Analysis of Ratio of One and Product of Two Rayleigh Random Variables and Its Application in Telecommunications

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Abstract: - In this paper, the ratio of product of two Rayleigh random variables and Rayleigh random variable (RV) is considered. The level crossing rate (LCR) of ratio of product of two Rayleigh random variables and Rayleigh random variable is evaluated. Using this result, the average fade duration (AFD) of wireless relay communication system with two sections operating over multipath fading channel in the presence of co-channel interference can be calculated. Then, the LCR of the ratio of Rayleigh RV and product of two Rayleigh RVs is determined. The formula is obtained in the closed form. The influence of Rayleigh fading average power on the LCR is analyzed. By using this formula, the AFD of wireless relay system with two sections working over Rayleigh multipath fading channel in the presence of two Rayleigh co-channel interferences could be evaluated.

Key-Words: - Level crossing rate (LCR); Ratio; Product; Rayleigh random variable; Fading; Co-channel interference

1 Introduction

Short term fading is caused by propagation of electromagnetic signals over more paths. Signal propagates by more paths because reflections, refractions and scattering of waves [1]. Refractions and reflections of electromagnetic waves cause variations of envelopes resulting in degradation of system performance [2]. These variations of signal envelope can be described by different distributions such as: Rayleigh, Nakagami-*m*, Weibull, Rician, κ-μ, η-μ, α-μ and other distributions. The Rayleigh distribution is frequently used to model multipath fading with no direct line-of-sight (LOS) path.

The first order performances of wireless communication system are: the outage probability (OP), system capacity and bit error probability (BEP) [3]. The second order performance provides a dynamic representation of a fading channel and has great significance in the analysis and design of the wireless communication systems. The second order performances of wireless communication system are: the level crossing rate (LCR) and the average fade duration (AFD) of random process. The LCR of random process is the number of crossing of random process on specified level. The AFD can be evaluated as the ratio of outage probability and LCR [1].

There is a lot of works in literature that handle the topics of ratios and products of random processes, because they have usage in wireless communication systems under fading and interference influence [4]-[9]. First, the probability density function (PDF) and cumulative distribution function (CDF) for different scenarios are derived, and after their results are used to evaluate the OP, capacity, and average BEP of digital communication systems over cascaded fading channels [5]-[7]. Also, some works cover the area of second order performances of wireless communication radio system such as LCR and AFD [10], [11]. The papers

[12]-[17] deal with ratios of variable and product of two variables.

In last decade, multiple Rayleigh or cascaded Rayleigh or n-Rayleigh distribution is introduced to describe the signal amplitude in wireless propagation. Physically, the n-Rayleigh model considers a cascade of n statistically independent Rayleigh fading processes, arise e.g. at diffraction. In [4], the exact PDF and CDF of a multiple Rayleigh, or n-Rayleigh, random variable (RV) are derived by using an inverse Mellin transform technique and given in terms of the Meijer G-function.

Approximate terms for PDFs and CDFs for the products of independent generalized Gamma, generalized Nakagami-m, and generalized Gaussian RVs have been derived in [5] based on new approximations with satisfactory accuracies. Using these formulas, closed-form expressions for the OP in the wireless cascaded channel is obtained.

Next distribution, signed as N∗Nakagami, constructed as the product of N statistically independent and not necessarily identically distributed Nakagami RVs is analyzed in [6]. This is used for performance considerations of digital communication systems employing different modulation schemes working over N∗Nakagami fading channel because proposed N∗Nakagami distribution model is realistic for wireless fading channels.

In [7], the distribution of the product of Rayleigh distributed random variables (RVs) is derived by the Mellin-Barnes inversion formula. An upper bound for the product distribution is obtained. The accuracy of this approximation increases as the number of RVs in the product increase.

The authors are derived in [8] the probability density function (PDF) of the product of Rayleigh, exponentially, Nakagami-m and Gamma distributed variables in a closed form by using the Mellin transform. It is known that Nakagami-m and Gamma are generic distributions that match a wide range of wireless communication channel fading conditions. Showed method can easy be use to study random variables for modeling of fading in cascaded channels for example the so-called N∗Nakagami channels within the area of cooperative communication.

The wireless cascaded channel, used in wireless multihop transmission, is an efficient technology for extending the coverage with respect to the channel path-loss and increasing the capacity of wireless communications especially in multipath fading channels [3]. Beside, the wireless cascaded channel appears when the received signals are generated by the product of a large number of rays reflected via *N* statistically independent scatters. For example, when nomadic transmitter communicates with another nomadic receiver in Rayleigh fading channels, the so-called double Rayleigh fading, i.e., multiplication of two Rayleigh fading is used. Furthermore, the double-Rayleigh fading channels received more interest, because they were a keyhole channel model for multiple-input multiple-output (MIMO) communications. For all these reasons, in [9], the authors analyzed the fading performance of a generic fading distribution, called the N-product Generalized Nakagami-m (*N**GNM) distribution, obtained as a product of the power of *N* statistically independent and non-identically distributed GNM RVs, for modelling the cascaded fading channels.

LCR of the product of two Nakagami-*m* random variables is evaluated in [10]. This result is possible to be used for evaluation AFD of wireless communication system working over Nakagami-*m**Nakagami-*m* short term fading channel.

In [11], the authors observed a cascaded channel modelled as the product of *N* RVs. By using the multivariate Laplace approximation theorem they derived analytical expressions for the average LCR and the AFD of the product of *N* Rayleigh fading envelopes. Such channel is called *N*∗Rayleigh channel. Thereafter, these results are used for determination the performance of an amplify-and-forward multihop Rayleigh fading channel. Likewise, they are useful in studying of the second order statistics of the cooperative diversity systems.

The authors in [12] give closed-form expressions for the PDF and CDF of the ratios of RV and product of two RVs for Rayleigh, Weibull, Nakagami-m and α-μ distributions. Results are applicable in performance analysis of multi-hop wireless communication systems in different transmission conditions.

The product of two Rician RVs, the ratio of two Rician RVs, the ratio of Rician RV and product of two Rician RVs, and the ratio of product of two Rician RVs and Rician RV are considered in [13]. The expressions for the LCR of all these quantities are evaluated. Those results are usable for calculation the AFD of wireless communication system operating over Rician multipath fading channels.

Further, in [14]-[16], the ratios of product of two random variables and random variable are analyzed. In [14], α-k-μ RV is presented. Obtained expression for LCR of the ratio of product of two α-k-μ RVs and α-k-μ RV can be used for calculation the AFD of wireless system operating over composite α-k-μ multipath fading channel in the presence of co-channel interference in the presence of α-k-μ multipath fading.

The described situation in [15] is an interference limited k-μ multipath fading line-of-sight domain. The ratio of product of two k-μ RVs and k-μ RV represent signal-to-interference (SIR) envelope ratio. Then, the formula for AFD of wireless system in composite multipath k-μ fading environment, in the presence of co-channel interference disturbed by multipath k-μ fading can be derived.

LCR of ratio of product of two k-μ RVs and Nakagami-m RV is processed in [16]. Performance of such ratio are applied in analysis of wireless relay communication system with two sections working in k-μ multipath fading environment in the presence of co-channel interference which suffers Nakagami-*m* fading.

The novel exact closed-form expressions for the PDF and CDF of the product and ratio of products of an arbitrary number of independent non-identically distributed (i.n.i.d) extended generalized-K (EGK) variables are delineated in [17]. Amount of fading is derived as useful system performance.

We think that ratios of product of two RVs and RV are not often presented in literature, and we will start filling the available literature in the field, first with evaluations of the ratio of product of two Rayleigh RVs and Rayleigh RV, and the ratio of Rayleigh RV and product of two Rayleigh RVs. The level crossing rate will be shown for different values of fading amplitudes and average powers.

2 Level Crossing Rate of Ratio of Product of Two Rayleigh Random Variables and Rayleigh Random Variable

Random variables y_1, y_2 and y_3 have Rayleigh distribution:

$$p_{y_1}(y_1) = \frac{2y_1}{\Omega_1} e^{-\frac{y_1^2}{\Omega_1}}, \quad y_1 \geq 0 \tag{1}$$

$$p_{y_2}(y_2) = \frac{2y_2}{\Omega_2} e^{-\frac{y_2^2}{\Omega_2}}, \quad y_2 \geq 0 \tag{2}$$

$$p_{y_3}(y_3) = \frac{2y_3}{\Omega_3} e^{-\frac{y_3^2}{\Omega_3}}, \quad y_3 \geq 0 \tag{3}$$

where Ω_1 is power of x_1, Ω_2 power of x_2, and Ω_3 is power of x_3.

Ratio of product of two Rayleigh random variables y_1 and y_2 and Rayleigh random variable y_3 is y:

$$y = \frac{y_1 \cdot y_2}{y_3}, \quad y_1 = \frac{y \cdot y_3}{y_2} \tag{4}$$

The first derivative of y is:

$$\dot{y} = \frac{y_2}{y_3}\dot{y}_1 + \frac{y_1}{y_3}\dot{y}_2 - \frac{y_1 y_2}{y_3^2}\dot{y}_3 = 0 \tag{5}$$

since

$$\dot{y}_1 = \dot{y}_2 = \dot{y}_3 = 0\,. \tag{6}$$

The variance of $\dot{y}$, $\sigma_{\dot{y}}$, is defined by:

$$\sigma_{\dot{y}}^2 = \frac{y_2^2}{y_3^2}\sigma_{\dot{y}_1}^2 + \frac{y_1^2}{y_3^2}\sigma_{\dot{y}_2}^2 + \frac{y_1^2 y_2^2}{y_3^4}\sigma_{\dot{y}_3}^2 \tag{7}$$

where is valid:

$$\sigma_{\dot{y}_1}^2 = \pi^2 f_m^2 \Omega_1$$
$$\sigma_{\dot{y}_2}^2 = \pi^2 f_m^2 \Omega_2 \tag{8}$$
$$\sigma_{\dot{y}_3}^2 = \pi^2 f_m^2 \Omega_3\,.$$

Here, f_m is maximal Doppler frequency.

After substituting, the expression for variance from (7) becomes:

$$\sigma_{\dot{y}}^2 = \pi^2 f_m^2 \left(\frac{y_2^2}{y_3^2}\Omega_1 + \frac{y_1^2}{y_3^2}\Omega_2 + \frac{y_1^2 y_2^2}{y_3^4}\Omega_3 \right) =$$
$$= \pi^2 f_m^2 \left(\frac{y_2^2}{y_3^2}\Omega_1 + \frac{y^2}{y_2^2}\Omega_2 + \frac{y^2}{y_3^2}\Omega_3 \right) =$$
$$= \pi^2 f_m^2 \frac{y_2^2}{y_3^2}\Omega_1 \left(1 + \frac{y^2}{y_2^2}\frac{y_3^2}{y_2^2}\frac{\Omega_2}{\Omega_1} + \frac{y^2}{y_3^2}\frac{y_3^2}{y_2^2}\frac{\Omega_3}{\Omega_1} \right) =$$
$$= \pi^2 f_m^2 \frac{y_2^2}{y_3^2}\Omega_1 \left(1 + \frac{y^2 y_3^2}{y_2^4}\frac{\Omega_2}{\Omega_1} + \frac{y^2}{y_2^2}\frac{\Omega_3}{\Omega_1} \right) \tag{9}$$

The first derivative of Rayleigh random variable is Gaussian random variable.

The joint probability density function of y_1 and $\dot{y}_1$ is:

$$p_{y_1\dot{y}_1}(y_1\dot{y}_1) = \frac{2y_1}{\Omega_1} e^{-\frac{y_1^2}{\Omega_1}} \frac{1}{\sqrt{2\pi}\beta_1} e^{-\frac{\dot{y}_1^2}{2\beta_1^2}}, \tag{10}$$

the joint probability density function of y_2 and $\dot{y}_2$ is:

$$p_{y_2\dot{y}_2}(y_2\dot{y}_2) = \frac{2y_2}{\Omega_2} e^{-\frac{y_2^2}{\Omega_2}} \frac{1}{\sqrt{2\pi}\beta_2} e^{-\frac{\dot{y}_2^2}{2\beta_2^2}}, \tag{11}$$

and the joint probability density function of y_3 and $\dot{y}_3$ is:

$$p_{y_3\dot{y}_3}\left(y_3\dot{y}_3\right)=\frac{2y_3}{\Omega_3}e^{-\frac{y_3^2}{\Omega_3}}\frac{1}{\sqrt{2\pi}\beta_3}e^{-\frac{\dot{y}_3^2}{2\beta_3^2}}, \tag{12}$$

where β_i , i =1, 2, 3 are variances of Gaussian distributions.

The joint probability density function of y_1, $\dot{y}$, y_2 and y_3 is:

$$p_{y\dot{y}y_2y_3}\left(y\dot{y}y_2y_3\right)=p_{\dot{y}}\left(\dot{y}/yy_2y_3\right)p_{yy_2y_3}\left(yy_2y_3\right)=$$
$$=p_{\dot{y}}\left(\dot{y}/yy_2y_3\right)p_{y_2}\left(y_2\right)p_{y_3}\left(y_3\right)p_y\left(y/y_2y_3\right) \tag{13}$$

where:

$$p_y\left(y/y_2y_3\right)=\left|\frac{dy_1}{dy}\right|p_{y_1}\left(y\frac{y_3}{y_2}\right) \tag{14}$$

$$\frac{dy_1}{dy}=\frac{y_3}{y_2} \tag{15}$$

$$p_{y\dot{y}}\left(y\dot{y}\right)=\int_0^\infty dy_2\int_0^\infty dy_3 p_{\dot{y}}\left(\dot{y}/yy_2y_3\right)p_{y_2}\left(y_2\right)p_{y_3}\left(y_3\right)\frac{y_3}{y_2}p_{y_1}\left(y\frac{y_3}{y_2}\right) \tag{16}$$

Level crossing rate of the ratio of product of two Rayleigh random variables y_1 and y_2 and Rayleigh random variable y_3, N_y, is:

$$N_y=\int_0^\infty d\dot{y}p_{y\dot{y}}\left(y\dot{y}\right)=$$
$$\int_0^\infty dy_2\int_0^\infty dy_3 p_{y_2}\left(y_2\right)p_{y_3}\left(y_3\right)\frac{y_3}{y_2}p_{y_1}\left(y\frac{y_3}{y_2}\right)\int_0^\infty d\dot{y}\,\dot{y}\,p_{\dot{y}}\left(\dot{y}/yy_2y_3\right)=$$
$$=\int_0^\infty dy_2\int_0^\infty dy_3 p_{y_2}\left(y_2\right)p_{y_3}\left(y_3\right)\frac{y_3}{y_2}p_{y_1}\left(y\frac{y_3}{y_2}\right)\frac{1}{\sqrt{2\pi}}\sigma_{\dot{y}}=$$
$$=\int_0^\infty dy_2\int_0^\infty dy_3 p_{y_2}\left(y_2\right)p_{y_3}\left(y_3\right)\frac{y_3}{y_2}p_{y_1}\left(y\frac{y_3}{y_2}\right)\cdot$$
$$\cdot\frac{1}{\sqrt{2\pi}}\pi f_m\frac{y_2}{y_3}\Omega_1^{1/2}\left(1+\frac{y^2y_3^2}{y_2^4}\frac{\Omega_2}{\Omega_1}+\frac{y^2}{y_2^2}\frac{\Omega_3}{\Omega_1}\right)^{1/2}=$$
$$=\frac{8}{\Omega_1\Omega_2\Omega_3}\pi f_m\Omega_1^{1/2}\frac{1}{\sqrt{2\pi}}y\cdot$$
$$\int_0^\infty dy_2y_2^{-1}y_2\int_0^\infty dy_3y_3^2e^{-\frac{1}{\Omega_1}y^2\frac{y_3^2}{y_2^2}-\frac{1}{\Omega_2}y_2^2-\frac{1}{\Omega_3}y_3^2}. \tag{17}$$

Last two fold integral can be solved by Laplace approximation formula [18]:

$$\int_0^\infty dy_2\int_0^\infty dy_3 g\left(y_{20},y_{30}\right)e^{\lambda f\left(y_{20},y_{30}\right)}=\frac{\pi}{\lambda}\frac{g\left(y_{20},y_{30}\right)}{B\left(y_{20},y_{30}\right)}e^{\lambda f\left(y_{20},y_{30}\right)} \tag{18}$$

where y_{20} and y_{30} are solution of the equations:

$$\frac{\partial f\left(y_{20},y_{30}\right)}{\partial y_{20}}=0\ ;\ \frac{\partial f\left(y_{20},y_{30}\right)}{\partial y_{30}}=0\ , \tag{19}$$

and B is matrix defined by:

$$B\left(y_{20},y_{30}\right)=\begin{vmatrix}\frac{\partial^2 f\left(y_{20},y_{30}\right)}{\partial y_{20}^2} & \frac{\partial^2 f\left(y_{20},y_{30}\right)}{\partial y_{20}\partial y_{30}}\\ \frac{\partial^2 f\left(y_{20},y_{30}\right)}{\partial y_{20}\partial y_{30}} & \frac{\partial^2 f\left(y_{20},y_{30}\right)}{\partial y_{30}^2}\end{vmatrix}. \tag{20}$$

In this section, ratio of product of two Rayleigh random variables and Rayleigh random variable is observed. LCR of wireless communication system with two sections operating over multipath fading channel in the presence of co-channel interference is evaluated.

Level crossing rate is calculated as LCR of product of two Rayleigh random variables and Rayleigh random variable. The first derivative of product of two Rayleigh random variables is calculated. Because the first derivative of Rayleigh random variable is Gaussian random variable, in this case, the first derivative of product of two Rayleigh random variables is linear transformation of Gaussian random variables. Under this condition, the first derivative of product of two Rayleigh random variables has conditional Gaussian distribution.

Derived expression for LCR can be applied for calculation average fade duration of wireless relay communication system with two sections operating over Rayleigh*Rayleigh multipath fading channel in the presence of Rayleigh co-channel interference. AFD of such wireless communication system could be evaluated as a ratio of OP and LCR.

3 Level Crossing Rate of Ratio of Rayleigh Random Variable and Product of Two Rayleigh Random Variables

Ratio z of Rayleigh random variable z_1 and product of two Rayleigh random variables z_2 and z_3 is:

$$z=\frac{z_1}{z_2z_3},\ z_1=zz_2z_3 \tag{21}$$

The first derivative of z is:

$$\dot{z}=\frac{\dot{z}_1}{z_2z_3}-\frac{z_1\dot{z}_2}{z_2^2z_3}-\frac{z_1\dot{z}_3}{z_2z_3^2}. \tag{22}$$

The first derivative of Rayleigh random variable is Gaussian random variable. Linear transformation of Gaussian random variables is Gaussian random variable. Therefore, $\dot{z}$ has conditional Gaussian distribution.

The main of $\dot{z}$ is:

$$\overline{\dot{z}} = \frac{1}{z_2 z_3}\overline{\dot{z}_1} - \frac{z_1}{z_2^2 z_3}\overline{\dot{z}_2} - \frac{z_1}{z_2 z_3^2}\overline{\dot{z}_3} = 0 \tag{23}$$

since: $\overline{\dot{z}_1} = \overline{\dot{z}_2} = \overline{\dot{z}_3} = 0$.

The variance of $\dot{z}$ is:

$$\sigma_{\dot{z}}^2 = \frac{1}{z_2^2 z_3^2}\sigma_{\dot{z}_1}^2 + \frac{z_1^2}{z_2^4 z_3^2}\sigma_{\dot{z}_2}^2 + \frac{z_1^2}{z_2^2 z_3^4}\sigma_{\dot{z}_3}^2, \tag{24}$$

where:

$$\sigma_{\dot{z}_1}^2 = \pi^2 f_m^2 \Omega_1$$
$$\sigma_{\dot{z}_2}^2 = \pi^2 f_m^2 \Omega_2 \tag{25}$$
$$\sigma_{\dot{z}_3}^2 = \pi^2 f_m^2 \Omega_3.$$

After substituting, formula for $\sigma_{\dot{z}}^2$ becomes:

$$\sigma_{\dot{z}}^2 = \pi^2 f_m^2 \left(\frac{1}{z_2^2 z_3^2}\Omega_1 + \frac{z_1^2}{z_2^4 z_3^2}\Omega_2 + \frac{z_1^2}{z_2^2 z_3^4}\Omega_3 \right) =$$
$$= \pi^2 f_m^2 \frac{1}{z_2^2 z_3^2}\Omega_1 \left(1 + \frac{z_1^2}{z_2^4 z_3^2}\Omega_2 z_2^2 z_3^2 \frac{1}{\Omega_1} + \frac{z_1^2}{z_2^2 z_3^4}\Omega_3 z_2^2 z_3^2 \frac{1}{\Omega_1} \right) =$$
$$= \pi^2 f_m^2 \frac{1}{z_2^2 z_3^2}\Omega_1 \left(1 + \frac{z_1^2}{z_2^2}\frac{\Omega_2}{\Omega_1} + \frac{z_1^2}{z_3^2}\frac{\Omega_3}{\Omega_1} \right) =$$
$$= \pi^2 f_m^2 \frac{1}{z_2^2 z_3^2}\Omega_1 \left(1 + z^2 z_3^2 \frac{\Omega_2}{\Omega_1} + z^2 z_2^2 \frac{\Omega_3}{\Omega_1} \right) \tag{26}$$

Joint probability density function of z, $\dot{z}$, z_2 and z_3 is:

$$p_{z\dot{z}z_2z_3}\left(z\dot{z}z_2z_3\right) = p_{\dot{z}}\left(\dot{z}/zz_2z_3\right) p_{zz_2z_3}\left(zz_2z_3\right) =$$
$$= p_{\dot{z}}\left(\dot{z}/zz_2z_3\right) p_z\left(z/z_2z_3\right) p_{z_2}\left(z_2\right) p_{z_3}\left(z_3\right). \tag{27}$$

Joint probability density function of z and $\dot{z}$ is:

$$p_{z\dot{z}}\left(z\dot{z}\right) = \int_0^\infty dz_2 \int_0^\infty dz_3\, p_{z\dot{z}z_2z_3}\left(z\dot{z}z_2z_3\right) =$$
$$= \int_0^\infty dz_2 \int_0^\infty dz_3\, p_{\dot{z}}\left(\dot{z}/zz_2z_3\right) p_z\left(z/z_2z_3\right) p_{z_2}\left(z_2\right) p_{z_3}\left(z_3\right) \tag{28}$$

Level crossing rate N_z of the ratio of Rayleigh random variable z_1 and product of two Rayleigh random variables z_2 and z_3 is:

$$N_z = \int_0^\infty d\dot{z}\, p_{z\dot{z}}\left(z\dot{z}\right) =$$
$$= \int_0^\infty dz_2 \int_0^\infty dz_3\, p_{z_2}\left(z_2\right) p_{z_3}\left(z_3\right) p_z\left(z/z_2z_3\right) \cdot \int_0^\infty d\dot{z}\, \dot{z}\, p_{\dot{z}}\left(\dot{z}/zz_2z_3\right) =$$
$$= \int_0^\infty dz_2 \int_0^\infty dz_3\, p_{z_2}\left(z_2\right) p_{z_3}\left(z_3\right) p_z\left(z/z_2z_3\right) \cdot \frac{1}{\sqrt{2\pi}}\sigma_{\dot{z}} =$$
$$= \int_0^\infty dz_2 \int_0^\infty dz_3\, p_{z_2}\left(z_2\right) p_{z_3}\left(z_3\right) p_z\left(z/z_2z_3\right) \cdot$$
$$\cdot \frac{1}{\sqrt{2\pi}} \pi f_m \frac{1}{z_2 z_3} \Omega_1^{1/2} \left(1 + z^2 z_3^2 \frac{\Omega_2}{\Omega_1} + z^2 z_2^2 \frac{\Omega_3}{\Omega_1} \right)^{1/2} \tag{29}$$

with:

$$p_z\left(z/z_2z_3\right) = \left|\frac{dz_1}{dz}\right| p_{z_1}\left(zz_2z_3\right) \tag{30}$$

$$\frac{dz_1}{dz} = z_2 z_3 \tag{31}$$

After new substitution, LCR of z is:

$$N_z = \int_0^\infty dz_2 \int_0^\infty dz_3\, p_{z_2}\left(z_2\right) p_{z_3}\left(z_3\right) z_2 z_3\, p_{z_1}\left(zz_2z_3\right) \cdot$$
$$\cdot \frac{1}{\sqrt{2\pi}} \pi f_m \frac{1}{z_2 z_3} \Omega_1^{1/2} \left(1 + z^2 z_3^2 \frac{\Omega_2}{\Omega_1} + z^2 z_2^2 \frac{\Omega_3}{\Omega_1} \right)^{1/2} =$$
$$= \int_0^\infty dz_2 \int_0^\infty dz_3\, p_{z_2}\left(z_2\right) p_{z_3}\left(z_3\right) p_{z_1}\left(zz_2z_3\right) \cdot$$
$$\cdot \frac{1}{\sqrt{2\pi}} \pi f_m \Omega_1^{1/2} \left(1 + z^2 z_3^2 \frac{\Omega_2}{\Omega_1} + z^2 z_2^2 \frac{\Omega_3}{\Omega_1} \right)^{1/2} \tag{32}$$

In this section, ratio of Rayleigh random variable and product of two Rayleigh random variables is considered. Actually, an expression for LCR of wireless system with two sections working in multipath fading channel under the influence of Rayleigh*Rayleigh co-channel interference is derived.

4 Numerical Results

Level crossing rate of ratio of product of two Rayleigh random variables and Rayleigh random variable is shown in Fig. 1 depending on the resulting variable y, for different values of average powers Ω_i, i=1, 2, 3.

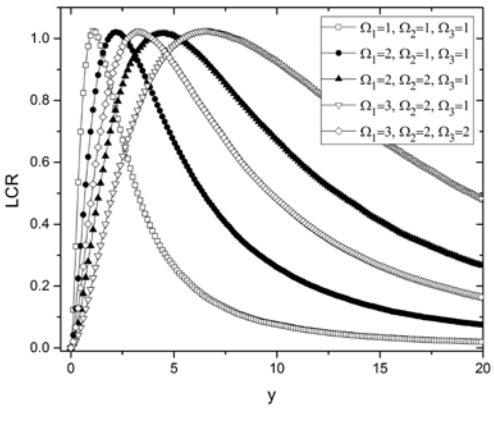

Fig. 1. LCR of ratio of product of two Rayleigh random variables and Rayleigh random variable.

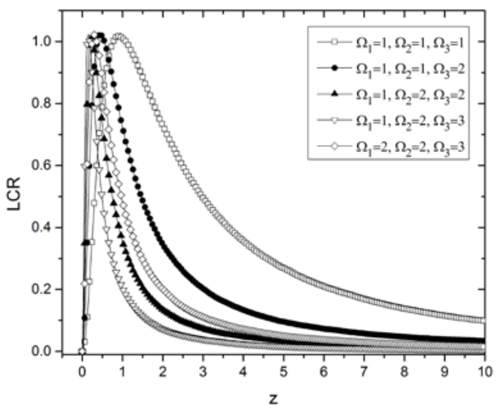

Fig. 2. LCR of ratio of Rayleigh random variable and product of two Rayleigh random variables.

It is visible from this figure that, for lower values of resulting variable, LCR increases as resulting variable increases. It achieves maximum and then, for higher values of resulting variable, LCR of ratio of product of two Rayleigh random variables and Rayleigh random variable decreases as resulting variable increases. The influence of resulting envelope on LCR is higher for higher values of resulting envelope.

It can be noticed from these graphs that LCR for small values of x increases for bigger average power Ω_i and LCR increases as average power decreases for bigger values of resulting amplitude y. Also, the influence of average power Ω_i, i=1,2,3, on LCR is lower for lower values of resulting envelope. Maximum of LCR goes to higher values of resulting envelope as average power takes higher values.

Level crossing rate of ratio of Rayleigh random variable and product of two Rayleigh random variables versus resulting envelope z is shown in Fig. 2. for some values of average powers Ω_i. LCR increases for small values of resulting amplitude, reaches maximum, and starts to decline. One can conclude from this figure that LCR is smaller for bigger magnitude of average power Ω_i in the region of small z, and conversely, LCR is bigger for higher Ω_i for the set of higher values of resulting amplitude z. The influence of average power Ω_i on LCR is higher for bigger values of resulting envelope z.

4 Conclusion

In this paper, the second order performance ratio of product of two Rayleigh variables and Rayleigh random variable as well as ratio of Rayleigh random variable and product of two Rayleigh random variables are calculated. The expression for the LCR of the ratio of product of two Rayleigh random variables and Rayleigh random variable can be used for evaluation the AFD of wireless relay communication system with two sections in the presence of co-channel interference. The expression for LCR of the ratio of Rayleigh random variable and product of two Rayleigh random variables can be used for evaluation AFD of wireless system operating over Rayleigh multipath fading channel in the presence of two Rayleigh co-channel interferences.

Derived analysis can be basis for calculation the LCR of the ratio of product of two Nakagami-m random variables and Nakagami-m random variable, as well as LCR of ratio of Nakagami-m random variable and product of two Nakagami-m random variables. All obtained results can be used for calculation the performance of wireless communication system with two sections in the presence of co-channel interference.

The achievements demonstrated in this paper can be applied in analysis of multi-hop relayed wireless communication systems where the signal level is much bigger than the noise level. Then, the noise level can be ignored and resulting signal can be presented as the product of two random variables.

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