Some Developments and Applications on Field Theory

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Abstract: First, when the dimension is extended to n, gradient, divergence, curl and corresponding Gauss's theorem and Stokes's theorem are researched. Next, field may be developed from scalar and vector to tensor, spinor, torsion and twistor, etc. Further, they are applied to physics, biology, earthquakes and social science, etc. Field theory has been widely applied in many regions of natural and social sciences, and its any development will necessarily inspire and apply to more aspects.

Key words: field theory, dimension, tensor, spinor, torsion, twistor, physics, biology, social science.

1. Introduction

In mathematics, physics and many regions the field theory is all a very important problem. A known important representation is tree in the graph theory. We extended tree, and proposed a new tree-field representation (Fig.1), which includes two parts: tree (trunk) and field [1].

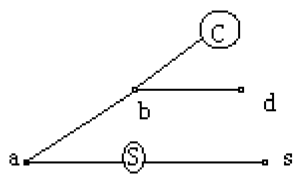

Fig.1. Tree-field representation

Here the fields (C and S, etc.) are some sets of much small trees, which are also trees under microscope. For a certainty condition the few small trees in the field can become a usual tree as trunk. These may be the directed graph or undirected graph. In Fig.1 the vertexes b, d, s and the fields C, S possess the same macroscopic property, and fields are only minute detail on vertex. In the tree-field representation there is a relation (for vertex n, edge m and field s) $m = n + s - 1$. Further, the field may have different weights and levels, and may be extended to the graph theory, i.e., G=(V,E,F), in which F is field and a set of the small graph, and represents region, grove and forests, etc.

It is known that the space-dimension of mathematics and calculus are extended to higher n-dimensions [2]. It may be Hilbert space. When the space-dimension of calculus is extended to n, fractal and complex, and various number-systems, the field theory and its formulas may be correspondingly extended. Its any development will necessarily inspire and apply to more aspects [3]. In this paper, we discuss some developments and applications of field theory.

2. Some Mathematics in n-dimensional Space

In the n-dimensional space, the gradient is:

$$Gradu = \sum_{i=1}^{n} \frac{\partial u}{\partial x_i} \vec{i}_i \quad . \tag{1}$$

The n-dimensional divergence is:

$$Div\vec{P} = \sum_{i=1}^{n} \frac{\partial P_i}{\partial x_i} \quad . \tag{2}$$

The n-dimensional curl is more complex, and has probably several different forms [3], for example:

$$Rot\vec{P} = \sum_{i,j=1}^{n} \left(\frac{\partial P_j}{\partial x_i} - \frac{\partial P_i}{\partial x_j}\right). \qquad (3)$$

But, according to the correspondence principle, these forms must be transformed into ordinary curl in the three dimensional space [3].

For conservative fields, there are the Stokes's theorem and Gauss's theorem. In this case, both theorems can be extended to the relations between n and n+1 multiple integral from geometry. The extensive Stokes's theorem represents the n-dimensional closed curve integral by n-dimensional surface integral:

$$\oint Pdl = \iint RotPdS \quad .$$

(4)

The extensive Gauss's theorem is:

$$\iint \ldots \int \left(\frac{\partial P_1}{\partial x_1} + \frac{\partial P_2}{\partial x_2} + \ldots + \frac{\partial P_{n+1}}{\partial x_{n+1}}\right) d\Omega =$$

$$\oiint \ldots \oint P_1 dx_2 dx_3 \ldots dx_{n+1} + P_2 dx_3 dx_4 \ldots dx_1 + P_{n+1} dx_1 dx_2 \ldots dx_n =$$

$$\oiint \ldots \oint PdV = \iint \ldots \int DivPd\Omega.$$

(5)

Further, the field in mathematics may be extended from usual scalar field, vector field to tensor field in general relativity, and spinor field of particles, torsion and twistor fields, etc.

3. Various Tensor and Spinor Fields

A classical vector field is the electromagnetic field. In electrodynamics Maxwell equations are [4]:

$$\frac{\partial F_{ik}}{\partial x_k} = \frac{4\pi}{c} j_i \qquad \text{(flow vector).}$$

(6)

In general relativity the gravitational field is tensor, in which there is Einstein symmetric tensor:

$$G_{\mu\nu} = R_{\mu\nu} - (Rg_{\mu\nu}/2) \quad .$$

(7)

$R_{ik,l}$ is 3-order tensor, etc. And the divergence of $G_{\mu\nu}$ is zero. In Riemann geometry an extensive form of curl is Christoffel symbols [4]:

$$\Gamma^i_{kl} = \frac{1}{2} g^{im} \left(\frac{\partial g_{mk}}{\partial x^l} + \frac{\partial g_{ml}}{\partial x^k} - \frac{\partial g_{kl}}{\partial x^m}\right).$$

(8)

It is not tensor. But, there have Riemann curvature tensor, Ricci tensor, scalar curvature and Contorsion tensor, etc.

Further, it may be extended to Brans-Dicke scalar-tensor theory [5], gravitational vector-tensor theory and tensor-tensor theory, both may be composed of the gravitational field and electromagnetic field and the electromagnetic general relativity [6], etc.

The gravitational field and electromagnetic field try to be unified by the gauge invariance geometry, the five-dimensional Kaluza-Klein theory, the projective theory, the affine field and the bivector fields, the non-symmetric field [7,8], and Utiyama field [9], Yilmaz theory [10] and Carmili theory [11], etc.

We proposed that the gravitational field and the source-free electromagnetic field can be unified preliminarily by the equations in the Riemannian geometry [12,13]:

$$R^i_{klm} = \kappa T^i_{klm} * \quad .$$

(9)

Both fields are contractions of im and ik, respectively. If $R^i_{klm} = \kappa T^i_{klm} *$=constant, so it will be equivalent to the Yang's gravitational equations [14]:

$$R_{km;l} - R_{kl;m} = 0 \quad ,$$

(10)

which include $R_{lm} = 0$. From $R_{lm} = 0$ we can obtain the Lorentz equations of motion, the first system and second source-free system of Maxwell field equations. This unification can be included in the gauge theory, and the unified gauge group is SL(2,C) $\times$ U(1)=GL(2,C), which is just the same as the gauge group of the Riemannian manifold. Another unification on the general nonsymmetric metric field with high-dimensional space-time is analyzed mathematically. Moreover, some possible unification ways on the gravitational field and electromagnetic field are discussed [13].

The physical 3-dimensional space has been developed to 4-dimensional space-time of relativity, and Kaluza-Klein's 5-dimensional space, and higher dimensional space of superstring, etc. Further, higher-dimensional space is related to black hole [15-17] and to Ruppeiner geometry and Weinhold geometry [18]. The physical space-time developed to fractal and complex dimension derive necessarily the fractal relativity [19,20] and fractal and complex dimensional time [21,22,1], etc.

The unification of various interactions is always an important question in physics. Their mathematical basis is mainly the gauge groups. Weinberg and Salam proposed a well-known electroweak theory unified the weak and electromagnetic interactions, whose unified gauge group is SU(2) $\otimes$ U(1)=U(2) [23,24]. Further, various grand unified theories of the strong, weak and electromagnetic interactions are researched, whose pioneer is Bars-Halpern-Yoshimura model [25,26].

Einstein gravitational Lagrangian possesses two invariances: the GL(4,R) invariance of Einstein under coordinate transformations, and the SL(2,C) gauge invariant of Weyl. The strong interaction of quarks possesses internal SU(3) symmetry. From these symmetries Isham, Salam and Strathdee proposed the unified scheme on gravitational and strong interactions, whose gauge group is SU(3) $\otimes$ SL(2,C)=SL(6,C) [27-29]. Based on the Weinberg-Salam unified electroweak theory and Isham-Salam-Strathdee strong-gravitational interactions unified scheme [27-29], in 1974 we proposed that the simplest unified gauge group of four-interactions must be GL(6,C) [12,30] , which is related to four- or six-dimensional Riemann geometry and nonsymmetric field [7,30]. In 1977 Terazawa proposed also a unified gauge symmetry of GL(32N,C) or GL(12+2n,C) for combining the SU(16N) or SU(6+n) group of lepton-quark internal symmetry and the SL(2,C) Lorentz group of space-time symmetry (where N=1,2,3… and n=0,1,2,3…N) [31]. Of course, this unified group is a non-compact group, and seems to construct a 'no-go' law. But, it is not only reasonable but also necessary, because the classical gravitational interaction corresponds to a non-compact group SL(2,C). Further, it should be that the gravitational theory is developed to the quantum gravity theory, for example, the supergravity and the loop quantum theory [32-34] and so on.

Based on Some new representations of the supersymmetric transformations and the supermultiplet, Graded Lie Algebras and various formulations (equations, commutation relations, propagators, Jacobi identities, etc.) of bosons and fermions may be unified. On the one hand, the mathematical characteristic of particles is proposed: bosons correspond to real number, and fermions correspond to imaginary number, respectively. Such fermions of even (or odd) number form bosons (or fermions), which is just consistent with a relation between imaginary and real number. The imaginary number is only included in the equations, forms, and matrixes of fermions. It is connected with relativity. On the other hand, the unified forms of supersymmetry are also connected with the statistics unifying Bose-Einstein and Fermi-Dirac statistics, and with the possible violation of Pauli exclusion principle; and a unified partition function is obtained. Moreover, three quarks may be described by the Borromean rings. The quantum statistics is unified by the nonlinear equations. A developed direction of particle physics and modern science is possibly the higher dimensional complex space [30].

Moreover, the unification is connected with the string model [35,36] and the grand unification theory [36], and the supersymmetry [37] and supergravity, etc.

It is believed that all the Lorentz covariant equations are the tensor equations. But, then it has been found that spins s of many particles are not zero, and in particular, the particle of spin s=1/2 is introduce into the two-component spin-field equation, which include Weyl equation, Dirac equations and Dirac-Fierz-Pauli equations with any spin, whose forms of spinor are [38] are:

$$\partial_{a\dot{b}}\phi^{aa_1a_2...a_l}_{\dot{b}_1\dot{b}_2...\dot{b}_k}=im\chi^{a_1a_2...a_l}_{\dot{b}\dot{b}_1\dot{b}_2...\dot{b}_k},$$

$$\partial^{a\dot{b}}\chi^{a_1a_2...a_l}_{\dot{b}\dot{b}_1\dot{b}_2...\dot{b}_k}=im\phi^{aa_1a_2...a_l}_{\dot{b}_1\dot{b}_2...\dot{b}_k}.$$

(11)

In 1932 van der Waerden proposed spinor analysis, whose two-component spinor differential operator is:

$$\partial_{1\dot{1}}=\partial_3-i\partial_4\ ,\quad \partial_{1\dot{2}}=\partial_1-i\partial_2\ ,$$

$$\partial_{2\dot{1}}=\partial_1+i\partial_2\,,\ \partial_{2\dot{2}}=-\partial_3-i\partial_4\,. \qquad (12)$$

At present, it is considered that the most common Lorentz covariant equation must be spinor equation [38]. It is clear that formula (12) is similar to curl.

Horvathy, et al., proposed the bosonized supersymmetry and massive higher-spin fields from (3+1)-dimensional linear vector equations and the Majorana-Dirac-Staunton theory [39]. Smilga discussed physics of crypto-Hermitian and crypto-supersymmetric field theories [40]. DeWolfe, et al., discussed gravity dual of metastable dynamical supersymmetry breaking [41]. Aguirre, et al., searched fractal structures of nonlinear dynamics in various different fields [42].

4. Various Torsion and Twistor Fields

Kleinert studies the multivalued fields in condensed matter, electromagnetism and gravitation [43]. In general metric-affine geometry

$$\Gamma^{\lambda}_{\mu\nu}=\frac{1}{2}(\Gamma^{\lambda}_{\mu\nu}+\Gamma^{\lambda}_{\nu\mu})+\frac{1}{2}(\Gamma^{\lambda}_{\mu\nu}-\Gamma^{\lambda}_{\nu\mu}),\qquad (13)$$

which divides into symmetrical part and antisymmetric part, and both are 40 and 24 degrees of freedom, respectively. Here antisymmetric part is called torsion [43]:

$$T^{\lambda}_{\mu\nu}=\frac{1}{2}(\Gamma^{\lambda}_{\mu\nu}-\Gamma^{\lambda}_{\nu\mu}).$$

(14)

Spin tensor corresponds to torsion [44].

General curved space has curvature and torsion. General relativity has curvature, and has not torsion (T=0). When Riemann-Cartan space-time is simplified to Weitzenbock space-time, it is 16 degrees of freedom, and is torsion and is not curvature (R=0) [45]. Ordinary flat space is neither curvature and torsion. Torsion field is possibly the theoretical foundation of quantum entanglement.

Twistor is proposed by Penrose [46]. It is a new type of algebra by description for Minkowski space-time, in terms of which it is possible to express any conformally covariant or Poincaré covariant operation. The elements of the algebra (twistors) are combined according to tensor-type rules, but they differ from tensors or spinors in that they describe locational properties in addition to directional ones. The representation of a null line by a pair of two-component spinors, one of which defines the direction of the line and the other, its moment about the origin, gives the simplest type of twistor, with four complex components.

Penrose, et al., proposed twistor theory as an approach to the quantisation of fields and space-time [47]. Penrose searched nonlinear gravitons and curved twistor theory. Twistor space is a parameter space-time with complex structure [48]. Twistor is:

$$Z_\alpha = (\lambda_A, \bar{\mu}^{\dot{A}}) , \quad \bar{Z}^\alpha = (\mu^A, \bar{\lambda}_{\dot{A}}) . \tag{15}$$

Here a pair of two-component spinors is:

$$Z^A_{;1} = (\omega^\alpha, \bar{\pi}_{\dot{\beta}}) , \quad Z^A_{;2} = (\lambda^\alpha, \bar{\eta}_{\dot{\beta}}) . \tag{16}$$

A 4-dimensional point is represented by 3-dimensional complex space, i.e., complex number in twistor. It is applied to quantization and curved space-time.

Penrose and Rindler discussed spinor and twistor methods in space-time geometry [49]. Twistor theory offers a new approach to the synthesis of quantum theory and relativity. Twistors for flat space-time are the SU(2,2) spinors of the twofold covering group O(2,4) of the conformal group. Penrose's basic relations are [49]:

$$\omega^\alpha = iz^{\alpha\dot{\beta}}\bar{\pi}_{\dot{\beta}} , \quad \bar{\omega}^{\dot{\alpha}} = -i\pi_\beta \bar{z}^{\beta\dot{\alpha}} . \tag{17}$$

They describe the momentum and angular momentum structure of zero-rest-mass particles. Space-time points arise as secondary concepts corresponding to linear sets in twistor space. Twistors are represented in two-component spinor terms. The generalisation to curved space can be accomplished in three ways; i) local twistors, a conformally invariant calculus, ii) global twistors, and iii) asymptotic twistors which provide the framework for an S-matrix approach in asymptotically flat space-times.

Further, Hayashi discussed general relativity as gauge field theory in curved twistor space [50]. Penrose summarized the central programme of twistor theory, which includes twistor quantization, self-dual and anti-self-dual fields, helicity 3/2 fields and the vacuum equations, etc [51]. Twistor may be related to string, superstring and supersymmetry [52-54], twistor space [55,56], twistor transform in d dimensions and a unifying role for twistors [57], particles and superparticles, instanton and gauge field, etc. Bandos, et al., even proposed supertwistors [58]. The variables of supertwistors are:

$$Z^I_L = (\lambda^\alpha_L, \mu^{\dot{\alpha}}_L, \psi^A_L) ,$$

$$Z^I_R = (\lambda^\alpha_R, \mu^{\dot{\alpha}}_R, \psi^A_R) . \tag{18}$$

These are a pair equations of three variables. Generally, these are n pair equations of m variables. Fedoruk, et al., researched unification of various string models from twistor [59], and twistor string, etc [60].

We researched applications of twistor and its extensions in biology, which may describe some biological duality, and proposed specially the twistor model of DNA [61]. It is well-known that space-time depend on matter and its movement in general relativity. Interactions between biological elements of different levels determine biological structures and shapes. For example, in DNA "horizontal" hydrogen bond interaction connects A-T, and "vertical" stacking interaction connects C-G [62]. Bena, et al., [63,64] elucidated the one-loop twistor-space structure corresponding to momentum-space maximally helicity-violating diagrams, and use the "holomorphic anomaly" to define modified differential operators which can be used to probe the twistor-space structure of one-loop amplitudes [63,64]. Bena, et al., searched loops in twistor space, and twistor transform in d dimensions and a unifying role for twistors [64]. We applied the loop quantum theory to biology, and proposed the model of protein folding and lungs, and obtain four approximate conclusions [65]. Further, it may combine twistor [61].

So far, biology applies only twister. Twistor as an extension of scalar, vector, spinor and tensor includes the self-dual Yang-Mills field. We may research generally twistor in biology. Based on complex number and the conformal transformation, twistor is [49]:

$$\begin{pmatrix} t+z & x+iy \\ x-iy & t-z \end{pmatrix} = t\sigma_0 + x\sigma_x + y\sigma_y + z\sigma_z \quad . \quad (19)$$

Here using quaternion or Pauli matrices are:

$$\sigma_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I \,, \quad \sigma_x == \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},$$

$$\sigma_y = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \quad (20)$$

Twistor (15) is two pair conjugate complex numbers, and includes (t, x, y, z) and Pauli matrices. Its space is C^4. This may be related to relativity. Pauli matrices describe particles with spin 1/2.

Some structures in biology are helix. RNA is a single link structure on A-U and G-C. Here A is adenine and G is guanine, while U is uracil and C is cytosine. It is known that a helical line is:

$$x = a\cos t, y = b\sin t, z = dt \quad . \quad (21)$$

Here d is a thread pitch. A form of the complex function is:

$$z = a\cos t + ib\sin t, w = dt \quad . \quad (22)$$

The double helix is:

$$z = a\cos(\varphi + \frac{C_1}{C_2}) \quad \text{and} \quad w = d\varphi \ . \quad (23)$$

In complex function $f^{x+iy} = f^x f^{iy}$. When f=e, so $e^x e^{iy}$ is a circle with radius $\rho = e^x$ and $\phi = y$ [66].

Twistor may describe biological extensive string with meander and twister. RNA as a string, and DNA as the double string of the double helix structure on A-T and G-C (here T is thymine) should be able to apply twistor. Therefore, we may research the twistor model described DNA, RNA, etc [61].

We may transform quaternion or Pauli matrices to other different forms:

1) 1→I= $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ is unit matrix, and i→C= $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ and $C^2 = -I$. Assume that a new complex positive number j, here j^2=1. Then, let j→B= $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, B^2=I, correspond to an inversion operator. k=ij→A= $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ =CB, here $A^2 = I$ and ji=-k→-A. This set (1,i,j,k) is another quaternion, and relates the Clifford fourfold quaternion. It corresponds to that a field is extended to a ring. Such new twistor is:

$$\begin{pmatrix} t+z & x+y \\ x-y & t-z \end{pmatrix} = tI + xB - yC - zA \quad . \quad (24)$$

2) We introduce two pair conjugate complex numbers

$$\begin{pmatrix} t+iz & x+iy \\ x-iy & t-iz \end{pmatrix} = tI + xB - iyC - izA$$

. (25)

This corresponds to two circles.

3) We may add four parameters

$$\begin{pmatrix} at+dz & bx+cy \\ bx-cy & at-dz \end{pmatrix} = atI + bxB - cyC - dzA$$

. (26)

Such their radii may be different, and may be determined. For DNA they correspond to a diameter 20A and the pitch 34A, etc.

These corresponding relations between new quaternion and DNA (t~A, z~T and x~G, y~C) are symmetry completely. It is usual B-DNA. Other is A-DNA and Z-DNA, etc. New forms of twistor correspond to develop matrix and tensor, etc.

Twistor is developed from general relativity, and may describe the curved space-time. It is also consistent with a basic idea: the matter determines the structure of space in biology [66]. Moreover, A-T(U), C-G in DNA (RNA) all are 2 square matrix.

Moreover, in biology there is duality. For example, duality and synergy in peptide antibiotic mechanisms by which peptide antibiotics disrupt bacterial DNA synthesis, protein biosynthesis, cell wall biosynthesis, and membrane integrity shown rich diversity, and involved in synergistic relations with antibiotics and proteins [67]. Two ideas in theoretical biology, ‘decomposition into functions’ and ‘gluing functions’, show a duality. They imply two biologically significant conditions: the existence of cycles in finite graphs and anticipatory diagrams [68]. Al-Sady, et al., searched mechanistic duality of transcription factor function in phytochrome signaling, and found that PIF3 acts positively as a transcription factor, exclusively requiring its DNA-binding capacity [69]. Twistor and spinor with double components may describe some biological duality, for example, excitatory synapse and inhibitory synapse, and above duality. The twistor model of DNA [61] is probably advantageous to development of DNA computing model [70].

5. Some Applications of Various Fields

New researches shown that the entangled state should be is a new fifth interaction [71,72]. Its action distance seem to be middle-rang, and its strength is also middle one. This likes the thought field [73]. Further, we proposed the entangled field, whose phase particle (phason) has some characters and corresponding equations. It is tachyon, and assume that it is similar to photon and J=1 and m=0 or mass is very small as similar neutrino, and may show the action at a distance. We researched that this field as wave has some characters, and discussed the superluminal quantum communication by a pair of entangled states is generated on both positions, or by preparing and transmitting a pair of entangled instruments, so the superluminal quantum communication. Assume that the entangled field has a similar magnetic theory, which may be a quantum cosmic field, or be the extensive quantum theory, or God or the Buddha-fields and so on. These are all macroscopic fields, which correspond to de Broglie-Bohm nonlinear “hidden variable” theory, but it is microscopic [74].

There are Earth’s magnetic field [75] and stress field of the Earth’s crust [76]. These fields are related to the plate tectonics and global mantle currents. They also determine earthquakes. Based on the nonlinear equations of fluid dynamics, we derived a simplified nonlinear solution of momentum, which correspond to the accumulation of the energy. Their values are excess a faulting threshold, earthquake will take place. From this a chaos equation is obtained, in which chaos corresponds to the earthquake, which shows complexity on seismology, and impossibility of exact prediction of earthquakes. But, combining the Carlson-Langer model and the Gutenberg-Richter relation, we derived approximately the magnitude-period formula of the earthquakes [12,73,77-80]:

$$T = 10^{-b(M_0 - M)}T_0. \quad (27)$$

By this formula some predictions can be calculated quantitatively and are already tested. We forecasted a series of earthquakes in California [73,78,79] since 2004. From this in California next earthquakes should take place in 2009, 2014 and 2019, etc. Earthquakes (M=6.9) occurred on 3 August of 2009, and M=6 on 24 August of 2014. A large earthquake of 2019 is more possible [73,78,79]. In fact, two earthquakes occurred July 4 (M=6.4) and July 5 (M=7.1) in California, which agrees completely with my prediction [80].

Mathematics is a powerful and important tool in modern ecology and environment science [81,82]. We researched field theory is applied to social sciences [3]. Ecological field is namely useful concept. Its complete mode is the unification field of human-nature in Chinese traditional culture. Using the similar formulas of the preference relation and the utility function, we proposed the confidence relations and the corresponding influence functions that represent various interacting strengths of different families, cliques and systems of organization. It produces a multiply connected topological economics. This model may describe a corruption field in usual economic system. Further, we discussed the binary periods of the political economy by the complex function and the elliptic functions [83].

We proposed the social extensive general relativity. Everyone possesses fate and luck, and both correspond to mass self and life orbit and movement. General relativity shows that matter and movement determine the space-time of everyone, and may derive the corresponding field equations and the evolutional orbits. In Fig. 2 the big mass of the center and its movement correspond to the great countries and great men that determine space-time and era, from which everyone's mass and efforts determine the orbits of life. Both determine the evolution of whole society and mankind. This as a universal physical representation of causality is a great contribution to modern social science. It is the causality field as a common basis of various natural sciences, Buddhism and some social sciences [84].

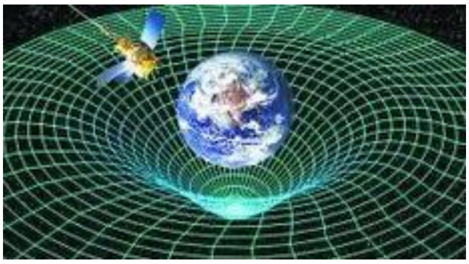

Fig. 2. Matter and movement determine the space-time, and space-time determines the evolutional orbits.

Based on the social structure we introduced the social individual-wave duality, and researched the social topology and the social strain field [82]. A variant of the damage field D(r,t) should agree with the damage field equation:

$$\frac{\partial D}{\partial t} + \frac{\partial (Dv)}{\partial r} = f. \quad (28)$$

Here f is dynamical function of damage.

In a word, in natural science and social science there are widely change and evolutionary fields. The development of mathematics often leads to the progress of physics and science. Field theory has been widely applied in many regions of natural and social sciences, and any development of field theory will necessarily inspire and apply to more aspects.

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