Estimation of the Three-Parameter Burr-XII Model Parameters based on the Integrable Estimation Method

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Abstract: - In parameter estimation techniques, there are many methods for estimating the distribution parameters in life data analysis. However, most of them are less efficient than the Bayes' method based on the informative prior. Thus, the main objective of this study is to introduce an optimal integrable estimation method for estimating the Burr type-XII distribution parameters and compare them with the Bayesian estimates based on the informative gamma and kernel priors. A comparison between these estimators is provided by using an extensive Monte Carlo simulation based on two criteria, namely, the absolute value bias and mean squared error. The simulation results indicated that the new integrable method is highly favorable, which provides better estimates and outperforms the Bayes' method using different loss functions based on the generalized progressive hybrid censoring scheme. Finally, two real datasets analyses are presented to illustrate the efficiency of the proposed methods.

Keywords: Bayes estimation; Compound LINEX loss function; Informative prior; Integrable estimation method; Kernel prior

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1 Introduction

In statistical inference, many methods have been used for estimating the distribution parameters. However, the most usable method in reliability analysis is the Bayes' method despite it depends on information other than data. Thus, in this study, we present an optimal integrable estimation method for estimating the distribution parameters that are more efficient compared to the Bayes' method based on the informative and kernel priors using different loss functions. To illustrate that, we employed the proposed methods on one of the employed distributions in lifetime distributions and reliability theory, which is the three-parameter Burr type-XII distribution. [3] and [4] showed that if one chooses the parameters appropriately, the Burr-XII distribution covers a large proportion of curve-shaped characteristics of types I, IV, and VI in the Pearson family of distributions. This distribution is one of the distribution families of survival and lifetime data that have the utility in survivorship applications.

Many authors have studied inferences on the Burr type-XII distribution. [25-29] have been described methods for fitting the Burr type-XII distribution on life test data based on type-II censoring using the maximum likelihood estimator (MLE) method. [23] derived the MLE based on censored and uncensored data. [17] studied the Bayes inference based on various loss functions. [13] derived the numerical estimation of the Burr type-XII distribution parameter based on Picard's method. [15] derived the estimation of the Burr-III parameters under the dual generalized progressive hybrid censoring scheme. [20, 21] derived the ML and Bayes estimators for some lifetime parameters, as well as the parameters of the Burr type-XII model based on progressive type-II censored samples.

The Burr type-XII distribution is the one that has a great part of applications based on progressive order statistics for its flexibility to describe the life test data. Consider the three-parameter Burr-XII distribution with the (CDF) F(x) and the (PDF) f(x), which are defined respectively as:

$$F(x) = 1 - [1 + \beta x^{\alpha}]^{-\gamma}, \quad x \geq 0, \alpha, \beta, \gamma > 0 \quad (1)$$

$$f(x) = \alpha\beta\gamma x^{\alpha-1}[1+\beta x^{\alpha}]^{-\gamma-1},\ x \geq 0, \alpha, \beta, \gamma > 0 \quad (2)$$

where α and γ are form factors and beta is the scale factor.

In reliability analysis, the progressive type-II censoring scheme is the most applicable in life test experiments, it is useful for both industrial life test applications and clinical trials and allows removing some of the surviving experimental units at different stages before testing is terminated. However, the trial time can be quite long due to some highly reliable units. Thus, [5-8] proposed the generalized progressive hybrid censoring scheme (GPHCS) that modifies the progressive hybrid censoring scheme. Let $\bar{X} = (X_1, X_2, \ldots, X_n)$ be the number of observations are subjected under the experiment, which allows to continue beyond defined time T to observe at least k failures, if the number of failures is less than m, where $k < m$ with $R_1, R_2, \ldots, R_m$ are the random removal units that are fixed at the beginning of the experiment such that $n = m + \sum_{i=1}^{m} R_i$. Let D denote the number of observed failures up to the time T. Generally, at the time of the i_th failure, R_i units are randomly removed from the remaining surviving units $S_i = n - i - \sum_{j=1}^{i-1} R_j$, where $i = 1,2,\ldots,m$. This process continues until, immediately following the terminated time $T^* = \max\{X_{k:m:n}, \min\{X_{m:n:n}, T\}\}$, where at this time all the remaining surviving units R_T^* are removed from the experiment. The algorithm of the GPHCS has been described in [10-11].

Thus, given a generalized progressive hybrid censored sample, the likelihood function for the three different cases can be written in a unified form as follows:

$$L(\bar{X};\theta) = C\prod_{i=1}^{n} f(x_i)\,[1-F(x_i)]^{R_i}[1-F(T)]^{R_T^*\delta}, \quad (3)$$

$$n = \begin{cases} m, & \delta = 0 \quad, \text{ if } \ X_{k:m:n} \leq X_{m:m:n} < T \quad, \\ k, & \delta = 0 \quad, \text{ if } \ T < X_{K:m:n} \leq X_{m:m:n}, \\ D, & \delta = 1 \quad, \text{ if } \ X_{k:m:N} < T < X_{m:m:n} \end{cases}$$

Thus, the log of the likelihood function of (3) can be written as follows:

$$H(\alpha,\beta,\gamma) = n\ln(\alpha\beta\gamma) + (\alpha-1)\sum_{i=1}^{n} \ln x_i - \sum_{i=1}^{n} \ln(1+\beta x_i^{\alpha})$$
$$-\gamma[\textstyle\sum_{i=1}^{n}(1+R_i)\ln(1+\beta x_i^{\alpha}) + \delta R_i^* \ln(1+\beta T^{\alpha})]. \quad (4)$$

The GPHCS has been applied for some distributions such as the Weibull distribution, see [7], the inverse Weibull distribution, see [10, 16], the Exponential distribution, see [6], the Rayleigh distribution, see [5], and some other distribution, see [12-16].

2 Estimation Methods

2.1 The Integrable Estimation Method

Theoretically, it is known that the traditional log likelihood function, $H(x,\theta)$, depends on the unknown parameter $\theta = (\alpha, \beta, \gamma)$ and the data x. Differentiating $H(x,\theta)$ with respect to θ, we get

$$\frac{dH(x,\theta)}{d\theta} = G(x,\theta). \quad (5)$$

Integrate (5) with respect to θ from θ_0 to θ_1 we get:

$$H(x,\theta_1) - H(x,\theta_0) = \int_{\theta_0}^{\theta_1} G(x,t)dt. \quad (6)$$

It is known that the second order approximation of the first derivative, which is defined by the centered differencing that can be written using (5) as follows:

$$\frac{dH(x,\theta)}{d\theta} = \frac{H(X,\theta_1) - H(x,\theta_0)}{\theta_1 - \theta_0} = G(x,\theta_m),\ \theta_0 < \theta_m < \theta_1. \quad (7)$$

From (6) and (7) we get the integral equation

$$\theta_1 = \theta_0 + \frac{1}{G(x,\theta_m)}\int_{\theta_0}^{\theta_1} G(x,t)dt, \quad \theta_0 < \theta_m < \theta_1.$$

Thus, we can get the successive approximation for the parameter θ from the following integral equation:

$$\theta_{n+1} = \theta_0 + \frac{1}{G(x,\theta_m)}\int_{\theta_0}^{\theta_n} G(x,t)dt, \quad \theta_0 < \theta_m < \theta_n \quad (8)$$

The iterative process is continued until two consecutive numerical solutions almost the same, that is if $|\theta_{n+1} - \theta_n| < 1E-05, n = 0,1,2,3,..$

2.2 The Bayes' Method

In this section, the Bayes estimators for the parameters will be derived using gamma and kernel prior distributions based on two different loss functions:

Firstly, the squared error loss function (SLF), $L(g(\theta), \hat{g}(\theta)) = (g(\theta) - \hat{g}(\theta))^2$. For this loss function the Bayes estimator that minimizes the risk function is given by $\hat{g}(\theta) = E_\theta(g(\theta)|x)$.

Secondly, the compound LINEX loss function which is defined as follows:

$$L(\Delta) = L_\delta(\Delta) + L_{-\delta}(\Delta) = e^{\delta\Delta} + e^{-\delta\Delta} - 2,\ \delta > 0.$$

It is named LINEX-based loss function, see [31], where

$L_{\delta}(\Delta) = exp[\delta\Delta] - \delta\Delta - 1,\ \Delta = \hat{g}(\theta) - g(\theta),\ \delta \neq 0,$

is the LINEX loss function (LLF) that has been introduced in [24] and [30]. The Bayes estimator of the parameter θthat minimizes the risk function can be derived as follows:

$$\hat{g}_L(\theta) = \frac{1}{2\delta}\log\left(\frac{E\left(e^{\delta g(\theta)}|X\right)}{E\left(e^{-\delta g(\theta)}|X\right)}\right).$$

2.2.1 The Informative prior

We consider the unknown parameters α, β and γ have independent gamma prior distributions with the joint probability density function, which is given by:

$$h(\alpha,\beta,\gamma) \propto \alpha^{a-1}\beta^{c-1}\gamma^{e-1}e^{-b\alpha-d\beta-f\gamma}, \quad (9)$$

where the hyper-parameter a, b, c, d, e and f are assumed to be known and non-negative and chosen to reflect the prior belief about the unknown parameters.

2.2.2 The Kernel Prior

For deriving the kernel prior, we introduce the trivariate kernel density estimator for the unknown probability density function $g(\alpha,\beta,\gamma)$ with support on ($0,\infty$), which is defined as follows:

$$\hat{g}(\alpha,\beta,\gamma) = \frac{1}{nh_1h_2h_3}\sum_{i=1}^{n} K\left(\frac{\alpha-\alpha_i}{h_1},\frac{\beta-\beta_i}{h_2},\frac{\gamma-\gamma_i}{h_3}\right), \quad (10)$$

$h_i, i = 1,2,3$ are called the bandwidths or smoothing parameters, which chosen such that $h_i \to 0$ and $nh_i \to \infty$ as $n \to \infty$, where n is the sample size. The influence of the smoothing parameter h is critical because it determines the amount of smoothing. However, the optimal choice for h_i that minimizes mean squared errors is given by $h_i = 1.06S_i n^{-0.2}$. where S_i the sample standard deviation. The optimal choice for the kernel function $K(.,.)$ can be used as the trivariate standard normal distribution for the parameters α, β and γ. Based on the properties of the maximum likelihood estimates (MLEs) of the parameters, which converge in probability to the original parameters, the algorithm for deriving the kernel prior estimate can be found [1] and [9].

The kernel prior has been used in [1] and [14, 15, 16]. Thus, using the joint priors (9) and (10) with the likelihood function of the GPHCS (3) the posterior density for the parameters α, β and γ can be written in a unified form as follows:

$f(\alpha,\beta,\gamma|\underline{x}) = Kq(\alpha,\beta,\gamma)L(\bar{X};\alpha,\beta,\gamma),$ where

$q(\alpha,\beta,\gamma) = h(\alpha,\beta,\gamma)\hat{g}(\alpha,\beta,\gamma) =$

$\hat{g}_1^{p_1}(\alpha)\hat{g}_2^{p_2}(\beta)\hat{g}_3^{p_3}(\gamma)\ \alpha^{a-1}\beta^{c-1}\gamma^{e-1}e^{-b\alpha-d\beta-f\gamma},$

is the general prior distribution function with the following cases:

1. $p_1 = p_2 = p_3 = 0$ for the informative prior (9),
2. $p_1 = p_2 = p_3 = 1,\ a = c = e = 1$, and
   $b = d = f = 0$ for the kernel prior (10).

Thus, the log posterior density can be written as follows:

$$\begin{aligned}H(\alpha,\beta,\gamma) &= p_1 \ln \hat{g}_1(\alpha) + p_2 \ln \hat{g}_2(\beta) + p_3\hat{g}_3 \ln(\gamma)\\ &+(n+a-1)ln\alpha + (n+c-1)\ln(\beta)\\ &+(n+e-1)\ln(\gamma) - \alpha b - \beta d - \gamma f\\ &-\sum_{i=1}^{n}\ln(1+\beta x_i^{\alpha}) + (\alpha-1)\sum_{i=1}^{n} lnx_i\\ &-\gamma[\sum_{i=1}^{n}(1+R_I)\ln(1+\beta x_i^{\alpha}) + \delta R_i^{*}\ln(1+\beta T^{\alpha})]\end{aligned} \quad (11)$$

approximate all the Bayes estimators for the unknown parameters. [22] introduced an easily computable approximation for the posterior mean and variance of a non-negative parameter or more generally, of a smooth function of the parameter that is non-zero on the interior of the parameter space. For detail, let $w(\alpha,\beta,\gamma)$ be a smooth, positive function on the parameter space. The posterior expectation of $w(\alpha,\beta,\gamma)$ can be obtained as follows:

$$w^* = E(w(\alpha,\beta,\gamma)|\bar{X})$$

$$= \frac{\int_0^\infty\int_0^\infty\int_0^\infty e^{nH^*(\alpha,\beta,\gamma)}d\alpha d\beta d\gamma}{\int_0^\infty\int_0^\infty\int_0^\infty e^{nH(\alpha,\beta,\gamma)}d\alpha d\beta d\gamma}, \quad (12)$$

where $H = \ln f(\alpha,\beta,\gamma|x)/n$, and

$H^* = H + \ln w(\alpha,\beta,\gamma)/n.$

For (α,β,γ) the Bayes estimator using Tierney-Kadane approximation for $w(\alpha,\beta,\gamma)$ can be obtained as follows:

$$w^* = \sqrt{|\Sigma^*|/|\Sigma|}\exp[n[H^*(\alpha,\beta,\gamma) - H(\alpha,\beta,\gamma)]$$

where $(\hat{\alpha},\hat{\beta},\hat{\gamma})$ and $(\hat{\alpha}^*,\hat{\beta}^*,\hat{\gamma}^*)$ maximize the $H(\hat{\alpha},\hat{\beta},\hat{\gamma})$ and $H^*(\hat{\alpha}^*,\hat{\beta}^*,\hat{\gamma}^*)$, respectively. Let

$|\Sigma| = \begin{vmatrix} H_{11} & H_{12} & H_{13}\\ H_{21} & H_{22} & H_{23}\\ H_{31} & H_{32} & H_{33}\end{vmatrix}^{-1}$ and

$$|\Sigma*| = \begin{vmatrix} H_{11}^* & H_{12}^* & H_{13}^* \\ H_{21}^* & H_{22}^* & H_{23}^* \\ H_{31}^* & H_{32}^* & H_{33}^* \end{vmatrix}^{-1}$$

denote the minus of inverse of Hessians of $H(\alpha,\beta,\gamma)$ and $H^*(\alpha,\beta,\gamma)$ at $(\hat{\alpha},\hat{\beta},\hat{\gamma})$ and $(\hat{\alpha}^*,\hat{\beta}^*,\hat{\gamma}^*)$ respectively. The derivatives of $H(\alpha,\beta,\gamma)$ and $H^*(\alpha,\beta,\gamma)$ have been derived in the Appendix A.

3 Simulation Study

The purpose of the simulation study is to compare the performance of the estimates using the integrable and Bayes methods based on the informative and the informative kernel priors with three different loss functions, through two criteria the average bias (AVB) and the mean squared error (MSE) as given by:

$\mathrm{AVB} = \frac{1}{L}\sum_{i=1}^{L}|\hat{\theta}_i - \theta|$, $\quad \mathrm{MSE} = \frac{1}{L}\sum_{i=1}^{L}\left(\hat{\theta}_i - \theta\right)^2$

$\hat{\theta}$ is the estimate of θ and L is the number of replications.

In our simulation study we choose the hyperparameters of α, β and γ as follows: $a = c = e = 5$, $b = d = f = 3$ and the values for the parameters are $\alpha = (1,2)$, two values for the parameter $\beta = (0.75,\ 1.75)$ and two values for the parameter $\gamma = (1.5,2.5)$ respectively. Using the above parameter values for generating different samples from the three-parameter Burr type-XII distribution with sizes $N = 20, 40$ and 60 to represent small, moderate and large sizes. To assess the performance of these estimates, the AVB and the MSE for each were calculated using 1000 replicates. An algorithm for generating the generalized progressive hybrid censoring scheme has been written, see [18,19].

From the simulation results in Tables 3, 4, 5, 6, 7 and 8, some of the points are quite clear based on these estimates and the others have been summarized in the following main points:

1. It is clear that generally for both parameters β and γ the average Bias values based on the integrable method outperform the corresponding values based on the Bayes method for the different loss functions. However, for the parameter α both methods have average bias almost the same especially based on the LINEX-based loss function.
2. In terms of MSE values, we can easily see that the integrable method has the smallest MSE values compared with their counterparts that are based on the Bayes method, but for the parameter α both methods have the same MSEs approximately.
3. It is evident that the estimated AVB and MSE values decrease with increasing the hyperparameters, the termination time of the experiment T, and the sample sizes as expected for all methods.
4. For the parameter α, the estimated MSE values increase with increasing the value of α, while decreasing as the value of β and γ increase.
5. For the parameters β and γ the estimated MSE values increase with increasing the values of β and γ, while decreasing as the value of α increases .
6. In general, the estimated MSE values for the Bayes method based on the LINEX-based loss function are less than those based on the squared error and the Stein loss functions.

In conclusion, the integrable method competes and outperforms the Bayes method based on the informative and kernel priors.

4 Real Data Analysis

In this section, we studied two real data sets to demonstrate the performance of the proposed methods on the Burr type XII model in practice and to illustrate that this distribution can be considered as a good lifetime model for some new area of applications, comparing with many known distributions such as the Weibull distribution. We have fitted these data sets using some goodness of fit tests such as the Kolmogorov-Smirnov (K-S), Anderson-darling (A-D) and Chi-Square (CH2) tests for significance level test equals 0.05.

4.1 The Reactor Pumps Data

In this section, a real data set for secondary nuclear pumps have been analyzed to illustrate the proposed methods. One of the most severe accidents in nuclear power generation is the loss of coolant, where the re-circulating coolant of the pressurized water reactor may flash into steam. Under such conditions, the reactor cooling pumps become unable to generate the same head as that of the single-phase flow case. Thus, the secondary reactor pump is a feedwater pump that takes from the desecrator storage tank feed water pressured up by the booster pump and pushes it into the steam generator through the high-pressure heater. Accordingly, the main feed pump must be high temperature and high-pressure pump since it requires a head larger than the pressure inside the steam generator. The secondary circulation pump differs slightly in design and has been developed specifically for cooling at higher temperatures. The following data set represents the times between the failure of the secondary reactor pumps. [18-20] have discussed the classical and Bayesian estimation methods under the Type-II censoring scheme of this data set. The times between failures of 23 secondary reactor pumps are as follows:

2.160, 0.746, 0.402, 0.954, 0.491, 6.560, 4.992, 0.347, 0.150, 0.358, 0.101, 1.359, 3.465, 1.060, 0.614, 1.921, 4.082, 0.199, 0.605, 0.273, 0.070, 0.062, 5.320.

We found the Burr type-XII model is a good fit for this dataset as shown in Table 1 and Figure (1 a). For studying the reliability of these reactor pumps based on this dataset we find the estimates for the parameters which represent the shape of the failures between pumps using our model to determine the behavior of the failure pumps. We noticed that the integrable and Bayes estimates for α lies in the interval [0.6, 0.7], for γ lies in the interval [0.9, 1.2] and for β lies in the interval [0.02, 0.13] indicating that the above dataset is heavily right-skewed and that means the failure rate decreases with increasing the time, see Figure (1 b), and that means decreasing the reliability of safety mechanism with increasing the time.

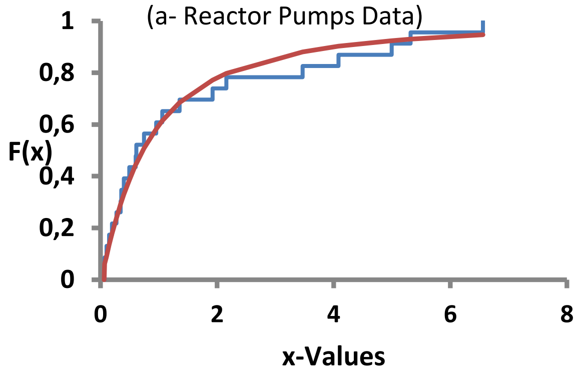

Fig. 1: a) The Empirical CDF and the fitted CDF.

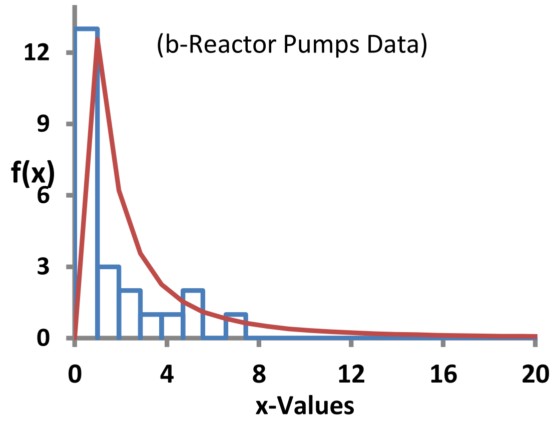

Fig.1 b) The Histogram and the fitted PDF.

4.2 The Vinyl Chloride Data

As vinyl chloride is a known human carcinogen, exposure to this compound should be avoided as far as practicable, and levels should be kept as low as technically feasible. Where it is known that a concentration of vinyl chloride in drinking water of 0.5 mg/liter was calculated as being associated with an excess risk of liver and Brain tumors for exposure beginning at adulthood and it would double cancer risk for continuous exposure from birth. Therefore, we consider the dataset used by [2], which represents 34 data points in mg/L from the vinyl chloride obtained from clean upgrade monitoring wells as follows:

5.1, 1.2, 1.3, 0.6, 0.5, 2.4, 0.5, 1.1, 8.0, 0.8, 0.4, 0.6, 0.9, 0.4, 2.0, 0.5, 5.3, 3.2, 2.7, 2.9, 2.5, 2.3, 1.0, 0.2, 0.1, 0.1, 1.8, 0.9, 2.0, 4.0, 6.8, 1.2, 0.4, 0.2.

We found the Burr type-XII model is a very good fit for this dataset as shown in Table 1 and Figure (2 a). For studying the concentration of the vinyl chloride in the water of these wells based on this dataset we find the estimates for the parameters which represent the shape of the concentration using our model to determine the average concentration in the water. We noticed that the integrable and Bayes estimates for α lies in the interval [0.7, 0.9] and for γ lies in the interval [1.2, 1.4] indicating that the above dataset is moderately right skewed and that means the concentration decreases with increasing time, see Fig .(2 b). Also, the integrable and Bayes estimates for β lies in the interval [0.09, 0.17], which ensures the dataset is right-skewed and the vinyl chloride concentration will decrease with increasing time, therefore, monitoring these wells is very significant.

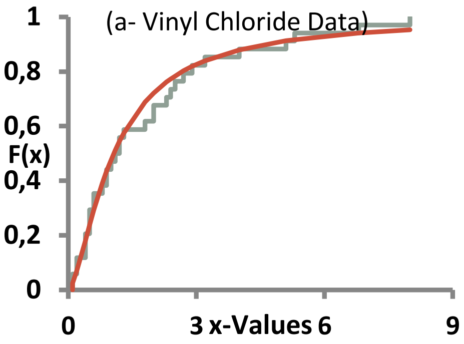

Fig. 2: a) The Empirical CDF and the fitted CDF.

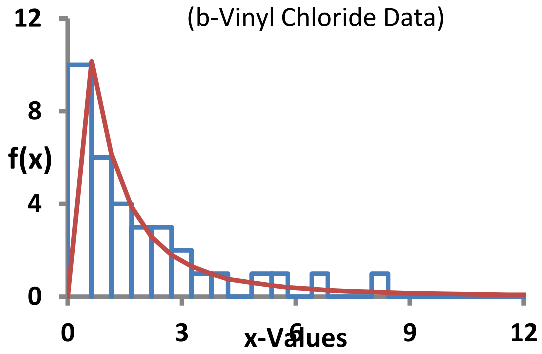

Fig. 2 b) The Histogram and the fitted PDF.

From the results in Table 1, the three-parameter Burr type-XII model is a very good fit for these data sets since the calculated values for the goodness of fit tests are less than the critical values. Moreover, the power of the tests is greater than the significance level 0.05 of the tests. From the results in Table 2, the estimated values of MSEs based on the integrable method are approximately smaller than those for the Bayes' method for these data sets, considering the MLEs are the true values of the parameters. Thus, the results of these data sets ensure the simulation results.

5 Conclusions

In this study, it is found that for the generalized progressive hybrid censored data, the integrable method is strongly unbiased and much more efficient than the Bayes' method based on the informative and kernel priors for the different loss functions. The Bayes estimates based on the nonparametric kernel prior are more efficient than those based on the informative gamma prior and are very close to those based on the integrable method. Thus, the statistical significance of the integrable method is its efficiency than most estimation methods, besides, it is reliable and easy to apply, especially for social sciences and psychology researchers.

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Contribution of the Author

I completely did this work.

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Appendix A:

$$\mathrm{H}(\alpha,\beta,\gamma) = p_1 \ln \hat{g}_1(\alpha) + p_2 \ln \hat{g}_2(\beta) + p_3 \hat{g}_3 \ln(\gamma)$$
$$+(n+a-1)ln\alpha + (\mathrm{n+c-1})\ln(\beta)$$
$$+(n+e-1)\ln(\gamma) - \alpha b - \beta d - \gamma f$$
$$-\sum_{i=1}^{n} \ln(1+\beta x_i^{\alpha}) + (\alpha-1)\sum_{i=1}^{n} ln x_i$$
$$-\gamma[\sum_{i=1}^{n}(1+R_I)\ln(1+\beta x_i^{\alpha}) + \delta R_i^{*}\ln(1+\beta T^{\alpha})]$$

$$\frac{\partial H}{\partial \alpha} = [p_1 \frac{\hat{g}_1^{/}(\alpha)}{\hat{g}_1(\alpha)} + (n+a-1)/\alpha - b$$
$$+\sum_{i=1}^{n} ln x_i - \sum_{i=1}^{n} \frac{\beta {x_i}^{\alpha} \ln x_i}{1+\beta {x_i}^{\alpha}}$$
$$-\gamma(\sum_{i=1}^{n}(1+R_i^{*})\frac{\beta {x_i}^{\alpha} \ln x_i}{1+\beta {x_i}^{\alpha}} + \frac{\beta T^{\alpha} \ln T}{1+\beta T^{\alpha}})]/n$$

$$\frac{\partial^2 H}{\partial \alpha^2} = [p_1 \frac{\hat{g}_1(\alpha)\hat{g}_1^{//}(\alpha) - {\hat{g}'}_1^{2}(\alpha)}{\hat{g}_1^2(\alpha)} - (n+a-1)/\alpha^2$$
$$-\sum_{i=1}^{n} \frac{(1+\beta {x_i}^{\alpha})\beta {x_i}^{\alpha}(\ln x_i)^2 - (\beta {x_i}^{\alpha} \ln x_i)^2}{(1+\beta {x_i}^{\alpha})^2}$$
$$-\gamma[\sum_{i=1}^{n}(1+R_i^{*})\frac{(1+\beta {x_i}^{\alpha})\beta {x_i}^{\alpha}(\ln x_i)^2 - (\beta {x_i}^{\alpha} \ln x_i)^2}{(1+\beta {x_i}^{\alpha})^2}$$
$$+\delta R_T^{*}\frac{(1+\beta T^{\alpha})\beta T^{\alpha}(\ln T)^2 - (\beta T^{\alpha} \ln T)^2}{(1+\beta T^{\alpha})^2}]/n$$

$$\frac{\partial^2 H}{\partial \alpha \partial \beta} = [-\sum_{i=1}^{n}\frac{(1+\beta {x_i}^{\alpha}){x_i}^{\alpha}\ln x_i - \beta {x_i}^{2\alpha}\ln x_i}{(1+\beta {x_i}^{\alpha})^2}$$
$$-\gamma[\sum_{i=1}^{n}(1+R_i^{*})\frac{(1+\beta {x_i}^{\alpha}){x_i}^{\alpha}\ln x_i - \beta {x_i}^{2\alpha}\ln x_i}{(1+\beta {x_i}^{\alpha})^2}$$
$$+\delta R_T^{*}\frac{(1+\beta T^{\alpha})T^{\alpha}\ln T - \beta T^{2\alpha}\ln T}{(1+\beta T^{\alpha})^2}]/n$$

$$\frac{\partial H}{\partial \beta} = [p_2 \frac{\hat{g}_2^{'}(\beta)}{\hat{g}_2(\beta)} + (n+c-1)/\beta - d - \sum_{i-1}^{n}\frac{x_i^{\alpha}}{1+\beta x_i^{\alpha}}$$
$$-\gamma[\sum_{i-1}^{n}(1+R_i)\frac{x_i^{\alpha}}{1+\beta x_i^{\alpha}} + \delta R_T^{*}\frac{T^{\alpha}}{1+\beta T^{\alpha}}]]/n$$

$$\frac{\partial^2 H}{\partial \beta^2} = [p_2 \frac{\hat{g}_2(\beta)g_2^{//}(\beta) - {\hat{g}_2^{/}}^{2}(\beta)}{{\hat{g}_2}^{2}(\beta)}$$
$$-\frac{n+c-1}{\beta^2} + \sum_{i-1}^{n}\frac{x_i^{2\alpha}}{(1+\beta x_i^{\alpha})^2}$$
$$+\gamma[\sum_{i-1}^{n}(1+R_i)\frac{x_i^{2\alpha}}{(1+\beta x_i^{\alpha})^2} + \delta R_T^{*}\frac{T^{2\alpha}}{(1+\beta T^{\alpha})^2}]]/n.$$

$$\frac{\partial H}{\partial \gamma} = [p_3 \frac{\hat{g}_3^{'}(\gamma)}{\hat{g}_3(\gamma)} + (n+e-1)/\gamma - f$$
$$-[\sum_{i=1}^{n}(1+R_I)\ln(1+\beta x_i^{\alpha}) + \delta R_i^{*}\ln(1+\beta T^{\alpha})]]/n$$

$$\frac{\partial^2 H}{\partial \gamma^2} = [p_3 \frac{\hat{g}_3(\gamma)g_3^{//}(\gamma) - {\hat{g}_3^{/}}^{2}(\gamma)}{{\hat{g}_3}^{2}(\gamma)} - (n+e-1)/\gamma^2]/n$$

$$\frac{\partial^2 H}{\partial \gamma^2} = [p_3 \frac{\hat{g}_3(\gamma)g_3^{//}(\gamma) - {\hat{g}_3^{/}}^{2}(\gamma)}{{\hat{g}_3}^{2}(\gamma)} - (n+e-1)/\gamma^2]/n$$

$$\frac{\partial \mathrm{H}}{\partial \gamma \, \partial \alpha} = -[\sum_{i=1}^{n}(1+\mathrm{R}_i^{*})\frac{\beta \mathrm{x_i}^{\alpha}\ln \mathrm{x_i}}{1+\beta \mathrm{x_i}^{\alpha}} + \delta \mathrm{R}_T^{*}\frac{\beta \mathrm{T}^{\alpha}\ln \mathrm{T}}{1+\beta \mathrm{T}^{\alpha}}]/\mathrm{n}$$
$$\frac{\partial \mathrm{H}}{\partial \gamma \, \partial \beta} = -[\sum_{i=1}^{n}(1+\mathrm{R}_i^{*})\frac{\mathrm{x_i}^{\alpha}}{1+\beta \mathrm{x_i}^{\alpha}} + \delta \mathrm{R}_T^{*}\frac{\mathrm{T}^{\alpha}}{1+\beta \mathrm{T}^{\alpha}}]/\mathrm{n}$$

where the r^{th} derivative of the kernel density estimation can be defined as

$$\frac{\mathrm{d^r}\hat{\mathrm{g}}_1(\alpha)}{\mathrm{d}\alpha^{\mathrm{r}}} = {\hat{\mathrm{g}}_1}^{\mathrm{r}}(\alpha) = \frac{1}{\mathrm{Dh}_1^{\mathrm{r+1}}}\sum_{\mathrm{i=1}}^{\mathrm{n}} \mathrm{K}^{\mathrm{r}}\left(\frac{\alpha - \hat{\alpha}_{\mathrm{i}}}{\mathrm{h}_1}\right), \quad \text{where}$$
r=0,1,2,3,-----. (13)

Using the Gaussian kernel and (13), we have

$$\hat{g}_1(\alpha) = \frac{1}{n\square_1\sqrt{2\pi}}\sum_{i=1}^{n} e^{-0.5(\frac{\alpha-\hat{\alpha}_i}{\square_1})^2},$$
$$\hat{g}_1^{'}(\alpha) = -\frac{1}{n\square_1^2\sqrt{2\pi}}\sum_{i=1}^{n}(\frac{\alpha-\hat{\alpha}_i}{\square_1})\, e^{-0.5(\frac{\alpha-\hat{\alpha}_i}{\square_1})^2}$$
$$\hat{g}_1^{''}(\alpha) = \frac{1}{n\square_1^3\sqrt{2\pi}}\sum_{i=1}^{n}[(\frac{\alpha-\alpha_i}{\square_1})^2 - 1]\, e^{-0.5(\frac{\alpha-\alpha_i}{\square_1})^2}.$$

Similarly for the kernel priors $\hat{g}_2(\beta)$ and $\hat{g}_3(\gamma)$ and the derivatives of H^{*}.

Table 1: The critical and calculated values for the K-S, A-D and CH2 tests and their powers (p-values) for the Burr type-XII model. The MLE for the parameters for these data sets have been calculated.

Data	The Tests	Calculated value	Critical value	p-values	MLES		
					α	β	γ
Reactor pumps Data N=23	K-S	0.4719	0.9001	0.8163	0.6944	0.0202	1.2035
	A-D	0.2466	0.7714	0.7563			
	CH2	9.5741	12.234	0.1355			
The vinyl Chloride data N=34	K-S	0.6064	0.9074	0.4872	0.9234	0.1238	1.4186
	A-D	0.3010	0.7711	0.6134			
	CH2	5.6472	15.086	0.3854			

Table 2: The estimate and the mean squared errors (MSEs) for the parameter α, β and γ based on the Integrable and Bayes methods under square error loss function based on the GPHCS: for $m = n/2$, $k = m/2$, (a=c=e=5; b=d=f=3).

Data	T	Param.	INT Estamite		Gamma Prior		Kernel prior	
			Estimate	MSE	Estimate	MSE	Estimate	MSE
The reactor Pumps Data n=23	0.25	α	0.5607	0.0179	0.5719	0.0150	0.5641	0.0169
		β	0.1330	0.0127	0.0164	0.0015	0.0159	0.0019
		γ	1.1921	0.0013	0.9751	0.0522	0.9634	0.0577
	3.5	α	0.6611	0.0011	0.5797	0.0132	0.5773	0.0137
		β	0.1345	0.0013	0.0167	0.0013	0.0165	0.0014
		γ	1.1884	0.0023	0.9922	0.0045	0.9892	0.0459
The vinyl Chloride Data n=34	0.5	α	0.9060	0.0029	0.7619	0.0261	0.7567	0.0278
		β	0.1880	0.0041	0.1008	0.0053	0.0985	0.0064
		γ	1.3751	0.0019	1.1564	0.0687	1.1501	0.0721
	5	α	0.8791	0.0019	0.7689	0.0239	0.7664	0.0247
		β	0.1727	0.0024	0.1010	0.0052	0.0989	0.0062
		γ	1.1670	0.0632	1.1614	0.0661	1.1559	0.0691

Table 3: The average bias (AVB) and the mean squared error (MSE) in parentheses for the Burr-XII parameter α using the integrable and Bayes methods with m = (n/2 and 3n/4) and k=(m/2 and 3m/4) at T=0.75 and $\delta = 2$ for LINEX loss function.

n	m	K	α	β	γ	Integrable Method	Gamma Prior		Kernel Prior	
							SQEL	LNXL	SQEL	LNXL
	10	5	1	0.75	1.5	0.1177(0.0360)	0.179(0.0392)	0.119(0.0149)	0.169(0.0295)	0.122(0.0156)
				1.75	2.5	0.2130(0.0487)	0.163(0.0301)	0.103(0.0114)	0.134(0.0186)	0.106(0.0119)
			2	0.75	1.5	0.1081(0.0120)	0.389(0.1521)	0.273(0.0750)	0.351(0.1314)	0.282(0.0795)
				1.75	2.5	0.2071(0.0430)	0.466(0.5340)	0.269(0.0727)	0.319(0.1028)	0.274(0.0752)
		8	1	0.75	1.5	0.1943(0.0413)	0.161(0.0266)	0.123(0.0157)	0.160(0.0261)	0.126(0.0163)
				1.75	2.5	0.2106(0.0449)	0.140(0.0203)	0.107(0.0120)	0.132(0.0179)	0.110(0.0124)
			2	0.75	1.5	0.1091(0.0166)	0.350(0.1234)	0.276(0.0765)	0.332(0.1106)	0.284(0.0807)
				1.75	2.5	0.2081(0.0434)	0.339(0.1172)	0.271(0.0735)	0.312(0.0992)	0.276(0.0761)

20	**15**	**8**	**1**	**0.75**	**1.5**	**0.1107(0.0164)**	**0.159(0.0260)**	**0.122(0.0155)**	**0.158(0.0255)**	**0.125(0.0161)**
				1.75	**2.5**	**0.2121(0.0477)**	**0.140(0.0205)**	**0.107(0.0120)**	**0.131(0.0176)**	**0.110(0.0125)**
			2	**0.75**	**1.5**	**0.1094(0.0128)**	**0.353(0.1330)**	**0.276(0.0762)**	**0.332(0.1102)**	**0.283(0.0804)**
				1.75	**2.5**	**0.2080(0.0434)**	**0.340(0.1166)**	**0.271(0.0738)**	**0.311(0.0970)**	**0.276(0.0763)**
		11	**1**	**0.75**	**1.5**	**0.1043(0.0115)**	**0.158(0.0283)**	**0.128(0.0167)**	**0.155(0.0243)**	**0.130(0.0172)**
				1.75	**2.5**	**0.2074(0.0431)**	**0.132(0.0180)**	**0.108(0.0122)**	**0.128(0.0170)**	**0.111(0.0127)**
			2	**0.75**	**1.5**	**0.1071(0.0119)**	**0.336(0.1154)**	**0.281(0.0789)**	**0.324(0.1051)**	**0.286(0.0821)**
				1.75	**2.5**	**0.2080(0.0434)**	**0.323(0.1199)**	**0.272(0.0738)**	**0.305(0.0933)**	**0.277(0.0765)**
40	**20**	**10**	**1**	**0.75**	**1.5**	**0.1099(0.0127)**	**0.147(0.0222)**	**0.117(0.0142)**	**0.146(0.0221)**	**0.120(0.0147)**
				1.75	**2.5**	**0.2098(0.0441)**	**0.132(0.0184)**	**0.104(0.0111)**	**0.122(0.0153)**	**0.106(0.0116)**
			2	**0.75**	**1.5**	**0.1088(0.0125)**	**0.335(0.1130)**	**0.273(0.0749)**	**0.320(0.1028)**	**0.280(0.0783)**
				1.75	**2.5**	**0.2060(0.0425)**	**0.351(0.1448)**	**0.271(0.0733)**	**0.301(0.0911)**	**0.274(0.0750)**
		15	**1**	**0.75**	**1.5**	**0.1076(0.0118)**	**0.142(0.0205)**	**0.121(0.0150)**	**0.142(0.0206)**	**0.123(0.0155)**
				1.75	**2.5**	**0.2100(0.0442)**	**0.126(0.0162)**	**0.108(0.0119)**	**0.123(0.0153)**	**0.110(0.0123)**
			2	**0.75**	**1.5**	**0.1084(0.0118)**	**0.318(0.1016)**	**0.275(0.0758)**	**0.312(0.0978)**	**0.281(0.0791)**
				1.75	**2.5**	**0.2074(0.0431)**	**0.308(0.0956)**	**0.270(0.0732)**	**0.296(0.0875)**	**0.274(0.0750)**
	30	**15**	**1**	**0.75**	**1.5**	**0.1080(0.0117)**	**0.141(0.0208)**	**0.120(0.0148)**	**0.142(0.0204)**	**0.122(0.0153)**
				1.75	**2.5**	**0.2099(0.0442)**	**0.125(0.0161)**	**0.107(0.0117)**	**0.122(0.0151)**	**0.109(0.0121)**
			2	**0.75**	**1.5**	**0.1081(0.0117)**	**0.333(0.3607)**	**0.275(0.0759)**	**0.311(0.0970)**	**0.281(0.0791)**
				1.75	**2.5**	**0.2075(0.0431)**	**0.307(0.0944)**	**0.270(0.0732)**	**0.295(0.0874)**	**0.274(0.0750)**
		23	**1**	**0.75**	**1.5**	**0.1035(0.0108)**	**0.141(0.0202)**	**0.128(0.0165)**	**0.142(0.0204)**	**0.129(0.0169)**
				1.75	**2.5**	**0.2063(0.0426)**	**0.122(0.0151)**	**0.111(0.0125)**	**0.121(0.0150)**	**0.112(0.0128)**
			2	**0.75**	**1.5**	**0.1068(0.0114)**	**0.307(0.0951)**	**0.281(0.0790)**	**0.306(0.0934)**	**0.285(0.0813)**
				1.75	**2.5**	**0.2071(0.0430)**	**0.294(0.0872)**	**0.272(0.0738)**	**0.291(0.0847)**	**0.275(0.0757)**
60	**30**	**15**	**1**	**0.75**	**1.5**	**0.1080(0.0117)**	**0.138(0.0194)**	**0.117(0.0140)**	**0.137(0.0191)**	**0.119(0.0144)**
				1.75	**2.5**	**0.2091(0.0438)**	**0.123(0.0155)**	**0.104(0.0111)**	**0.119(0.0173)**	**0.106(0.0115)**
			2	**0.75**	**1.5**	**0.1084(0.0118)**	**0.322(0.1412)**	**0.273(0.0747)**	**0.308(0.0951)**	**0.278(0.0776)**
				1.75	**2.5**	**0.2068(0.0428)**	**0.317(0.1012)**	**0.270(0.0731)**	**0.293(0.0860)**	**0.273(0.0744)**
		23	**1**	**0.75**	**1.5**	**0.1078(0.0117)**	**0.134(0.0183)**	**0.120(0.0148)**	**0.135(0.0184)**	**0.122(0.0152)**
				1.75	**2.5**	**0.2090(0.0438)**	**0.119(0.0144)**	**0.107(0.0117)**	**0.118(0.0140)**	**0.109(0.0120)**
			2	**0.75**	**1.5**	**0.1084(0.0118)**	**0.305(0.0947)**	**0.275(0.0759)**	**0.302(0.0921)**	**0.280(0.0786)**
				1.75	**2.5**	**0.2072(0.0430)**	**0.296(0.0875)**	**0.271(0.0735)**	**0.289(0.0837)**	**0.274(0.0750)**
	45	**23**	**1**	**0.75**	**1.5**	**0.1078(0.0117)**	**0.135(0.0183)**	**0.121(0.0147)**	**0.135(0.0185)**	**0.122(0.0151)**
				1.75	**2.5**	**0.2094(0.0439)**	**0.120(0.0146)**	**0.108(0.0119)**	**0.119(0.0143)**	**0.110(0.0122)**
			2	**0.75**	**1.5**	**0.1085(0.0118)**	**0.304(0.0923)**	**0.276(0.0761)**	**0.301(0.0910)**	**0.281(0.0788)**
				1.75	**2.5**	**0.2069(0.0429)**	**0.294(0.0868)**	**0.271(0.0733)**	**0.289(0.0839)**	**0.273(0.0748)**
		34	**1**	**0.75**	**1.5**	**0.1036(0.0108)**	**0.138(0.0192)**	**0.128(0.0165)**	**0.138(0.0192)**	**0.129(0.0168)**
				1.75	**2.5**	**0.2064(0.0427)**	**0.117(0.0139)**	**0.109(0.0121)**	**0.117(0.0139)**	**0.111(0.0124)**
			2	**0.75**	**1.5**	**0.1073(0.0115)**	**0.299(0.0896)**	**0.281(0.0789)**	**0.299(0.0893)**	**0.284(0.0809)**
				1.75	**2.5**	**0.2072(0.0430)**	**0.286(0.0824)**	**0.271(0.0736)**	**0.285(0.0815)**	**0.274(0.0752)**

Table 4: The average bias (AVB) and the mean squared error (MSE) in parentheses for the Burr-XII parameter α using the integrable and Bayes methods with m = (n/2 and 3n/4) and k=(m/2 and 3m/4) at T=2 and $\delta = 2$ for LINEX loss function.

n	m	K	α	β	γ	Integrable Method	Gamma Prior		Kernel Prior	
							SQEL	LNXL	SQEL	LNXL
20	**10**	**5**	**1**	**0.75**	**1.5**	**0.1057(0.0113)**	**0.167(0.0290)**	**0.119(0.0149)**	**0.164(0.0276)**	**0.122(0.0155)**
				1.75	**2.5**	**0.2158(0.0544)**	**0.151(0.0247)**	**0.105(0.0117)**	**0.133(0.0184)**	**0.108(0.0122)**
			2	**0.75**	**1.5**	**0.1363(0.8098)**	**0.371(0.1387)**	**0.273(0.0751)**	**0.342(0.1183)**	**0.282(0.0797)**
				1.75	**2.5**	**0.2082(0.0434)**	**0.378(0.1486)**	**0.270(0.0732)**	**0.316(0.1014)**	**0.275(0.0757)**
		8	**1**	**0.75**	**1.5**	**0.1157(0.0515)**	**0.160(0.0264)**	**0.123(0.0156)**	**0.159(0.0256)**	**0.126(0.0163)**
				1.75	**2.5**	**0.2295(0.3439)**	**0.139(0.0205)**	**0.106(0.0119)**	**0.131(0.0179)**	**0.109(0.0124)**
			2	**0.75**	**1.5**	**0.1082(0.0118)**	**0.351(0.1240)**	**0.276(0.0766)**	**0.333(0.1109)**	**0.284(0.0807)**
				1.75	**2.5**	**0.2078(0.0433)**	**0.338(0.1152)**	**0.271(0.0734)**	**0.311(0.0967)**	**0.276(0.0760)**
		8	**1**	**0.75**	**1.5**	**0.1072(0.0126)**	**0.160(0.0288)**	**0.122(0.0155)**	**0.157(0.0253)**	**0.125(0.0161)**
				1.75	**2.5**	**0.2111(0.0478)**	**0.139(0.0202)**	**0.107(0.0120)**	**0.131(0.0179)**	**0.109(0.0125)**
			2	**0.75**	**1.5**	**0.1082(0.0120)**	**0.352(0.1257)**	**0.276(0.0764)**	**0.332(0.1104)**	**0.284(0.0806)**

	15			1.75	2.5	0.2080(0.0434)	0.336(0.1139)	0.270(0.0731)	0.310(0.0961)	0.275(0.0758)
		11	1	0.75	1.5	0.1041(0.0115)	0.155(0.0245)	0.127(0.0165)	0.155(0.0246)	0.129(0.0170)
				1.75	2.5	0.2199(0.1371)	0.133(0.0188)	0.110(0.0125)	0.130(0.0172)	0.112(0.0129)
			2	0.75	1.5	0.1086(0.0141)	0.338(0.1169)	0.281(0.0789)	0.325(0.1056)	0.286(0.0821)
				1.75	2.5	0.2086(0.0436)	0.317(0.1030)	0.271(0.0736)	0.305(0.0931)	0.276(0.0764)
40	20	10	1	0.75	1.5	0.1049(0.0111)	0.142(0.0206)	0.121(0.0149)	0.143(0.0209)	0.123(0.0154)
				1.75	2.5	0.2083(0.0435)	0.127(0.0167)	0.107(0.0117)	0.122(0.0153)	0.108(0.0120)
			2	0.75	1.5	0.1078(0.0117)	0.322(0.1042)	0.275(0.0755)	0.314(0.0988)	0.281(0.0789)
				1.75	2.5	0.2076(0.0432)	0.316(0.1006)	0.271(0.0735)	0.298(0.0891)	0.274(0.0752)
		15	1	0.75	1.5	0.1051(0.0111)	0.142(0.0205)	0.121(0.0150)	0.142(0.0206)	0.123(0.0155)
				1.75	2.5	0.2083(0.0435)	0.127(0.0171)	0.108(0.0119)	0.123(0.0153)	0.110(0.0123)
			2	0.75	1.5	0.1077(0.0116)	0.316(0.1002)	0.275(0.0758)	0.311(0.0968)	0.281(0.0791)
				1.75	2.5	0.2079(0.0433)	0.308(0.0948)	0.271(0.0733)	0.296(0.0875)	0.274(0.0751)
	30	15	1	0.75	1.5	0.1042(0.0109)	0.141(0.0202)	0.121(0.0150)	0.141(0.0202)	0.123(0.0155)
				1.75	2.5	0.2077(0.0432)	0.124(0.0158)	0.107(0.0117)	0.121(0.0149)	0.109(0.0120)
			2	0.75	1.5	0.1073(0.0116)	0.318(0.1057)	0.276(0.0765)	0.312(0.0986)	0.282(0.0796)
				1.75	2.5	0.2075(0.0431)	0.306(0.0949)	0.270(0.0732)	0.294(0.0868)	0.274(0.0750)
		23	1	0.75	1.5	0.1037(0.0108)	0.142(0.0205)	0.128(0.0167)	0.143(0.0207)	0.130(0.0170)
				1.75	2.5	0.2067(0.0428)	0.121(0.0149)	0.109(0.0121)	0.120(0.0146)	0.111(0.0124)
			2	0.75	1.5	0.1068(0.0114)	0.308(0.0953)	0.281(0.0791)	0.354(0.7658)	0.285(0.0814)
				1.75	2.5	0.2076(0.0432)	0.294(0.0874)	0.272(0.0738)	0.291(0.0846)	0.275(0.0757)
60	30	15	1	0.75	1.5	0.1052(0.0111)	0.134(0.0182)	0.122(0.0150)	0.135(0.0185)	0.124(0.0154)
				1.75	2.5	0.2075(0.0431)	0.118(0.0143)	0.107(0.0117)	0.117(0.0139)	0.109(0.0119)
			2	0.75	1.5	0.1076(0.0116)	0.301(0.0905)	0.276(0.0761)	0.300(0.0902)	0.281(0.0788)
				1.75	2.5	0.2074(0.0430)	0.292(0.0851)	0.270(0.0730)	0.287(0.0826)	0.273(0.0745)
		23	1	0.75	1.5	0.1054(0.0112)	0.134(0.0183)	0.122(0.0150)	0.135(0.0185)	0.123(0.0154)
				1.75	2.5	0.2078(0.0432)	0.120(0.0147)	0.109(0.0119)	0.118(0.0142)	0.110(0.0122)
			2	0.75	1.5	0.1080(0.0117)	0.302(0.0912)	0.275(0.0759)	0.301(0.0905)	0.280(0.0786)
				1.75	2.5	0.2072(0.0430)	0.293(0.0864)	0.271(0.0734)	0.289(0.0833)	0.274(0.0749)
	45	23	1	0.75	1.5	0.1037(0.0108)	0.135(0.0183)	0.122(0.0151)	0.135(0.0185)	0.124(0.0155)
				1.75	2.5	0.2067(0.0428)	0.120(0.0148)	0.108(0.0119)	0.119(0.0144)	0.110(0.0122)
			2	0.75	1.5	0.1073(0.0116)	0.302(0.0916)	0.276(0.0763)	0.300(0.0903)	0.281(0.0789)
				1.75	2.5	0.2076(0.0431)	0.293(0.0860)	0.271(0.0732)	0.288(0.0828)	0.273(0.0747)
		34	1	0.75	1.5	0.1034(0.0107)	0.137(0.0188)	0.128(0.0164)	0.138(0.0192)	0.129(0.0167)
				1.75	2.5	0.2067(0.0428)	0.117(0.0139)	0.109(0.0121)	0.117(0.0139)	0.111(0.0124)
			2	0.75	1.5	0.1071(0.0115)	0.299(0.0893)	0.281(0.0790)	0.299(0.0895)	0.284(0.0809)
				1.75	2.5	0.2071(0.0429)	0.287(0.0829)	0.272(0.0738)	0.286(0.0816)	0.274(0.0753)

Table 5: The average bias (AVB) and mean squared error (RMSE) in parentheses for the Burr-XII parameter β using the integrable and Bayes methods with m = (n/2 and 3n/4) and k=(m/2 and 3m/4) at T=0.75 and $\delta = 2$ for LINEX loss.

n	m	K	α	β	γ	Integrable Method	Gamma Prior		Kernel Prior	
							SQEL	LNXL	SQEL	LNXL
20	10	5	1	0.75	1.5	0.1551(0.0241)	0.293(0.2442)	0.126(0.0158)	0.240(0.0738)	0.124(0.0154)
				1.75	2.5	0.1595(0.0254)	0.741(0.8654)	0.388(0.1507)	0.483(0.2466)	0.335(0.1125)
			2	0.75	1.5	0.4109(0.1695)	0.269(0.1475)	0.119(0.0142)	0.245(0.1296)	0.118(0.0139)
				1.75	2.5	0.4340(0.1884)	0.743(0.6574)	0.297(0.0882)	0.448(0.2302)	0.281(0.0789)
		8	1	0.75	1.5	0.1638(0.0268)	0.246(0.1242)	0.121(0.0147)	0.209(0.0529)	0.120(0.0144)
				1.75	2.5	0.1664(0.0277)	0.643(0.8997)	0.346(0.1197)	0.425(0.1881)	0.313(0.0983)
			2	0.75	1.5	0.4369(0.1909)	0.262(0.2381)	0.118(0.0138)	0.207(0.0655)	0.117(0.0137)
				1.75	2.5	0.4420(0.1954)	0.612(0.9324)	0.289(0.0835)	0.392(0.1648)	0.278(0.0772)
	15	8	1	0.75	1.5	0.1637(0.0268)	0.249(0.1277)	0.121(0.0147)	0.207(0.0669)	0.120(0.0145)
				1.75	2.5	0.1665(0.0277)	0.681(0.8943)	0.346(0.1196)	0.425(0.1880)	0.314(0.0983)
			2	0.75	1.5	0.4369(0.1909)	0.248(0.2368)	0.118(0.0138)	0.200(0.0490)	0.117(0.0137)
				1.75	2.5	0.4419(0.1953)	0.3462(0.7658)	0.289(0.0836)	0.392(0.1633)	0.278(0.0773)
		11	1	0.75	1.5	0.1698(0.0288)	0.226(0.0917)	0.119(0.0143)	0.189(0.0520)	0.119(0.0141)
				1.75	2.5	0.1714(0.0294)	0.596(0.6758)	0.328(0.1076)	0.389(0.1547)	0.304(0.0922)
			2	0.75	1.5	0.4449(0.1980)	0.223(0.1241)	0.117(0.0137)	0.184(0.0500)	0.116(0.0136)

				1.75	2.5	0.4484(0.2010)	0.596(0.7684)	0.285(0.0812)	0.365(0.1353)	0.276(0.0763)
40	20	10	1	0.75	1.5	0.1564(0.0245)	0.236(0.1342)	0.125(0.0157)	0.205(0.0559)	0.124(0.0154)
				1.75	2.5	0.1604(0.0257)	0.612(0.6175)	0.387(0.1499)	0.451(0.2099)	0.348(0.1208)
			2	0.75	1.5	0.4277(0.1829)	0.254(0.3152)	0.119(0.0141)	0.213(0.0941)	0.118(0.0140)
				1.75	2.5	0.4349(0.1891)	0.585(0.9723)	0.297(0.0880)	0.426(0.4693)	0.285(0.0810)
		15	1	0.75	1.5	0.1622(0.0263)	0.205(0.1003)	0.122(0.0148)	0.185(0.0873)	0.121(0.0146)
				1.75	2.5	0.1653(0.0273)	0.554(0.5182)	0.349(0.1221)	0.402(0.1633)	0.324(0.1049)
			2	0.75	1.5	0.4350(0.1892)	0.209(0.1035)	0.118(0.0139)	0.176(0.0576)	0.117(0.0138)
				1.75	2.5	0.4405(0.1940)	0.558(0.6498)	0.289(0.0838)	0.365(0.1380)	0.281(0.0789)
	30	15	1	0.75	1.5	0.1621(0.0263)	0.228(0.4255)	0.122(0.0148)	0.173(0.0329)	0.121(0.0146)
				1.75	2.5	0.1652(0.0273)	0.556(0.6224)	0.349(0.1221)	0.402(0.1641)	0.324(0.1049)
			2	0.75	1.5	0.4349(0.1892)	0.245(0.6574)	0.118(0.0138)	0.173(0.0337)	0.117(0.0137)
				1.75	2.5	0.4405(0.1941)	0.607(0.8794)	0.289(0.0837)	0.364(0.1355)	0.281(0.0789)
		23	1	0.75	1.5	0.1704(0.0291)	0.192(0.1094)	0.119(0.0142)	0.154(0.0253)	0.119(0.0141)
				1.75	2.5	0.1721(0.0296)	0.464(0.2937)	0.325(0.1054)	0.363(0.1359)	0.308(0.0947)
			2	0.75	1.5	0.4454(0.1984)	0.173(0.0470)	0.117(0.0136)	0.153(0.0275)	0.116(0.0136)
				1.75	2.5	0.4491(0.2017)	0.478(0.5447)	0.284(0.0808)	0.336(0.1138)	0.278(0.0772)
60	30	15	1	0.75	1.5	0.1610(0.0259)	0.257(0.5643)	0.125(0.0157)	0.180(0.0415)	0.124(0.0155)
				1.75	2.5	0.1633(0.0267)	0.565(0.6064)	0.387(0.1497)	0.437(0.1950)	0.354(0.1252)
			2	0.75	1.5	0.4320(0.1866)	0.232(0.2433)	0.119(0.0141)	0.186(0.0619)	0.118(0.0140)
				1.75	2.5	0.4382(0.1920)	0.538(0.9512)	0.296(0.0879)	0.388(0.1703)	0.286(0.0819)
		23	1	0.75	1.5	0.1627(0.0265)	0.191(0.1506)	0.121(0.0147)	0.164(0.0425)	0.121(0.0146)
				1.75	2.5	0.1655(0.0274)	0.513(0.4854)	0.348(0.1208)	0.385(0.1487)	0.327(0.1069)
			2	0.75	1.5	0.4355(0.1897)	0.183(0.0515)	0.118(0.0138)	0.161(0.0336)	0.117(0.0138)
				1.75	2.5	0.4409(0.1944)	0.461(0.3567)	0.289(0.0835)	0.347(0.1217)	0.282(0.0795)
	45	23	1	0.75	1.5	0.1625(0.0264)	0.182(0.0991)	0.121(0.0147)	0.163(0.0350)	0.121(0.0146)
				1.75	2.5	0.1655(0.0274)	0.499(0.4124)	0.348(0.1208)	0.388(0.1543)	0.327(0.1069)
			2	0.75	1.5	0.4355(0.1897)	0.191(0.0992)	0.118(0.0138)	0.163(0.0674)	0.117(0.0138)
				1.75	2.5	0.4408(0.1943)	0.538(0.8954)	0.289(0.0835)	0.351(0.1305)	0.282(0.0795)
		34	1	0.75	1.5	0.1703(0.0290)	0.157(0.0279)	0.119(0.0142)	0.145(0.0220)	0.119(0.0141)
				1.75	2.5	0.1717(0.0295)	0.454(0.6220)	0.325(0.1057)	0.350(0.1230)	0.311(0.0967)
			2	0.75	1.5	0.4453(0.1983)	0.154(0.0259)	0.117(0.0136)	0.141(0.0203)	0.117(0.0136)
				1.75	2.5	0.4487(0.2013)	0.456(0.5056)	0.284(0.0807)	0.324(0.1061)	0.279(0.0778)

Table 6: The average bias (AVB) and mean squared errors (MSE) in parentheses for the Burr-XII parameter β using the integrable and Bayes methods with m = (n/2 and 3n/4) and k=(m/2 and 3m/4) at T=2 and $\delta = 2$ for LINEX loss function.

n	M	K	α	β	γ	Integrable Method	Gamma Prior		Kernel Prior	
							SQEL	LNXL	SQEL	LNXL
20	10	5	1	0.75	1.5	0.1662(0.0276)	0.277(0.8922)	0.124(0.0153)	0.235(0.0939)	0.122(0.0149)
				1.75	2.5	0.1684(0.0284)	0.680(0.8139)	0.370(0.1381)	0.460(0.2290)	0.326(0.1063)
			2	0.75	1.5	0.4400(0.1936)	0.301(0.6570)	0.118(0.0140)	0.228(0.0851)	0.118(0.0138)
				1.75	2.5	0.4445(0.1976)	0.644(0.9926)	0.294(0.0864)	0.420(0.1904)	0.280(0.0783)
		8	1	0.75	1.5	0.1656(0.0274)	0.246(0.1193)	0.121(0.0147)	0.209(0.0733)	0.120(0.0144)
				1.75	2.5	0.1665(0.0277)	0.637(0.8604)	0.346(0.1195)	0.422(0.1824)	0.313(0.0983)
			2	0.75	1.5	0.4368(0.1908)	0.244(0.1515)	0.118(0.0139)	0.211(0.0815)	0.117(0.0137)
				1.75	2.5	0.4422(0.1956)	0.637(0.6843)	0.289(0.0836)	0.421(0.6311)	0.278(0.0773)
	15	8	1	0.75	1.5	0.1661(0.0276)	0.238(0.1090)	0.121(0.0147)	0.206(0.0728)	0.120(0.0144)
				1.75	2.5	0.1684(0.0284)	0.620(0.8569)	0.346(0.1195)	0.424(0.1851)	0.313(0.0982)
			2	0.75	1.5	0.4395(0.1932)	0.245(0.1165)	0.118(0.0138)	0.204(0.0567)	0.117(0.0137)
				1.75	2.5	0.4444(0.1975)	0.704(0.8943)	0.289(0.0834)	0.391(0.1599)	0.278(0.0772)
		11	1	0.75	1.5	0.1698(0.0288)	0.222(0.0796)	0.120(0.0143)	0.188(0.0418)	0.119(0.0141)
				1.75	2.5	0.1715(0.0294)	0.597(0.7394)	0.327(0.1071)	0.387(0.1507)	0.303(0.0921)
			2	0.75	1.5	0.4448(0.1979)	0.228(0.1290)	0.117(0.0137)	0.182(0.0383)	0.116(0.0136)
				1.75	2.5	0.4484(0.2011)	0.574(0.8885)	0.285(0.0812)	0.364(0.1348)	0.276(0.0762)
			1	0.75	1.5	0.1681(0.0283)	0.207(0.0720)	0.122(0.0148)	0.179(0.0442)	0.121(0.0147)
				1.75	2.5	0.1700(0.0289)	0.568(0.5306)	0.361(0.1304)	0.422(0.1874)	0.331(0.1098)

40	20	10	2	0.75	1.5	0.4423(0.1956)	0.225(0.1065)	0.118(0.0139)	0.177(0.0345)	0.118(0.0138)
				1.75	2.5	0.4463(0.1992)	0.555(0.7568)	0.292(0.0852)	0.383(0.1540)	0.282(0.0796)
		15	1	0.75	1.5	0.1680(0.0282)	0.206(0.0916)	0.121(0.0147)	0.173(0.0341)	0.121(0.0146)
				1.75	2.5	0.1700(0.0289)	0.556(0.5927)	0.350(0.1222)	0.403(0.1653)	0.324(0.1049)
			2	0.75	1.5	0.4423(0.1957)	0.207(0.0858)	0.118(0.0138)	0.186(0.0961)	0.117(0.0137)
				1.75	2.5	0.4464(0.1993)	0.639(0.8954)	0.289(0.0838)	0.367(0.1409)	0.281(0.0789)
	30	15	1	0.75	1.5	0.1699(0.0289)	0.225(0.2787)	0.121(0.0147)	0.173(0.0353)	0.120(0.0145)
				1.75	2.5	0.1707(0.0292)	0.566(0.9381)	0.345(0.1189)	0.398(0.1812)	0.321(0.1031)
			2	0.75	1.5	0.4435(0.1967)	0.197(0.0692)	0.118(0.0138)	0.175(0.0507)	0.117(0.0137)
				1.75	2.5	0.4474(0.2001)	0.561(0.8331)	0.288(0.0831)	0.357(0.1301)	0.280(0.0786)
		23	1	0.75	1.5	0.1707(0.0291)	0.170(0.0355)	0.119(0.0142)	0.157(0.0349)	0.119(0.0141)
				1.75	2.5	0.1721(0.0296)	0.530(0.8932)	0.325(0.1054)	0.359(0.1297)	0.308(0.0948)
			2	0.75	1.5	0.4457(0.1987)	0.185(0.1333)	0.117(0.0136)	0.156(0.0298)	0.116(0.0136)
				1.75	2.5	0.4492(0.2018)	0.487(0.8768)	0.284(0.0808)	0.337(0.1181)	0.278(0.0772)
60	30	15	1	0.75	1.5	0.1681(0.0282)	0.174(0.0452)	0.121(0.0146)	0.155(0.0258)	0.120(0.0145)
				1.75	2.5	0.1700(0.0289)	0.484(0.4113)	0.342(0.1168)	0.378(0.1453)	0.323(0.1043)
			2	0.75	1.5	0.4423(0.1956)	0.179(0.0874)	0.117(0.0138)	0.156(0.0321)	0.117(0.0137)
				1.75	2.5	0.4463(0.1992)	0.529(0.7843)	0.288(0.0827)	0.344(0.1258)	0.281(0.0791)
		23	1	0.75	1.5	0.1681(0.0282)	0.174(0.0452)	0.121(0.0146)	0.153(0.0244)	0.120(0.0145)
				1.75	2.5	0.1700(0.0289)	0.481(0.3270)	0.345(0.1187)	0.382(0.1474)	0.325(0.1055)
			2	0.75	1.5	0.4423(0.1956)	0.180(0.1042)	0.117(0.0138)	0.154(0.0304)	0.117(0.0137)
				1.75	2.5	0.4463(0.1992)	0.507(0.9330)	0.288(0.0832)	0.343(0.1193)	0.282(0.0793)
	45	23	1	0.75	1.5	0.1707(0.0291)	0.180(0.0590)	0.121(0.0146)	0.154(0.0249)	0.120(0.0144)
				1.75	2.5	0.1714(0.0294)	0.456(0.2632)	0.342(0.1168)	0.376(0.1433)	0.323(0.1043)
			2	0.75	1.5	0.4446(0.1977)	0.182(0.0827)	0.117(0.0138)	0.155(0.0317)	0.117(0.0137)
				1.75	2.5	0.4483(0.2009)	0.517(0.8943)	0.288(0.0828)	0.389(0.7843)	0.281(0.0791)
		34	1	0.75	1.5	0.1706(0.0291)	0.151(0.0251)	0.119(0.0142)	0.143(0.0212)	0.119(0.0141)
				1.75	2.5	0.1719(0.0295)	0.482(0.9996)	0.325(0.1057)	0.351(0.1231)	0.311(0.0967)
			2	0.75	1.5	0.4453(0.1983)	0.153(0.0264)	0.117(0.0136)	0.142(0.0205)	0.117(0.0136)
				1.75	2.5	0.4489(0.2015)	0.445(0.4512)	0.284(0.0807)	0.324(0.1058)	0.279(0.0778)

Table 7: The Average bias (AVB) and mean square error (RMSE) in parentheses for the Burr-XII parameter γ using the integrable and Bayes methods with m = (n/2 and 3n/4) and k=(m/2 and 3m/4) at T=0.75 and $\delta = 2$ for LINEX loss function.

N	m	N	α	β	γ	Integrable Method	Gamma Prior		Kernel Prior	
							SQEL	LNXL	SQEL	LNXL
20	10	5	1	0.75	1.5	0.1714(0.0294)	0.530(0.9134)	0.243(0.0592)	0.405(0.3557)	0.237(0.0562)
				1.75	2.5	0.2920(0.0853)	0.573(0.6784)	0.510(0.2599)	0.547(0.3014)	0.433(0.1875)
			2	0.75	1.5	0.1701(0.0289)	0.491(0.5028)	0.241(0.0583)	0.382(0.2088)	0.236(0.0555)
				1.75	2.5	0.2912(0.0848)	0.564(0.7843)	0.442(0.1950)	0.525(0.2900)	0.402(0.1619)
		8	1	0.75	1.5	0.1736(0.0301)	0.437(0.5634)	0.236(0.0555)	0.337(0.1284)	0.233(0.0541)
				1.75	2.5	0.2939(0.0864)	0.735(0.9694)	0.456(0.2080)	0.506(0.2637)	0.414(0.1718)
			2	0.75	1.5	0.1707(0.0292)	0.423(0.2895)	0.236(0.0557)	0.345(0.1717)	0.233(0.0542)
				1.75	2.5	0.2917(0.0851)	0.837(0.9843)	0.421(0.1773)	0.489(0.2451)	0.397(0.1575)
	15	8	1	0.75	1.5	0.1737(0.0302)	0.446(0.5385)	0.236(0.0556)	0.337(0.1381)	0.233(0.0541)
				1.75	2.5	0.2939(0.0864)	0.761(0.8943)	0.456(0.2083)	0.516(0.4031)	0.415(0.1719)
			2	0.75	1.5	0.1707(0.0291)	0.438(0.6345)	0.236(0.0557)	0.342(0.1642)	0.233(0.0542)
				1.75	2.5	0.2917(0.0851)	0.801(0.7864)	0.421(0.1774)	0.487(0.2388)	0.397(0.1576)
		11	1	0.75	1.5	0.1783(0.0318)	0.386(0.2833)	0.232(0.0540)	0.313(0.1314)	0.230(0.0531)
				1.75	2.5	0.2973(0.0884)	0.665(0.6776)	0.432(0.1869)	0.472(0.2250)	0.405(0.1643)
			2	0.75	1.5	0.1719(0.0295)	0.427(0.6758)	0.233(0.0543)	0.310(0.1072)	0.231(0.0534)
				1.75	2.5	0.2926(0.0856)	0.715(0.8943)	0.410(0.1677)	0.463(0.2154)	0.393(0.1544)
		10	1	0.75	1.5	0.1714(0.0294)	0.396(0.2733)	0.243(0.0589)	0.340(0.1380)	0.238(0.0567)
				1.75	2.5	0.2920(0.0853)	0.818(0.9673)	0.510(0.2599)	0.530(0.2827)	0.447(0.1998)
			2	0.75	1.5	0.1701(0.0289)	0.434(0.6182)	0.242(0.0584)	0.345(0.2221)	0.237(0.0563)
				1.75	2.5	0.2912(0.0848)	0.768(0.8546)	0.443(0.1960)	0.506(0.2662)	0.410(0.1680)

40	20	15	1	0.75	1.5	0.1728(0.0299)	0.374(0.4089)	0.236(0.0559)	0.314(0.1220)	0.234(0.0547)
				1.75	2.5	0.2932(0.0859)	0.774(0.8943)	0.462(0.2133)	0.491(0.2434)	0.426(0.1811)
			2	0.75	1.5	0.1705(0.0291)	0.361(0.2206)	0.237(0.0561)	0.303(0.1019)	0.234(0.0549)
				1.75	2.5	0.2915(0.0850)	0.673(0.8214)	0.424(0.1797)	0.475(0.2411)	0.403(0.1623)
	30	15	1	0.75	1.5	0.1727(0.0298)	0.380(0.3225)	0.236(0.0559)	0.316(0.2080)	0.234(0.0548)
				1.75	2.5	0.2931(0.0859)	0.689(0.8275)	0.462(0.2134)	0.507(0.6094)	0.426(0.1811)
			2	0.75	1.5	0.1705(0.0291)	0.370(0.2492)	0.237(0.0561)	0.310(0.1680)	0.234(0.0549)
				1.75	2.5	0.2915(0.0850)	0.693(0.7684)	0.424(0.1796)	0.473(0.2305)	0.403(0.1623)
		23	1	0.75	1.5	0.1783(0.0318)	0.301(0.1011)	0.232(0.0538)	0.276(0.0813)	0.231(0.0532)
				1.75	2.5	0.2975(0.0885)	0.578(0.4972)	0.429(0.1842)	0.452(0.2048)	0.409(0.1676)
			2	0.75	1.5	0.1719(0.0295)	0.327(0.2945)	0.233(0.0542)	0.271(0.0752)	0.231(0.0535)
				1.75	2.5	0.2927(0.0857)	0.585(0.5602)	0.408(0.1667)	0.441(0.1951)	0.396(0.1568)
60	30	15	1	0.75	1.5	0.1724(0.0297)	0.367(0.1931)	0.243(0.0591)	0.311(0.1229)	0.239(0.0572)
				1.75	2.5	0.2926(0.0856)	0.727(0.8832)	0.509(0.2596)	0.526(0.2813)	0.455(0.2070)
			2	0.75	1.5	0.1703(0.0290)	0.416(0.4854)	0.242(0.0584)	0.321(0.1390)	0.238(0.0567)
				1.75	2.5	0.2913(0.0849)	0.730(0.7563)	0.443(0.1960)	0.487(0.2419)	0.414(0.1713)
		23	1	0.75	1.5	0.1728(0.0299)	0.321(0.1539)	0.236(0.0558)	0.285(0.0969)	0.234(0.0549)
				1.75	2.5	0.2932(0.0859)	0.616(0.6121)	0.460(0.2116)	0.495(0.5368)	0.430(0.1846)
			2	0.75	1.5	0.1705(0.0291)	0.323(0.1361)	0.237(0.0560)	0.291(0.0984)	0.235(0.0550)
				1.75	2.5	0.2915(0.0850)	0.825(0.6573)	0.423(0.1791)	0.459(0.2114)	0.405(0.1644)
	45	23	1	0.75	1.5	0.1727(0.0298)	0.313(0.1133)	0.236(0.0558)	0.283(0.0849)	0.234(0.0549)
				1.75	2.5	0.2931(0.0859)	0.632(0.7159)	0.460(0.2114)	0.480(0.2330)	0.429(0.1845)
			2	0.75	1.5	0.1705(0.0291)	0.352(0.2695)	0.237(0.0560)	0.284(0.0853)	0.235(0.0550)
				1.75	2.5	0.2915(0.0850)	0.613(0.6896)	0.423(0.1790)	0.459(0.2148)	0.405(0.1643)
		34	1	0.75	1.5	0.1777(0.0316)	0.282(0.0870)	0.232(0.0538)	0.263(0.0718)	0.231(0.0533)
				1.75	2.5	0.2968(0.0881)	0.563(0.8452)	0.430(0.1851)	0.447(0.2000)	0.413(0.1706)
			2	0.75	1.5	0.1718(0.0295)	0.285(0.0930)	0.233(0.0542)	0.263(0.0723)	0.232(0.0537)
				1.75	2.5	0.2925(0.0856)	0.540(0.4336)	0.409(0.1671)	0.434(0.1896)	0.398(0.1584)

Table 8: The Average bias (AVB) and mean square error (MSE) in parentheses for the Burr-XII parameter γ using the integrable and Bayes methods with m = (n/2 and 3n/4) and k=(m/2 and 3m/4) at T=2 and $\delta = 2$ for LINEX loss function.

n	m	K	α	β	γ	Integrable Method	Gamma Prior		Kernel Prior	
							SQEL	LNXL	SQEL	LNXL
20	10	5	1	0.75	1.5	0.1750(0.0306)	0.452(0.4017)	0.240(0.0575)	0.365(0.1709)	0.235(0.0552)
				1.75	2.5	0.2946(0.0868)	0.896(0.7685)	0.486(0.2366)	0.527(0.2811)	0.425(0.1808)
			2	0.75	1.5	0.1710(0.0292)	0.483(0.8004)	0.239(0.0572)	0.370(0.2018)	0.235(0.0550)
				1.75	2.5	0.2919(0.0852)	0.995(0.8796)	0.433(0.1878)	0.514(0.2740)	0.400(0.1603)
		8	1	0.75	1.5	0.1747(0.0305)	0.601(0.8976)	0.235(0.0555)	0.344(0.1629)	0.232(0.0540)
				1.75	2.5	0.2937(0.0862)	0.765(0.6785)	0.456(0.2081)	0.501(0.2533)	0.415(0.1718)
			2	0.75	1.5	0.1707(0.0291)	0.418(0.4050)	0.236(0.0557)	0.353(0.1829)	0.233(0.0542)
				1.75	2.5	0.2917(0.0851)	0.738(0.7854)	0.421(0.1772)	0.486(0.2395)	0.397(0.1575)
	15	8	1	0.75	1.5	0.1748(0.0306)	0.401(0.2422)	0.236(0.0555)	0.334(0.1586)	0.233(0.0541)
				1.75	2.5	0.2946(0.0868)	0.775(0.8954)	0.456(0.2080)	0.505(0.2579)	0.414(0.1718)
			2	0.75	1.5	0.1709(0.0292)	0.460(0.6486)	0.236(0.0557)	0.360(0.2648)	0.233(0.0542)
				1.75	2.5	0.2919(0.0852)	0.745(0.9087)	0.421(0.1771)	0.485(0.2388)	0.397(0.1575)
		11	1	0.75	1.5	0.1786(0.0319)	0.377(0.3869)	0.232(0.0540)	0.310(0.1085)	0.230(0.0531)
				1.75	2.5	0.2972(0.0883)	0.708(0.8954)	0.432(0.1868)	0.472(0.2230)	0.405(0.1642)
			2	0.75	1.5	0.1719(0.0295)	0.403(0.3543)	0.233(0.0543)	0.324(0.1663)	0.231(0.0534)
				1.75	2.5	0.2925(0.0855)	0.709(0.7854)	0.409(0.1676)	0.471(0.3042)	0.393(0.1544)
	20	10	1	0.75	1.5	0.1757(0.0309)	0.356(0.1780)	0.237(0.0562)	0.302(0.0984)	0.234(0.0549)
				1.75	2.5	0.2953(0.0872)	0.758(0.8943)	0.477(0.2273)	0.508(0.2817)	0.432(0.1870)
			2	0.75	1.5	0.1712(0.0293)	0.404(0.6054)	0.238(0.0568)	0.326(0.1828)	0.235(0.0554)
				1.75	2.5	0.2921(0.0853)	0.697(0.9381)	0.430(0.1851)	0.495(0.3252)	0.405(0.1644)
		15	1	0.75	1.5	0.1755(0.0308)	0.364(0.4275)	0.236(0.0557)	0.312(0.1553)	0.234(0.0546)
				1.75	2.5	0.2953(0.0872)	0.672(0.6537)	0.462(0.2133)	0.491(0.2430)	0.426(0.1811)
			2	0.75	1.5	0.1712(0.0293)	0.371(0.3149)	0.237(0.0561)	0.305(0.1039)	0.234(0.0548)

40				1.75	2.5	0.2920(0.0853)	0.679(0.8643)	0.424(0.1797)	0.475(0.2434)	0.403(0.1623)
	30	15	1	0.75	1.5	0.1774(0.0315)	0.358(0.4785)	0.235(0.0555)	0.304(0.1248)	0.233(0.0544)
				1.75	2.5	0.2958(0.0875)	0.703(0.8435)	0.456(0.2082)	0.482(0.2337)	0.423(0.1789)
			2	0.75	1.5	0.1714(0.0294)	0.358(0.1977)	0.236(0.0558)	0.301(0.1042)	0.234(0.0546)
				1.75	2.5	0.2922(0.0854)	0.627(0.5230)	0.421(0.1774)	0.464(0.2171)	0.402(0.1614)
		23	1	0.75	1.5	0.1786(0.0319)	0.321(0.3593)	0.232(0.0538)	0.275(0.0786)	0.231(0.0532)
				1.75	2.5	0.2973(0.0884)	0.565(0.4689)	0.430(0.1846)	0.453(0.2065)	0.410(0.1679)
			2	0.75	1.5	0.1719(0.0296)	0.303(0.1024)	0.233(0.0542)	0.275(0.0792)	0.231(0.0535)
				1.75	2.5	0.2926(0.0856)	0.573(0.5245)	0.408(0.1667)	0.444(0.2113)	0.396(0.1567)
60	30	15	1	0.75	1.5	0.1753(0.0307)	0.305(0.1060)	0.235(0.0553)	0.280(0.0892)	0.233(0.0545)
				1.75	2.5	0.2951(0.0871)	0.594(0.5217)	0.453(0.2049)	0.473(0.2264)	0.426(0.1812)
			2	0.75	1.5	0.1712(0.0293)	0.325(0.2149)	0.236(0.0556)	0.283(0.0914)	0.234(0.0547)
				1.75	2.5	0.2920(0.0853)	0.598(0.6201)	0.420(0.1761)	0.456(0.2140)	0.404(0.1629)
		23	1	0.75	1.5	0.1753(0.0307)	0.332(0.3457)	0.235(0.0552)	0.281(0.0882)	0.233(0.0544)
				1.75	2.5	0.2951(0.0871)	0.599(0.6540)	0.456(0.2078)	0.474(0.2255)	0.427(0.1826)
			2	0.75	1.5	0.1712(0.0293)	0.327(0.3408)	0.236(0.0557)	0.281(0.0988)	0.234(0.0548)
				1.75	2.5	0.2920(0.0853)	0.641(0.7648)	0.421(0.1776)	0.456(0.2087)	0.405(0.1637)
	45	23	1	0.75	1.5	0.1781(0.0317)	0.317(0.1225)	0.235(0.0552)	0.277(0.0794)	0.233(0.0544)
				1.75	2.5	0.2963(0.0878)	0.612(0.7994)	0.453(0.2048)	0.472(0.2245)	0.426(0.1811)
			2	0.75	1.5	0.1716(0.0294)	0.358(0.6537)	0.236(0.0556)	0.293(0.2257)	0.234(0.0547)
				1.75	2.5	0.2923(0.0855)	0.632(0.6573)	0.420(0.1762)	0.455(0.2090)	0.404(0.1630)
		34	1	0.75	1.5	0.1779(0.0317)	0.273(0.0768)	0.232(0.0538)	0.261(0.0712)	0.231(0.0534)
				1.75	2.5	0.2968(0.0881)	0.534(0.3298)	0.430(0.1851)	0.446(0.1986)	0.413(0.1706)
			2	0.75	1.5	0.1717(0.0295)	0.323(0.4563)	0.233(0.0542)	0.262(0.0690)	0.232(0.0537)
				1.75	2.5	0.2925(0.0855)	0.618(0.7853)	0.409(0.1671)	0.433(0.1875)	0.398(0.1584)