

# Compromise Solutions: System Approach

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*Abstract:* - Currently, in the scientific literature [1], considerable attention is being drawn to the development of models and methods of decision theory, which allow one to efficiently solve multicriteria problems that arise in various subject areas. This applies, for example, to important and responsible negotiation processes, tasks of multi-criteria evaluation and optimization of complex technical objects, etc. Such problems are usually poorly formalized. Modern means of solving them, as a rule, are associated with the need for a compromise. Therefore, research in this direction is relevant.

*Key-Words:* - system approach, compromise, nonlinear compromise scheme, formalization.

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## 1 Introduction

The concept of “compromise” provides for the presence of several competing (having different goals) subjects integrated into a system. The task of this system is to develop a common solution for all subjects. Such a solution, being the best for the entire system as a whole, necessarily infringes on the interests of each of the subjects, that is, in principle it is a compromise. The researcher's task is to choose a compromise scheme in which the general solution of the system will be acceptable to all subjects.

Thus, the object of research is a system designed to develop a compromise solution. In the study, we will use the method of a systematic approach [2]. The term “systematic approach” means that a real object represented as a system is described as a set of interacting components that implements a specific goal. In our case, the subjects entering the system pursue *different* goals. If the same object can realize several goals, then with respect to each it acts as an independent system. Therefore, it is possible to *decompose* the problem being solved.

Each independent (partial) system is a model of a real object only in the aspect of the goal that it realizes. The goal, requiring certain functions for its achievement, determines through them the consist and structure of the partial system. This model will include from the real object only what is necessary and sufficient to achieve this goal.

Having determined the consist and structure of each of the partial systems, thereby completing the decomposition stage, in order to obtain a common

compromise solution, it is necessary to proceed to combining various object models into a single whole, that is, to perform the act of *composition*.

## 2 Problem Formulation

Let us consider a system for developing a compromise solution in the form of an object of study. The system includes  $s$  equal subjects, each of which seeks to realize its goal. The effectiveness of achieving this goal is quantified by the partial optimality criterion  $y_k, k \in [1, s]$ . Partial (particular) criteria  $y_1, y_2, \dots, y_s$  form a vector

$y = \{y_k\}_{k=1}^s \in M$ , where  $M$  is the domain of definition of the vector  $y$ . Its components quantitatively express the effectiveness of achieving partial goals for a given set of optimization arguments  $x = \{x_i\}_{i=1}^n \in X$ , where  $X$  is the domain of the vector of independent variables (optimization arguments). The external influences  $r$  do not depend on us, but it is known that they can take their values from the compact set  $R$ . Usually, it is believed that the calculations are carried out for a given and known vector of external influences  $r^0 \in R$ , on which, ultimately, the decision-making *situation* depends.

In the framework of the  $k$ -th particular system, the effectiveness of achieving the goal is quantitatively expressed by the partial criterion of optimality  $y_k, k \in [1, s]$ . The solution to the optimization problem involves achieving an extreme value of the

criterion by choosing a set of optimization arguments.

Extremization of the optimality criterion is often identified with the conception of goal realization, while in reality these are different conceptions. We can say that the criterion and the goal are related to each other as a model and an original with all the ensuing consequences. In any case, the criterion is just a substitute for the goal. Criteria characterize the goal only indirectly, sometimes better, sometimes worse, but always *approximately*.

At the decomposition stage, the concept and structure of each of the particular systems is determined. This means that functions  $y_k = f_k(x), k \in [1, s]$  are defined that link partial quality criteria with optimization arguments. In estimation problems, the function  $f(x)$  is called the *evaluation* function, and in optimization problems, it is called the *objective* function.

To obtain a general compromise solution, it is necessary to proceed to the stage of *composition* of the criteria. Various approaches to the implementation of this stage are possible. This is finding a compromise-optimal solution in an interactive procedure, a lexicographic approach, etc. In practice, the approach most often used is the choice of an adequate scalar convolution of partial criteria and finding a solution in the process of extremizing this convolution. We will consider here a method in which the act of composition is a *scalar convolution* of partial criteria [3]. Scalar convolution is a mathematical technique for compressing information and quantifying its integral properties with a single number.

As an objective function, we consider a certain function  $Y[y(x)]$  that has the meaning of scalar convolution of the vector of partial criteria, the form of which depends on the chosen compromise scheme. Its extremization leads to the desired compromise solution.

**The task is:** to choose a compromise scheme and determine the type of function  $Y(y)$  in which a compromise solution will be acceptable to all subjects of the decision-making system.

The formalization of these qualitative conceptions is very difficult, since the problems of vector optimization are conceptual in nature and formalization can be difficult.

### 3 Problem Solution

With some reservations, the optimization problem is formulated as finding such a combination of arguments from the domain of their definition, in which the objective function in a given situation takes on extreme value:

$$x^* = \arg \operatorname{extr}_{\substack{x \in X \\ y \in M}} Y[y(x)] \Big|_{r^O \in R}$$

The solution of optimization problems involves the presence of some assessment of the quality of the system, based on which it can be said that one system works better and the other worse, and how much concrete. The root problem of the quantitative assessment of objects and processes is that the qualitative concepts of “better” and “worse” are brought into line with the concepts of “more” and “less”. For definiteness, it is believed that, for example, “better” means “less.” Then all partial criteria are considered to be minimized (or reduced to this form). If so, then in practice, with a fixed  $r^O \in R$  and guaranteed  $y_k \in M, k \in [1, s]$  expression is used

$$x^* = \arg \min_{x \in X} Y[y(x)].$$

The choice of a compromise scheme and, therefore, the form of a scalar convolution of partial criteria is conceptual. The most commonly used is additive (linear) scalar convolution

$$Y(y) = \sum_{k=1}^s \frac{y_k}{A_k},$$

where  $A_k$  are the utterly permissible values of the criteria (restrictions). The Laplace principle in decision theory consists in the extremization of linear scalar convolution. The disadvantage (specificity) of using linear scalar convolution is the possibility of “compensation” of one criterion at the expense of others.

*Multiplicative convolution*

$$Y(y) = \prod_{k=1}^s y_k$$

is free from this flaw. Pascal's principle is the extremization of multiplicative scalar convolution.

Historically, the principle of Blaise Pascal [4] was first set forth in the work “Pensees”, published in 1670. It is believed that this work laid the foundation for the whole theory of decision making. Two key concepts of the theory are

introduced here: 1) partial criteria, each of which evaluates any one side of the effectiveness of the solution, and 2) the optimality principle, i.e. a rule that allows using the values of the criteria to calculate some unified numerical measure of the effectiveness of the solution.

Logically justified is a compromise scheme in which the vector of a compromise solution closest to the ideal (utopian) vector in the criterion space is considered as the preferred one (Charnes-Cooper concept [5]). This is the principle of optimality "closer to the utopian point."

Under the given conditions and restrictions, in the criteria space the a priori unknown ideal vector  $y^{id}$  is determined, for which the optimization problem is solved  $s$  times (by the number of partial criteria), each time with one (next) criterion, as if there were no others at all. The sequence of "single-criterion" solutions of the original multicriteria problem gives the coordinates of an unattainable ideal vector

$$y^{id} = \left\{ y_k^{id} \right\}_{k=1}^s.$$

After that, the objective function (scalar convolution of criteria)  $Y(y)$  is introduced as a measure of approach to the ideal vector in the space of optimized criteria in the form of some non-negative function of the vector  $y^{id}-y$ , for example, in the form of a square of the Euclidean norm of this vector

$$Y(y) = \left\| \frac{y^{id} - y}{y^{id}} \right\| = \sum_{k=1}^s \left[ \frac{y_k^{id} - y_k}{y_k^{id}} \right]^2,$$

after which the task of minimizing this objective function is solved. This approach provides a fairly complete formalization in the formulation and solution of multicriteria problems and has a clear physical meaning.

This scheme of compromises allows one to obtain a common compromise solution for the subjects, which approaches the utopian point as much as possible. The disadvantage of this approach is the cumbersomeness of the computational process, since it is necessary to solve  $s+1$  optimization problems. And most importantly, approaching the ideal point is carried out only in a generalized sense, i.e. there is always the possibility of violating restrictions on one or more components of the vector criterion.

## 4 Limitations

In decision theory, in addition to criteria, limitations play an equally important role. They can be superimposed both on optimization arguments  $x \in X$  and on the criteria for the effectiveness of the solution  $y \in M$ . For example, during important negotiations, the subjects of the negotiation process establish the so-called "red lines" according to decision-making criteria, approaching which, and even more so violating them, is not allowed under any circumstances.

The restrictions imposed on the characteristics of the system under certain circumstances can often be the reason for introducing one or another criterion. To illustrate this point, let us turn to an example. Under normal ground conditions, it is not customary to evaluate the quality of an ergatic system by the amount (or flow rate) of oxygen consumed by a human operator when performing a given job. It is a completely different matter when a system functions without contact with an "unlimited" terrestrial atmosphere (in space, under water, etc.). In this case, oxygen resources are limited and the efficiency of its consumption becomes a very important quality. A reflection of this requirement is the introduction of an appropriate criterion. In such cases, we can say that the restriction generates a criterion.

Even small changes in restrictions can significantly affect the decision. And absolutely serious consequences can be obtained by removing some restrictions and adding others with the same compromise scheme. In 1956, Brazilian entomologists found that bees do not produce enough honey. They crossed several European species and added a species of African bees. Hybrid bees, indeed, gave more honey, were resistant to diseases, tolerated heat, but at the same time they became incredibly aggressive and very poisonous. From their bites in Brazil and the southern United States killed more than 150 people and hundreds of animals, both domestic and wild.

Therefore, there is a great danger in the formal optimization of complex systems, as N. Wiener drew attention to in the very first publications on cybernetics. The fact is that, without setting all the necessary restrictions, we can simultaneously receive unforeseen and undesirable accompanying effects along with extremizing the objective function.

To illustrate, N. Wiener loved to cite the English tale of a monkey's paw. The owner of this talisman could fulfill any desire with its help. When he once wished to receive a large sum of money, it turned out that he paid for it with the life of his

beloved son. We agree that it is often very difficult, and sometimes simply impossible, to foresee all the consequences of the formal adoption of multicriteria decisions.

The thought of N. Wiener that in complex systems we are fundamentally unable to determine in advance all the conditions and limitations that guarantee the absence of undesirable optimization effects, allowed him to make a gloomy assumption about the catastrophic consequences of society's cybernetization.

Nevertheless, from the point of view of system analysis, the attitude to optimization can be formulated as follows: this is a powerful tool to increase decision-making efficiency, but it should be used more carefully as the complexity of the problem increases.

### 5 Non-Linear Compromise Scheme

Considering the fundamental role of restrictions in solving multicriteria problems, we formulate, in contrast to the Charnes-Cooper concept, the following optimality principle: "away from restrictions" [2]. In accordance with this principle, a non-linear compromise scheme is determined that allows one to obtain a common compromise solution that removes partial criteria from their "red lines" (restrictions) to the greatest extent possible. Requirements for the scalar convolution of criteria in a non-linear scheme of compromises:

1. It should "penalize" partial criteria for approaching their limitations.

2. It should be differentiable in its arguments.

Of the possible functions that meet the above requirements, we choose the simplest:

$$Y[y(x)] = \sum_{k=1}^s A_k [A_k - y_k(x)]^{-1},$$

where  $A_k$  are the restrictions from above on the minimized criteria:  $y_k \leq A_k, k \in [1, s]$ .

When an improvement in the quality of a solution is reflected by an increase in criteria, the scalar convolution of criteria according to a nonlinear compromise scheme has the form

$$Y[y(x),] = \sum_{k=1}^s B_k [y_k(x) - B_k]^{-1},$$

where  $B_k$  are the lower limits on the maximized criteria:  $y_k \geq B_k, k \in [1, s]$ .

Minimization of the objective function in both cases leads to a common compromise solution,

which removes partial criteria from its limitations to the greatest extent. For minimized criteria

$$x^* = \arg \min_{x \in X} \sum_{k=1}^s A_k [A_k - y_k(x)]^{-1}.$$

In the case of maximized criteria

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The scalar convolution according to a nonlinear scheme of compromises penalizes partial criteria for approaching their limit values. Indeed, let any criterion  $y_m(x)$  from among  $[1, s]$  dangerously approaches its limitation. This means that in the case of minimized criteria, the difference  $[A_m - y_m(x)] \rightarrow 0$  tends to zero and the

corresponding term  $\frac{A_m}{A_m - y_m(x)}$  in the

minimized sum rapidly increases. The situation is similar in the case of maximized criteria, for  $[y_m(x) - B_m] \rightarrow 0$ .

With a substantial increase  $y_m(x)$  and, consequently, of a member  $\frac{A_m}{A_m - y_m(x)}$ ,

minimizing the entire amount reduces to minimizing only this, the most "unsuccessful" member. This means that a nonlinear compromise scheme with a dangerous increase in one or more minimized partial criteria acts as a minimax (Chebyshev) optimization model

$$x^* = \arg \min_{x \in X} \max_{k \in [1, s]} y_k(x).$$

Accordingly, in the case of maximized criteria, the Chebyshev optimization model is maximin:

$$x^* = \arg \max_{x \in X} \min_{k \in [1, s]} y_k(x).$$

A situation in which partial criteria are dangerously close to their limitations is considered tense. On the contrary, if the criteria are far from limitations, the situation is calm. In a calm situation, a non-linear compromise scheme acts as an integral optimization model:

$$x^* = \arg \min_{x \in X} \sum_{k=1}^s \frac{y_k(x)}{A_k}$$

for minimized criteria and

$$x^* = \arg \min_{x \in X} \sum_{k=1}^s \frac{[-y_k(x)]}{B_k}$$

for criteria to be maximized.

Thus, the non-linear scheme of compromises adapts to the decision-making situation. The optimization model ranges from integral in calm situations to egalitarian (Chebyshev) in tense situations. In intermediate situations, compromise schemes are obtained that satisfy partial criteria to the extent that they are removed from their limitations. This means that instead of choosing a compromise scheme in various situations, we can use a single universal nonlinear compromise scheme that automatically gives a scheme that is adequate to a given specific situation.

From this point of view, traditional schemes of compromises can be considered as the result of "linearization" of the nonlinear scheme in various "working points" - situations. This, by the way, explains the name of the proposed *non-linear* scheme of compromises, since in other respects it is no more "non-linear" than other schemes considered in decision theory. We emphasize that the nonlinear scheme is adapted to the situation continuously, while the traditional choice of the compromise scheme is made discretely, which adds errors associated with the quantization of compromise schemes to subjective errors.

## 6 Weights

If the decision maker (DM) considers that the subjects of partial systems are unequal, then he assigns weighting factors for partial criteria of the system for developing compromise decisions. In fact, the decision-maker becomes the only subject for this system. The scalar convolution according to the nonlinear compromise scheme takes the form

$$Y[y(x), \alpha] = \sum_{k=1}^s \alpha_k A_k [A_k - y_k(x)]^{-1}, \alpha_k \geq 0, \sum_{k=1}^s \alpha_k = 1,$$

where  $\alpha = \{\alpha_k\}_{k=1}^s$  is the vector of weights.

The fundamental difference between a convolution in a nonlinear scheme from other known scalar convolutions is an organic connection with the situation of making a multi-criteria decision. In fact, this convolution is a nonlinear regression function (linear in the parameters  $\alpha$ ), selected for physical reasons and therefore effective. The coefficients  $\alpha$  in the expression for the nonlinear scalar convolution have the meaning of the parameters of the nonlinear content regression function, therefore, being found in the nominal mode, they do not change from situation to situation, as in the case of linear and other known convolutions that do not adapt to the situation.

Weights reflect the individual preferences of the decision maker according to individual partial criteria. Subjectivity is permissible and even desirable if the multicriteria problem is solved in the interests of a particular person. So, a suit made in an atelier by the individual measure of a customer is usually better than a suit bought at a ready-made clothing store. Therefore, the mechanism of individual preferences is quite intensively applied in the practice of solving multicriteria problems.

However, subjectivity in their solution is permissible and desirable only as long as the result is intended for specific decision makers or narrow groups of people with similar preferences. If it is intended for general use, then it must be completely objective, unified. In these cases, the mechanism of individual preferences from the methodology for solving multicriteria problems should be excluded in order to avoid arbitrariness and ambiguity of the results of the solution [3].

## 7 Unification

When the result of solving a multicriteria problem is intended for widespread use, then it is unified, and individual preferences are leveled according to statistics; the Bernoulli-Laplace principle of insufficient foundation becomes applicable: if the a priori probabilities of possible hypotheses are unknown, then they should be set equal, i.e. all hypotheses should be considered equally probable. As applied to the multicriteria problem, this means that all the weighting coefficients  $\alpha_k, k \in [1, s]$  in the expression for the scalar convolution of partial criteria should be equal, unless there are no

preliminary data on the difference in the criteria. So, the subjects of the negotiation process are usually considered the same in importance (equal).

Therefore, during unification we must take all the weighting factors equal:  $\alpha_k \equiv 1/s, \forall k \in [1, s]$ . Then

$$Y[y(x), \alpha] = \sum_{k=1}^s \alpha_k A_k [A_k - y_k(x)]^{-1} = \frac{1}{s} \sum_{k=1}^s A_k [A_k - y_k(x)]^{-1}.$$

Considering that multiplication by  $1/s$  is a monotone transformation, which, according to Germeier's theorem, does not change the comparison results, we pass to a unified expression for scalar convolution of criteria with minimized criteria:

$$Y[y(x)] = \sum_{k=1}^s A_k [A_k - y_k(x)]^{-1}.$$

Similarly, with maximized criteria

$$Y[y(x)] = \sum_{k=1}^s B_k [y_k(x) - B_k]^{-1}.$$

This formula is recommended to be used in all cases when the multicriteria problem is solved not in the interests of any one particular decision-maker, but for widespread use.

### 8 Dual approach

If the multicriteria problem is solved in the interests of a particular decision-maker, then it is necessary to determine the weighting coefficients  $\alpha_k, k \in [1, s]$  in the expression of the scalar convolution, reflecting his individual preferences. Various approaches can be used to determine the coefficients  $\alpha$ . The most reasonable is the dual approach [3].

We will present a scheme for finding compromise solutions for minimized criteria in the form of a model

$$x^{(\alpha)} = \operatorname{argmin}_{x \in X} \sum_{k=1}^s \alpha_k A_k [A_k - y_k(x)]^{-1}, \alpha \in X_\alpha$$

Choosing different values of the parameters  $\alpha$  from the admissible region

$$X_\alpha = \left\{ \alpha \mid \alpha_k \geq 0, \sum_{k=1}^s \alpha_k = 1 \right\},$$

according to this scheme, we obtain various compromise solutions  $x^{(\alpha)}$ . The task is to organize the interactive procedure so that the sequence of generated points is *improved* from the point of view of the decision maker.

Such a method, based on a comparison of preferences with specially calculated alternatives, is an ordinal analog of the simplex planning method [6].

The dual procedure begins with finding the first ("general") solution for  $\alpha_k^0 = 1/s, k \in [1, s]$ , which corresponds to a unified model

$$x^{(0)} = \operatorname{argmin}_{x \in X} \sum_{k=1}^s A_k [A_k - y_k(x)]^{-1}.$$

The resulting solution and the corresponding values of partial criteria are presented to the decision maker for evaluation. If the decision maker believes that the solution  $x^{(0)}$  does not satisfy him and a correction is required according to his individual preferences, then an interactive simplex planning procedure is organized. Recall that a simplex is a set of  $s+1$  vertices of the simplest figure (polyhedron) in an  $s$ -dimensional criterial space.

At each vertex of the initial simplex, starting from  $S_0$ , compromise solutions  $x^{(\alpha)}$  are calculated and the corresponding values of the particular criteria  $y(x^{(\alpha)})$  are presented to the decision makers to select the worst, from his point of view, vertex. According to the idea of the simplex planning method, the value of the utility function is more likely to improve if a solution is found at a new point directly opposite the worst peak in the sense of the initial simplex. The simplex planning mechanism consists in replacing the current simplex with a new one at each iteration: the worst vertex is discarded and a new one is introduced into the set, obtained by mirroring the worst point relative to the center of the opposite face.

So we get a sequence of simplexes  $S_0, S_1, S_2, \dots$ . The search stops when the decision maker considers that the solutions cease to improve significantly. According to the above, the sequence of generated points  $x^{(\alpha)}$  is improving from the point of view of the decision maker and converges to the

best, in his opinion, solution  $x^* = x^{(\alpha^*)}$ . Thus, at the same time, the vector  $\alpha^*$  is determined, which reflects the preferences of this particular decision-maker.

An important factor determining the effectiveness of the described method seems to be that the initial search point is chosen not as an arbitrary point in the criteria space, but as a reasonable basic solution, which should only be adjusted in accordance with the informal preferences of a particular decision-maker. Using our analogy, a suit bought at a ready-made clothing store is only slightly customized according to the customer's figure (if necessary). The adjustment process is dual,

it provides mutual adaptation: a person adapts to this particular multicriteria task, and the model of a non-linear scheme of compromises becomes a reflection of the individual preferences of this person.

The problem of determining the coefficients  $\alpha$  in a dual procedure can be considered as the task of synthesizing a *decision rule*, which, when formally applied, adequately reflects the logic of a particular decision-maker in any possible situation. Such a problem arises, for example, when a multicriteria system operates in the mode of an adviser to an operator in a time-constrained environment. Here it is desirable that the system in any situation makes the same decision as the given operator, if he had the opportunity to calmly think. Similar problems have to be solved when developing a decisive system of an intelligent robot that functions in changing and uncertain dynamic environments, if you want it to do the same as the person who trained it would do in its place, etc.

### 9 Illustrative example

As an example of using a nonlinear compromise scheme, we consider the problem of the compromise-optimal distribution of limited resources [3]. To carry out two flights ( $n = 2$ ), the airport has fuel with a total limited volume  $R = 12$  tons (conventional figures). The minimum requirement for the first flight is  $r_1 \geq B_{1\min} = 2$  tons, the second flight  $r_2 \geq B_{2\min} = 5$  tons. These are the bottom limits for partial resources. It makes no sense to emit less fuel: airplanes simply do not reach their destination. Thus the "red lines" are introduced, which must not be crossed under any circumstances.

The task is: to obtain an analytical solution  $r^*$  of compromise-optimal distribution of fuel between flights.

The problem is solved by minimizing the objective function

$$Y(r) = \sum_{i=1}^n B_{i\min} (r_i - B_{i\min})^{-1}$$

under isoperimetric constraint  $\sum_{i=1}^n r_i = R$ . Here

$Y(r)$  is a scalar convolution of maximized partial criteria according to a nonlinear compromise scheme.

We will solve the problem by the method of indefinite Lagrange multipliers. We construct the Lagrange function

$$L(r, \lambda) = B_{1\min} (r_1 - B_{1\min})^{-1} + B_{2\min} (r_2 - B_{2\min})^{-1} + \lambda (r_1 + r_2 - R)$$

where  $\lambda$  is the indefinite Lagrange multiplier.

The necessary minimum condition for this function leads to a system of equations

$$\frac{\partial L(r, \lambda)}{\partial r_1} = -B_{1\min} (r_1 - B_{1\min})^{-2} + \lambda = 0;$$

$$\frac{\partial L(r, \lambda)}{\partial r_2} = -B_{2\min} (r_2 - B_{2\min})^{-2} + \lambda = 0;$$

$$r_1 + r_2 - R = 0.$$

Substituting numeric data

$$-2(r_1 - 2)^{-2} + \lambda = 0;$$

$$-5(r_2 - 5)^{-2} + \lambda = 0;$$

$$r_1 + r_2 - 12 = 0$$

and solving this system by the Gauss method (sequential exclusion of variables), we obtain

$$r_1^* = 3,94 \text{ tons}, r_2^* = 8,06 \text{ tons}.$$

The problem was solved under the assumption that the relative importance of both flights for the DM is the same. If not, then the weighting coefficients  $\alpha_1$  and  $\alpha_2$ , reflecting the individual preferences of the decision maker, are introduced into the objective function. These coefficients should be normalized and determined on a simplex:

$$\alpha_1, \alpha_2 \in X_\alpha = \left\{ \alpha_i \mid \alpha_i \geq 0, \sum_{i=1}^{n=2} \alpha_i = 1, i \in [1;2] \right\}$$

### 10 Conclusion

An advantage of the concept of a nonlinear compromise scheme is the possibility of making a multi-criteria decision formally, without the direct involvement of a person. At the same time, on a unified ideological basis, both tasks that are important for general use and those, which main substantive essence is the satisfaction of individual preferences of decision makers, are solved. The apparatus of a nonlinear compromise scheme, designed as a formalized tool for studying decision-making and control systems with conflicting criteria, allows one to practically solve multicriteria problems of a wide class.

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