda \varepsilon_{\Delta \varphi})^2},$$

which was calculated as the total differential. The maximum uncertainty can be estimated as $\varepsilon_\lambda = 0.01$ nm for the used laser sources. Usual phase field uncertainty is $\varepsilon_{\Delta \varphi} = \pi/5$.

Figure 5: Reconstructed candle flame interference phase measured on (a) Mach-Zehnder interferometer, (b) Twyman-Green interferometer with double sensitivity

Figure 6: (a) Initial temperature field, (b) Reconstructed temperature field in 5° increments, (c) Map of deviation between (a) and (b), RMS map value is 0.2 °C, [9]
Considering $L = 2 \times 30$ mm (Twyman-Green interferometer) and measured phase change $\Delta \varphi = 10 \pi$, the uncertainty in determining the temperature change correspond to $\Delta T = 0.3 \degree C$.

Reconstruction of 3D temperature fields is based on the inverse Radon transformation. Its accuracy depends mainly on the range of projection angles and their total number. This effect can be estimated by comparing the known field with the results of this field after the reconstruction process. A known temperature field (Fig. 6a) was transformed into a sinogram using direct Radon transformation with projections of $(0^\circ \div 179^\circ)$ with step $1^\circ$. The inverse Radon transformation was also performed with projections from $0^\circ$ to $179^\circ$ but with reduced number of projections. The reconstructed temperature field is shown in Fig. 6b. The difference between the phase fields (a) and (b) is visible in Fig. 6c. RMS differential map corresponds to $0.2 \degree C$. The dependence of the RMS variation of the temperature fields on the number of projections is plotted in Fig. 7.

Temperature change causes the refractive index change within the measured area. It consequently leads to deflection of the waves passing through the measured area. Equation (5) is considering a change in the optical wave path dependence on refractive index change. Changing the optical path caused by the divergence of the wave from the original trajectory brings uncertainty to the measurement. This phenomenon is negligible for lightly refractive fields, as is the case of experiments devoting the measurement of temperature in the air. In this case the approximation $\frac{dn}{dt} = -1 \times 10^{-4}$ could be applied. The uncertainty caused by the deflection of the wave is at least one order smaller than the uncertainty described above. On the other hand, for highly refractive fields (e.g. in water, when $\frac{dn}{dt} = -1 \times 10^{-4}$) it can play a major role.

**Abbreviation:**

CCA constant current anemometry  
DHI digital holographic interferometry  
HI holographic interferometry  
M-Z Mach-Zehnder  
T-G Twyman-Green

**References:**


**Acknowledgement:**

We gratefully acknowledge the support of the Grant Agency of the Czech Republic (Project No. 16-16596S).