Effectiveness Of Melin Integral and Fractal Dimensional Integral in Capital Market Efficiencies

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Abstract: -Melin integral is an integral transform which has been proposed for many types of physical and economical system even in perpetual option pricing. In financial market price stability, we observed that it is weakly efficient. We herein presented harmonized dimensional integral transform as a generalized state of the Melin transform with the aim of testing for market efficiency.
The effect of our approach is that, the technique is stable and a strongly efficient model for capital market, having a significant impact on returns and taking account of issues of positive bias natures of investors which the Melin integral transform ignores. In addition, the model allows one to define an optimal positioning between a reduction in market and likelihood of a bankruptcy within the given time horizon.

Key words: Melin Transform, Fractal measure, Capital Market efficiency, Normal Gussian and Raydon Nicodyn.

1 Introduction

Efficiency refers only to historical information which is contained in every private trading agent's information set. In a financial market system, there are several reasons for using a stable model to describe an efficient market system. The first is where there are solid theoretical reasons for expecting a non-Gaussian model. Example, reflection of a rotating mirror yielding a Cauchy distribution, hitting times for a Brownian motion yielding a levy distribution, the gravitation field of stars yielding the Holtsmark distribution (Barndorff-Nielsen et al 2000/2001). The second reason is empirical because many large data sets exhibit heavy tails and skewness. The strong empirical evidence for these features is used by many to justify the use of stable and efficient models. Examples in finance and economics are given in Mandelbrot (1965), Esser (2003), McKean and Fama (1965), Franke et al (2004) and Carr et al (2000). Such data sets are poorly described by Normal Gaussian model, but can be well describe by a stable distribution.
The Melin transform has been successful in perpetual option pricing theory both in physical and economic system but the applicability to problem in modern finance theory have not been studied extensively yet suggested by Panini and Srivastav (2004). To this, it is observed that in complete financial market efficiency, it has no significant on interest rate fluctuation. In practice, random fluctuation of interest rate has a significant contribution to change of returns and even though interest rate fluctuates randomly in market, many models do not fully consider their bias natures owing to their generally limited impact on returns. However, in this paper, we show that harmonized dimension transform technique is an efficient financial market model having a significant impact on price returns as it takes account of issue of positive bias natures of investors.

2 Efficient and Inefficient Market

According to Fama (1970), financial market system is separated into standard and exotic market. Standard market is subject to certain regularity, standardization conditions and organized exchange while exotic have more complicated payoff traded in the over- the-counted-market and the leading players are the commercial and investment ba
The more complicated payoff, however, introduces much greater complexity to the valuation problem, thus demanding a sophisticated mathematical machinery. Efficiency refers to when the relevant information set is reflected in the market price and weakly efficient implies that successive price changes or returns are independently

distributed. Formally, let the market model be described by a probability triple $(\Omega, \mathcal{F}, \mathbb{P})$. Suppose that trading takes place in continuous time, and that there is one risky security.

Let $h > 0, t \geq 0$ and let $rh(t + h)$ denote the return of security from t to $t + h$, where h is a fixed time lag and let $S(t)$ be the price of the risky security at time t. Also let $\mathcal{F}(t)$ be the

Then the market is **weakly efficient** based on the following axioms:

AXIOM 1:
   i) if $x\mathcal{F}(t), exist$ $\forall$ $x \in R$.
   ii) $ii)$ $\mathbb{P}[rh(t + h) \leq x\mathcal{F}(t)]$ $\forall$ $x \in R$. $h > 0, t \geq 0$.

Here, the information $\mathcal{F}(t)$ which is publicly available at time t is nothing other than the generated of the price returns with the random variable, were

$$r_h(t + h) = log \frac{S(t+h)}{S(t)}$$

is the rate of returns.

On the other hand, a market is **strongly efficient** if

AXIOM 2:
   i) if $x\mathcal{F}(t)$ $exist$ $\forall$ $x \in R$
   ii) $\mathbb{P}[rh(t + h) \geq x\mathcal{F}(t)]$ $\forall$ $x \in R$. $h > 0, t \geq 0$.

Note that Dimension is defined $\lim\limits_{size \to 0} \frac{log\, bulk}{log\, size}$ as which is equivalent to rate of return

$$r_h(t + h) = log \frac{S(t+h)}{S(t)}.$$

2.1 Financial Instruments

Financial instruments are defined in terms of other underlying quantities such as stock, indices, currency and interest rates or volatilities. Here $S_t = S(t)$ is the price of underlying asset, subscripts are used to emphasize on evolutions of underlying asset process through time and due to the time dependency, $h = T - t$ is the remaining time of maturity. S_T is normally distributed $(X \sim N(\mu, \sigma2)\,)$ if it has a density. Normal or Gaussian distribution if it has a density

$$S_T = \frac{1}{\sqrt{2\pi}\sigma} e^{-\left(\frac{(x-\mu)^2}{2\sigma^2}\right)}, \qquad (1)$$

$-\infty < x < \infty \Rightarrow S(x, \mu, \sigma, \pi)$

$$= \frac{1}{\sqrt{2\pi}\sigma} e^{-\left(\frac{in\, S_t^2}{4\theta}\right)},$$

where; $\ln S_t = x - \mu$, $\theta = 2$, and $\sigma^2 = 4$.

Take $S_T = F(x)$. The cumulative distribution function for which there is a close form expression is $F(x) = P(X \leq x)$,
where

$F(x)$ is the $probability$ $that$ a $standard$ $mormal$ $random$ $variable$ is $less$ $than$ or $equal$ to x.

For a given pay off function

$g = R^+ \to R$ written as S can be regarded as a financial instrument that pays the holder$g(S(T))$ at expiry T for all $S \in R^+$ then we take $f(x)$ to be

$$S_T = S_t e^{\left(\left(\mu - q - \frac{\sigma^2}{2}\right)(T-t) + \sigma(W_T - W_t)\right)}, \qquad (2)$$

for all $t \in [0, T]$,

where $T - t = h$ and $W_T - W_t = \beta$, so that equation (2) reduces to

$$S_T = S_t e^{\left(\left(\mu - q - \frac{\sigma^2}{2}\right)h + \sigma\beta\right)}, \qquad (3a)$$

for all $t \in [0, T] \Rightarrow S(\mu, \sigma, q, h, \beta)$.

Here, S(t) is the price of the risky security at time t, μ is the appreciation rate of price or riskless interest rate, q is the dividend yield, β is one dimensional Brownian motion and $\sigma > 0$ is the volatility and it is well known that the logarithm of S grows linearly in the long-run. The Radon Nicodyn derivative given as

$$S_T = S_t e^{-\frac{1}{2}\left(\left(\frac{(\mu-q)^2}{\sigma^2}\right)T - \frac{u-q}{\sigma}\beta\right)}, \qquad (3b)$$

for all $t \in [0, T] \Rightarrow S(\mu, \sigma, q, \beta, T)$.

2.2 The Melin Transform

In mathematics, the Melin transform is an integral transform that may be regarded as the multiplicative version of the two-sided laplace transform closely related to Dirichlet Series, Fourier transform, theory of Gamma and Allied Special Function and often used in number theory, Mathematical Statistics, and theory of asymptotic expansions.

Robert Hjalmar Melin (1854-1933) gave his name to the Melin transform that associate to the locally Lebesgue integral function $f(x)$ defined over positive real number and the complex function $M(f(x), w)$ defined by

$$F(x) = M(f(x), w) = f(w) \int_o^\infty f(x) x^{w-1} dx \qquad (4a)$$

Its probability density function is $f(x) = \frac{1}{\gamma(r)} X^{r-1} e^{-x}, r > 0$.

Where X^{r-1} is the Melin kernel and the expectation of the continuous random variable X is given by

$$E[X] = \int_{-\infty}^{\infty} XF(x)dx = 1 \quad (4b)$$

$$\int_{o}^{\infty} F(x)dx = \int_{o}^{\infty} \frac{1}{\gamma(r)} X^{r-1} e^{-x} dx$$

$$= \frac{1}{\gamma(r)} \int_{o}^{\infty} X^{r-1} e^{-x} dx = 1 \quad (4c)$$

The Melin transform is defined on a vertical strip in the w-plane, whose boundaries are determined by the asymptotic behavior of $f(x)$ as $x \to 0^+$ and $x \to \infty$ the largest strip (a, b) in which the integral converges is called the fundamental strip. The condition

$$f(x) = 0(x^u) for\ x \to 0^+$$

and

$$f(x) = 0(x^v) for\ x \to \infty$$

When $u > v$, guarantee the existence of $M(f(x), w)$ in the strip $(-u, -v)$. Thus, the existence is granted for locally integrable functions, whose exponent in the order of 0 is strictly larger than the exponent of order at infinity.

Conversely $F(x)$ is an integrable function with fundamental strip (a, b), then if c is such that $a < c < b\ and\ f(c + i\infty)$ is integrable, the equality

$$\frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} \tilde{f}(x)\, x^{-w} dw = f(x), \quad (4d)$$

Holds almost everywhere. Moreover, if $f(x)$ is continuous, then the equality holds everywhere on $(0, \infty)$. Simple changes of variables in definition of the Melin transforms yield to a whole set of transformation rules and facilitates the computations. In particular, if $f(x)$ admits the Melin transform on the strip (a, b) and α, β are positive reals, then the following relation holds.

$$M(f(\alpha x), w) = \alpha^{-w} \tilde{f}(w)\ on\ (a, b)$$

$$M(f(x^\alpha), w) = \tilde{f}(w + \alpha)\ on\ (a, b)$$

$$M(f(x^\alpha), w) = \frac{1}{\alpha} \tilde{f}(w/\alpha), \alpha > 0\ on\ (a\alpha, b\alpha).$$

$$M\left(f\left(\frac{1}{x}\right), w\right) = -\tilde{f}(-w)\ on\ (-b, a).$$

$$M(x^\beta f(x^\alpha), w) = \frac{1}{\alpha} \tilde{f}(w + \beta/\alpha)\ \alpha > 0\ on\ (a\alpha, b\alpha),$$

$$M\left(x \frac{d}{dx} f(x), w\right) = -\tilde{f}(w)\ on\ (a^*, b^*).$$

$$M\left(\frac{d}{dx} f(x), w\right) = -(w - 1)\tilde{f}(w - 1),$$
on $(a^* - 1, b^* - 1)$.

$$M\left(\frac{d^n}{dx^n} f(x), w\right) = (-1)^n \frac{\Gamma(w)s}{\Gamma(w-n)} \tilde{f}(w - n)\ on\ (a^* - n, b^* - n).$$

For a proof of some of these relations we refer to Titchmarsh (1986) and Sneddon (1972). The change of variables $x = e^s$ shows that the Melin transform is closely related to the Laplace-Fourier version. In particular, if $F(f(x), w)\ and\ L(f(x), w)$ denote the two-sided Fourier and Laplace transform, respectively, then we have

$$M(f(x), w) = L(f(e^{-x}), w) =$$
$$F(f(e^{-x}), -iw). \quad (5)$$

However, there are numerous applications where it proves to be more convenient to operate directly with the Melin rather than the Laplace and Fourier transform.

3 Application of Melin Transform and Radon Nikodyn Derivatives

According to Robert and Rainer (2008) application of Melin transform using the Gaussian distribution technique and the Radon Nikodyn derivatives, from their application it is confirmed that the result does represent an efficient market in the weak sense. The result is thus.

Given $\Phi(w) = xF(t)$ as the Melin transform of $\phi(S) = f(x)$ were

$$\phi(S) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\left(\frac{(In\ s)^2}{4\theta}\right)}$$

$$S_T = \frac{1}{\sqrt{2\pi\sigma}} e^{-\left(\frac{(x-\mu)^2}{2\sigma^2}\right)},$$

from equation (1).

Here, $In\ s = x - \mu, and\ \theta = 2, and\ \sigma^2 = 4,$.

$$\Phi(w) = \int_{o}^{\infty} \phi(S) S^{w-1} dS$$

from equation (4).

From there result, using the transformation in Erdelyi et. al. (1954).

$$\Phi(w) = \int_o^\infty \frac{1}{\sqrt{2\pi\sigma}} e^{-\left(\frac{(In\, s)^2}{4\theta}\right)} S^{w-1} dS,$$
$$Re(\theta) \geq 0. \qquad .$$

The result yield $\frac{S^{\beta_2}}{n\sigma\sqrt{2\pi(T-t)}} e^{-\frac{1}{2}\left(\frac{Ins}{n\sigma\sqrt{T-t}}\right)^2}$.

TESTING FOR EFFICIENCY: we can see that from the result

$$F(t) = \frac{S^{\beta_2}}{n\sigma\sqrt{2\pi(T-t)}} e^{-\frac{1}{2}\left(\frac{Ins}{n\sigma\sqrt{T-t}}\right)^2} \qquad (6)$$

and

$$r_h(t+h) = log\frac{S(t+h)}{S(t)} = -\frac{(In\, s)^2}{4\theta} \qquad (7)$$

$$-\frac{(In\, s)^2}{4\theta} \leq \left(\frac{S^{\beta_2}}{n\sigma\sqrt{2\pi(T-t)}} e^{-\frac{1}{2}\left(\frac{Ins}{n\sigma\sqrt{T-t}}\right)^2}\right)$$

.   Axiom 1 satisfied.

3.1 Weakly Efficient Market

In order to establish that a market is weakly efficient we have the following derivation:

a. From equation (3a) and (4a) we have

$$S_T = S_t e^{\left(\left(\mu-q-\frac{\sigma^2}{2}\right)h+\sigma\beta\right)}, for\ all\ t \in$$
$$[0,T] \Rightarrow S(\mu,\sigma,q,h,\beta) . \quad \text{From equation (3a).}$$

Where $f(x) = S_t e^{\left(\left(\mu-q-\frac{\sigma^2}{2}\right)h+\sigma\beta\right)}$ Equation (4a).

Let $x = \left(\left(\mu-q-\frac{\sigma^2}{2}\right)h+\sigma\beta\right)$, then $f(x) = S_t e^x$.

Then equation (4a) gives

$$f(X) = \int_o^\infty f(x)x^{w-1}dx$$
$$= \int_o^\infty S_t e^x x^{w-1}dx$$
$$= S_t \int_o^\infty e^x x^{w-1}dx$$
$$F(t) = \frac{S_t \tan\frac{\pi}{2}\left(\left(\mu-q-\frac{1}{2}\sigma^2\right)h+\sigma(\beta)\right)}{\left(\left(\mu-q-\frac{1}{2}\sigma^2\right)h+\sigma(\beta)\right)(w-1)}. \qquad (8)$$

$$r_h(t+h) = log\frac{S(t+h)}{S(t)}$$
$$= \left(\left(\mu-q-\frac{\sigma^2}{2}\right)h+\sigma\beta\right). \qquad (9)$$

Here, the information $\mathcal{F}(t)$ which is publicly available at time t is nothing other than the generated of the price returns.

We have

$$\left(\left(\mu-q-\frac{\sigma^2}{2}\right)h+\sigma\beta\right) \leq$$
$$x\left(\frac{S_t \tan\frac{\pi}{2}\left(\left(\mu-q-\frac{\sigma^2}{2}\right)h+\sigma\beta\right)}{\left(\left(\mu-q-\frac{\sigma^2}{2}\right)h+\sigma\beta\right)(w-1)}\right).$$

Axiom 1 satisfied

b. Using equation (3b) and (4a) we have

$$S_T = e^{-\frac{1}{2}\left(\left(\frac{(\mu-q)^2}{\sigma^2}\right)T-\frac{u-q}{\sigma}\beta\right)}$$
$$for\ all\ t \in [0,T] \Rightarrow S(\mu,\sigma,q,h,\beta) \qquad \text{from}$$
equation (3b),

$$f(X) = e^{-\frac{1}{2}\left(\left(\frac{(\mu-q)^2}{\sigma^2}\right)T-\frac{u-q}{\sigma}\beta\right)}$$

Let $x = \left(\left(\frac{(\mu-q)^2}{\sigma^2}\right)T-\frac{u-q}{\sigma}\beta\right)$

$$F(t) = \int_o^\infty f(x)x^{w-1}dx \quad \text{from equation (4a),}$$
$$= \int_o^\infty e^{-x}x^{w-1}dx$$

$$F(t) = \frac{\tan\frac{\pi}{2}\left(\frac{1}{2}\left(\frac{(\mu-q)^2}{\sigma^2}\right)T-\frac{u-q}{\sigma}\beta\right)}{\left(\frac{1}{2}\left(\frac{(\mu-q)^2}{\sigma^2}\right)T-\frac{u-q}{\sigma}\beta\right)(w-1)} \qquad (10).$$

$$r_h(t+h) = log\frac{S(t+h)}{S(t)}$$
$$= -\left(\frac{1}{2}\left(\frac{(\mu-q)^2}{\sigma^2}\right)T-\frac{u-q}{\sigma}\beta\right) \qquad (11) .$$

We have that

$$-\left(\frac{1}{2}\left(\frac{(\mu-q)^2}{\sigma^2}\right)T-\frac{u-q}{\sigma}\beta\right) \leq$$
$$x\left(\frac{\tan\frac{\pi}{2}\left(\frac{1}{2}\left(\frac{(\mu-q)^2}{\sigma^2}\right)T-\frac{u-q}{\sigma}\beta\right)}{\left(\frac{1}{2}\left(\frac{(\mu-q)^2}{\sigma^2}\right)T-\frac{u-q}{\sigma}\beta\right)(w-1)}\right) .$$

Axiom 1 satisfied.

3.2 The Harmonized Dimensional Transform

According to Ying and Lawrence (2008), the average fractal dimension is the Pth fractal distribution and the basis of fractal geometry is the idea of self-similarity, hence fractal dimension average all over the fractal properties. It is a natural dimensional measure relevant for dynamic system given as

$$\mu(\beta) = \lim_{T\to\infty}\frac{1}{T}\int_0^T I_\beta(\varphi_t(X_0))\,dt \Rightarrow$$
$$\frac{1}{\Delta\alpha}\int_{min}^{max} F(\alpha)\,d\alpha, \qquad (12)$$

by Zhang (2011), harmonized as

$$\Rightarrow F(t) = \lim_{T\to\infty}\frac{1}{T}\int_0^T I_\beta\left(\varphi_t(X_0)\right)^\lambda dt \Rightarrow$$

$$F(t) = \frac{1}{\Delta\alpha}\int_0^\infty \left(f(x)\right)^\gamma dx \qquad (13)$$

by Osu and Ogwo (2015).

The evolution operator φ_t tells how the state of the system changes with time and it is a family of function $\varphi_t: R^m \to R^m$ that maps the current

state of the system into the future state at a time units later and φ_t $satisfy$

$$\varphi_0(X) = X \ and \varphi_{t+s}(X) = \varphi_t\big(\varphi_s(X)\big).$$

φ_t Can be defined either as a discrete map or in terms of ordinary differential equation.

$\alpha \in (0,1]$ is the extent of short selling in the market or "going long". Here an increased is associated with an increased tendency to short sell or going long. In this model, we presume that the returns evolve according to the strength of the various agent trading in the market at any given time, each agent determines a threshold which signals whether the market is overbought or oversold and the agent becomes more risk averse in there trading strategies when these overbought or oversold threshold are breached.

γ is the maximum displacement parameter which can be defined as the market bias from the unbiased value. Our assumption γ means that the presence of the bias increases the average size of the departure of returns from the trend growth rate. Here, $\mathcal{F}$ is assumed continuous and odd on R so that the positive and negative returns are treated symmetrically. Moreover, in order to make the bias be modest, we require

$$\lim_{|x|\to\infty} xF(x) \to R \in (0, \tfrac{\sigma^2}{2}].$$

Moreover, we assume that the investors can estimate the value of σ by tracking the size of any large deviations.

3.2.1. Strongly Efficient Market

In order to establish that a market is strongly efficient we have the following derivation:

a. Using equation (1) and (13)   we have

$$F(t) = \frac{1}{\Delta\alpha}\int_0^\infty \big(f(x)\big)^\gamma \ dx$$

$$= \frac{1}{\Delta\alpha}\int_0^\infty \left(\frac{1}{\sqrt{2\pi\sigma}} e^{-\left(\frac{(In\,s)^2}{4\theta}\right)}\right)^\gamma \ dx.$$

If $x = \frac{(In\,s)^2}{4\theta}$ then

$$F(t) = \frac{1}{\Delta\alpha}\int_0^\infty \left(\frac{1}{\sqrt{2\pi\sigma}} e^{-x}\right)^\gamma \ dx$$

$$F(t) = \frac{1}{\gamma(\sqrt{2\pi\sigma})^\gamma\Delta\alpha} x^{1-\gamma}. \tag{14}$$

Then Testing for efficiency, we can see that

$$F(t) = \frac{1}{\gamma(\sqrt{2\pi\sigma})^\gamma\Delta\alpha} x^{1-\gamma} \text{ a.s on } \quad \gamma \le 0.$$

$$\frac{1}{\gamma(\sqrt{2\pi\sigma})^\gamma\Delta\alpha} x^{1-\gamma} \Rightarrow \frac{\left(\frac{(In\,s)^2}{4\theta}\right)^{1-\gamma}}{\gamma(\sqrt{2\pi\sigma})^\gamma\Delta\alpha}$$

from (8),

$$r_h(t + h) = log\frac{S(t+h)}{S(t)} = -\frac{(In\,s)^2}{4\theta}$$

from (7).

Hence $\quad -\frac{(In\,s)^2}{4\theta} \ge \dfrac{\left(\frac{(In\,s)^2}{4\theta}\right)^{1-\gamma}}{\gamma(\sqrt{2\pi\sigma})^\gamma\Delta\alpha}\quad$ a.s on the event

.   Axiom 2 satisfied.

Here, the information $\mathcal{F}(t)$ which is publicly available at time t is nothing other than the generated $\sigma - algebrae$ of the price returns.

b. Using Equation (3a) and (13) gives

$$S_T = S_t e^{\left(\left(\mu-q-\frac{\sigma^2}{2}\right)h+\sigma\beta\right)}, for \ all \ t \in$$

$[0,T] \Rightarrow S(\mu,\sigma,q,h,\beta)$. From equation (3a),

$f(X) = S_t e^{\left(\left(\mu-q-\frac{\sigma^2}{2}\right)h+\sigma\beta\right)}$ in equation (5).

Let $x = \left(\mu - q - \frac{\sigma^2}{2}\right)h + \sigma\beta$
then equation (5) gives

$$F(t) = \frac{1}{\Delta\alpha}\int_0^\infty (S_t e^x)^\gamma \ dx$$

$$F(t) = -\frac{(S_t)^\gamma}{\gamma x^{\gamma-1}\Delta\alpha} \tag{15}$$

$$= -\frac{(S_t)^\gamma}{\gamma\big(\left(\mu-q-\frac{\sigma^2}{2}\right)h+\sigma\beta\big)^{\gamma-1}\Delta\alpha} \text{ , from } (15),$$

$$r_h(t + h) = log\frac{S(t+h)}{S(t)}$$

$$= \left(\mu - q - \frac{\sigma^2}{2}\right)h + \sigma\beta, \quad \text{from (7).} \tag{16}$$

Hence,

$$\left(\left(\mu - q - \frac{\sigma^2}{2}\right)h + \sigma\beta\right) \ge$$

$$-x\frac{(S_t)^\gamma}{\gamma\big(\mu-q-\frac{\sigma^2}{2}\big)h+\sigma\beta)^{\gamma-1}\Delta\alpha}$$

a.s Axiom 2 satisfied.

c. Using Equation (3b) and (13) we have

$$S_T = e^{-\left(\frac{1}{2}\left(\frac{(\mu-q)^2}{\sigma^2}\right)T-\frac{u-q}{\sigma}\beta\right)}$$

$for \ all \ t \in [0,T] \Rightarrow S(\mu,\sigma,q,\beta,T)$
from equation (3b),

$F(x) = e^{-\left(\frac{1}{2}\left(\frac{(\mu-q)^2}{\sigma^2}\right)T-\frac{u-q}{\sigma}\beta\right)}$ in equation (5),

and
$$x = \left(\frac{1}{2}\left(\frac{(\mu-q)^2}{\sigma^2}\right)T - \frac{u-q}{\sigma}\beta\right)$$

Equation (5) gives

$$F(t) = \frac{1}{\Delta\alpha}\int_0^\infty (e^{-x})^\gamma \, dx$$

$$F(t) = \frac{1}{\gamma x^{\gamma-1}\Delta\alpha}. \qquad (15)$$

$$= \frac{1}{\gamma\left(\frac{1}{2}\left(\frac{(\mu-q)^2}{\sigma^2}\right)T - \frac{u-q}{\sigma}\beta\right)^{\gamma-1}\Delta\alpha} \qquad \text{from (15),}$$

$$r_h(t+h) = \log\frac{S(t+h)}{S(t)}$$

$$= -\left(\frac{1}{2}\left(\frac{(\mu-q)^2}{\sigma^2}\right)T - \frac{u-q}{\sigma}\beta\right) \qquad (16)$$

Hence,

$$\left(\frac{1}{2}\left(\frac{(\mu-q)^2}{\sigma^2}\right)T - \frac{u-q}{\sigma}\beta\right) \geq$$

$$x\frac{1}{\gamma\left(\frac{1}{2}\left(\frac{(\mu-q)^2}{\sigma^2}\right)T - \frac{u-q}{\sigma}\beta\right)^{\gamma-1}\Delta\alpha} \qquad \text{a.s on the event}$$

$\gamma \leq 0$, Axiom 2 satisfied.

3.2.2 Harmonized Dimensional Integral Transform as A Generalized State of Melin Integral Transform

Two measures are said to be equivalent when they are equal to zero but if they have the same dimension then it is fractal. (Falcon ,1985).

$$F(t) = \frac{1}{\Delta\alpha}\int_0^\infty \left(f(X)\right)^\gamma \, dx$$

$$= \frac{1}{\Delta\alpha}\int_0^\infty \left(f(X)\right)^\gamma (\Delta\alpha)^{-1} \, dx$$

$$= \frac{1}{\Delta\alpha}\int_0^\infty f(x)x^{w-1} \, dx \qquad (17)$$

If $x = \Delta\alpha, w = 0 \text{ and } \gamma = 1$.

4   Conclusions

Finally, we confirm that the Melin integral transform does represent an inefficient market in the weak sense. The presence of the indicator function γ in the harmonized dimensional integral transform indicates that, the stronger the bias nature of investors, the larger the average values of prices excursions and consequently the smaller the volatility that arises around the average values. This persistence could make the investors believe that the cumulative returns are

close to their true values and are unbiased, the agents become more risk averse in there trading strategies when their overbought or oversold threshold are breached. Hence, allowing one to define an optimal positioning between a reduction in market and likelihood of a bankruptcy, addressing the issue of inefficiency and making the model strongly efficient.

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