

A NSGA-II Approach to the Bi-objective Multi-vehicle Allocation of Customers to Distribution Centers

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Abstract:- This study proposes a bi-objective model for capacitated multi-vehicle allocation of customers to potential distribution centers (DCs). The optimization objectives are to minimize transit time and total cost including opening cost, assumed for opening potential DCs and shipping cost from DCs to the customers where considering heterogeneous vehicles lead to a more realistic model and cause more conflicting in the two objectives. An evolutionary algorithm named non-dominated sorting genetic algorithm (NSGA-II) is used as the optimization tool. To aid the decision maker choosing the best compromise solution from the Pareto front, the fuzzy-based mechanism is employed. For ensuring the robustness of the proposed method and giving a practical sense of this study, the computational results in some small cases are compared with those obtained by LP-metric method. Results show the percentage errors of objective functions compared to the LP-metric method are less than 2%. Furthermore, it can be seen that with increasing size of the problems, while the time of problem solving increases exponentially by using the LP-metric method, the running time of NSGA-II is more stable, so these show the advantages and effectiveness of NSGA-II in reporting the Pareto optimal solutions in large scale.

Key-Words:- bi-objective optimization; capacitated location-allocation; multi-vehicle; LP-metric; evolutionary algorithm; non-dominated sorting genetic algorithm II;

1 Introduction

In today's business environment, the competitiveness of a firm heavily depends on its ability to handle the challenges of reducing cost, increasing customer service and improving product quality. In this competitive market, customer satisfaction is the most important factor for the success of the firm. In this regard, the supply chain network among different business entities like manufacturers, suppliers and distribution centers (DCs) needs to be effective enough to handle the changing demand patterns. Nowadays, efforts have been made to design and develop a more conducive and profitable supply chain network. Efficient allocation of customers to DCs always plays an important role in developing a flawless and reliable supply network.

One of the most active topics in manufacturing research over the last 10 years has been supply chain management (SCM). SCM is the management of material and information flows both in and between facilities, such as vendors, manufacturing, assembly

plants and distribution centers [1]. Transportation network design is one of the most important fields of SCM. It offers great potential to reduce costs. In the several decades, there have been many researchers reported new models or methods to determine the transportation or the logistics activities that can lead to the least cost [2]. One of the important factors which influences on logistic system is to decide regarding the number of distribution centers. Geoffrion and Graves (1974) were the first researchers studied on two-stage distribution problem [3]. Pirkul and Jayaraman (1998) presented a new mathematical formulation called PLANWAR to locate a number of production plants and warehouses and to design distribution network, so that the total operating cost can be minimized. They developed an approach based on Lagrangian relaxation to solve the problem [4]. Hindi et al. (1998) stated a two-stage distribution planning problem. They supposed that each customer must be served from a single distribution center. The authors gave mathematical model for the problem and developed a branch and bound algorithm to solve the problem [5]. Zhou et al.

(2003) proposed a mathematical model and an efficient solution procedure for the bi-criteria allocation problem involving multiple warehouses with different capacities [6]. Hajiaghayi-Keshteli (2011) considered two stages of supply chain network including distribution centers (DCs) and customers. His proposed model selects some potential places as distribution centers in order to supply demands of all customers and in order to solve the given problem, two algorithms, genetic algorithm and artificial immune algorithm, were developed [7]. Chan and Kumar (2009) discussed a multiple ant colony optimization (MACO) approach in an effort to design a balanced and efficient supply chain network that maintains the best balance of transit time and customers service. The focus of their paper is on the effective allocation of the customers to the DCs with the two-fold objective of minimization of the transit time and degree of imbalance of the DCs [8]. Arabzad et al. (2015) proposed a multi-objective optimization algorithm for solving the new multi-objective location-inventory problem (MOLIP) in a distribution center (DC) network with the presence of different transportation modes and third-party logistics (3PL) providers. In their paper, NSGA-II algorithm was proposed to perform high-quality search using two-parallel neighborhood search procedures for creating initial solutions [9].

2 Preliminaries

2.1 Multi-objective optimization problems

Multi-objective optimizations concerned with mathematical optimization problems involving more than one objective function to be optimized simultaneously. To obtain the optimal solution, there will be a set of optimal trade-offs between the conflicting objectives, where the set of optimal solution is known as Pareto front. A multi-objective optimization problem is defined as the maximization or the minimization of many objectives subject to equality and inequality constraints. The multi-objective optimization problem can be formulated as follows:

$$\text{Max. /Min. } f_i(x), i=1, \dots, N_{obj} \quad (1)$$

Subject to constraints:

$$\begin{aligned} g_j(x) &= 0, j=1, \dots, M \\ h_k(x) &\leq 0, k=1, \dots, K \end{aligned} \quad (2)$$

where f_i is the i th objective function, x is the decision vector, N_{obj} is the number of objectives, g_j is the j th equality constraint and h_k is the k th inequality constraint.

2.2 Multi-objective evolutionary algorithms

There are techniques such as weighting methods and ε -constraint method which transfer multi-objective problems to a single-objective one, using different combinations of a weighting vector and constraints. Thus, each optimal solution can be assigned to a specific combination of weighting vector and constraint. Hence, in each run of the algorithm, a single solution can be achieved. However, multi-objective evolutionary algorithms are capable of finding almost all candidate solutions (Pareto) in a single run. Evolutionary algorithms are based on evolutionary computations which can perform optimal/near-optimal solutions in all types of problems (linear/nonlinear, discrete/continuous, convex/ non-convex) using validated experimental theories of biological evolution and natural processes, particularly through activities of different species of animals.

A set of solutions resulting from a program run, without using any techniques such as the weighting approach that are directly related to decision-makers' opinions, is the most important advantage of evolutionary algorithms in the field of multi-objective optimization. In this paper, we used an evolutionary algorithm, NSGA-II, to tackle the problem. In contrast to traditional multiple objective programming techniques such as goal programming that requires the decision maker to arbitrarily determine weighting coefficients and/or preferences on multiple criteria and consequently produces a dominated solution, the proposed algorithm was designed to generate a wide range of non-dominated solutions without the arbitrary determination of weights. Bandyopadhyay and Bhattacharya (2014) solved a tri-objective supply chain problem with modified NSGA-II algorithm. They have introduced a mutation algorithm which has been embedded in the proposed algorithm [10].

In this paper, two-stage supply chain network including the distribution centers and the customers, are considered. There are potential places which are candidate to be as distribution centers, called potential DCs, and customers with particular demands. Each of the potential DCs can ship to any of the customers. The two optimization objectives are to minimize

transit time and total cost involving opening cost, assumed for opening a potential DC and shipping cost per unit from DC to the customers. The proposed model selects some potential places as distribution centers in order to supply demands of all customers, i.e. the model selects some potential DCs in such a way that the customer demand can be satisfied at minimum DCs' opening cost and minimum shipping cost with minimum transit time. It is assumed that distribution centers have unequal capacities and each customer must be served from a single distribution center. Also in this paper, considering different types of vehicles caused more conflicting in these two objectives. We proposed an evolutionary algorithm, Non-dominated Sorting Genetic Algorithm (NSGA-II) to tackle with the problem in this paper. In contrast to the traditional multiple objective programming techniques such as LP-metric, the proposed algorithm was designed to generate a wide range of non-dominated solutions without the arbitrary determination of weights.

3 Description of model

In this paper, a location-allocation model for multi-vehicle single product in two-stage supply chain network is developed. This model includes distribution centers (DCs) and customers with respect to two conflicting objectives consist of minimizing total transit time and total cost. The total cost here, involves opening cost, assumed for opening potential DCs and shipping cost from DCs to the customers. The proposed model selects some potential places as distribution centers in order to supply demands of all customers. It is assumed that distribution centers have unequal capacities and each customer must be served from a single distribution center. Considering heterogeneous vehicles lead to a more realistic model and cause more conflicting in the two objectives of the proposed problem, since a fast vehicle (because of high technology or having low capacity) has more cost and a vehicle with low cost can lead to higher transit time. Let us denote I as a set of nodes representing m customers, J as a set of nodes representing p potential distribution centers, V as a vehicle type number for transferring process so that it is assumed that there are sufficient numbers of each type of vehicle, and E as a set of edges representing a connection between customers and DCs. d_i denotes the demand of customer i , f_j the fixed cost for opening

a potential DC at site j , q_v the capacity of type of vehicle v , $v \in V$, and the associated capacity q_j for such DC; d_{ij} the distance between DC j and customer i ; c_{ij}^v is the cost of assigning customer i to DC located at site j with type of vehicle v , t_{ij}^v is the transit time between customer i to DC located at site j with type of vehicle v . All parameters introduced above are assumed to be non-negative. The binary variable y_j is 1 if a DC is located at site j and 0 otherwise. Similarly, binary variable x_{ij}^v is equal to 1 if customer i is served by the DC located at site j with type of vehicle $v \in V$ and 0 otherwise. The bi-objective capacitated multi-vehicle allocation of customers to distribution centers problem can be formulated as the following binary integer programming:

$$\min z_1 = \sum_{v=1}^V \sum_{j=1}^p \sum_{i=1}^m d_i d_{ij} c_{ij}^v x_{ij}^v + \sum_{j=1}^p f_j y_j \quad (3)$$

$$\min z_2 = \sum_{v=1}^V \sum_{j=1}^p \sum_{i=1}^m t_{ij}^v x_{ij}^v \quad (4)$$

Subject to:

$$\sum_{v=1}^V \sum_{j=1}^p x_{ij}^v = 1, i = 1, \dots, m \quad (5)$$

$$\sum_{v=1}^V \sum_{i=1}^m d_i x_{ij}^v \leq q_j y_j, j = 1, \dots, p \quad (6)$$

$$\sum_{i=1}^m \sum_{j=1}^p d_i x_{ij}^v \leq q_v, v = 1, \dots, V \quad (7)$$

$$x_{ij}^v, y_j \in \{0,1\}, \forall i = 1, \dots, m, \forall j = 1, \dots, p, \forall v = 1, \dots, V \quad (8)$$

The first objective function Eq. (3) minimizes the total cost of opening distribution centers and assigning customers to such distribution centers, while the second objective function Eq. (4) minimizes total transit time between distribution centers and customers allocated to them. Constraints Eq. (5) guarantee that each customer is served by exactly one DC and also guarantee that each customer's demand on each edge between a customer and a DC is transferred by a vehicle type and exactly one of it, and capacity constraints Eq. (6) ensure that the total demand assigned to a DC cannot exceed its capacity. The constraints Eq. (7) ensure that the total demand transferred by a vehicle cannot exceed its capacity. In this paper, capacity constraints of DCs have been relaxed considering penalty function. In general, a penalty function approach is as follows. Given an optimization problem:

$$\begin{aligned} &\text{Min } f(\mathbf{X}) \\ &\text{s.t. } \mathbf{X} \in A \\ &\quad \mathbf{X} \in B \end{aligned} \quad (9)$$

where \mathbf{X} is a vector of decision variables, the constraints " $\mathbf{X} \in A$ " are relatively easy to satisfy, and

the constraints “ $X \in B$ ” are relatively difficult to satisfy, the problem can be reformulated as:

$$\begin{aligned} \min & f(X) + p(d(X, B)) \quad (10) \\ \text{s.t.} & X \in A \end{aligned}$$

where $d(X, B)$ is a metric function describing the distance of the solution vector X from the region B , and $p(0)$ is a monotonically non-decreasing penalty function such that $p(0) = 0$. Furthermore, any optimal solution of Eq. (10) will provide an upper bound on the optimum for Eq. (9), and this bound will in general be tighter than that obtained by simply optimizing $f(X)$ over A .

In this paper, the objective functions are as follows:

$$\begin{aligned} \min \widehat{z}_1 &= z_1 + \delta_1 \cdot Vi \quad (11) \\ \min \widehat{z}_2 &= z_2 + \delta_2 \cdot Vi \end{aligned}$$

where $\delta_1 \cdot Vi$ and $\delta_2 \cdot Vi$ are penalty functions. δ_1 and δ_2 are two positive coefficients where usually are considered greater than $\max(z_1)$ and $\max(z_2)$, respectively. Also, Vi represents relatively violation value of capacity constraints related to DCs Eq. (6):

$$Vi = \left(\sum_{v=1}^V \sum_{i=1}^m d_i x_{ij}^v - q_j y_j \right) / q_j y_j \quad \text{if} \quad \sum_{v=1}^V \sum_{i=1}^m d_i x_{ij}^v > q_j y_j \quad \forall j=1, \dots, p \quad \text{and also}$$

$$Vi = 0 \quad \text{if} \quad \sum_{v=1}^V \sum_{i=1}^m d_i x_{ij}^v \leq q_j y_j \quad \forall j=1, \dots, p \quad (12)$$

Besides fulfilling other constraints (Eq. (5) and Eq. (7)), the solutions with $Vi = 0$ are feasible and otherwise the solutions are infeasible.

4 Solution approach

In this paper, MATLAB platform, along with an evolutionary algorithm, NSGA-II is used as the optimization tool in extracting the solution of the bi-objective capacitated multi-vehicle allocation of customers to distribution centers problem. In order to test the validity of the proposed algorithm, the LP-metric method is used. In this section, the LP-metric and NSGA-II approach are described to solve the problem.

4.1 LP-metric method

In this approach, the decision maker must define the reference point z to attain. Then, a distance metric between the referenced point and the feasible region of the objective space is minimized. The aspiration levels of the reference point are introduced into the formulation of the problem, transforming it into a mono-objective problem. For instance, the objective function can be defined as a weighted norm that minimizes the deviation from the reference point.

Using the LP-metric, the problem can be formulated in the following way:

$$\begin{aligned} \text{MOP}(\lambda, z) &= \min \left(\sum_{j=1}^n \lambda_j |f_j(x) - z_{j1}|^p \right)^{1/p} \\ \text{s.t.} & x \in S \end{aligned} \quad (13)$$

where $1 \leq p \leq \infty$, λ_j is the weight of j th objective function and z is the reference point [11;12].

4.2 Non-dominated sorting genetic algorithm (NSGA-II)

GA is a type of evolutionary algorithm in which a population set of solutions (chromosomes) move toward a better set. The evolution usually starts from a random population in the first generation (iteration). In each generation, the fitness of all chromosomes in the population is evaluated. Chromosomes are then stochastically selected from the current population (based on their fitness), and modified using the GA operators (crossover and mutation) to perform a new population. The new population is then used in the next iteration of the algorithm. Commonly, the algorithm process stops in the maximum number of generations. By increasing the number of objectives in different optimization problems, applications of the multi-objective GA (MOGA) have been widely developed from concepts borrowed from the single-objective GA (SOGA). Srinivas and Deb (1994) used the non-dominated sorting concept on the GA (NSGA) [13]. Then, NSGA-II, which is proposed by Deb et al. (2002), is one of the most efficient and famous multi-objective evolutionary algorithms [14]. Steps of the NSGA-II are as follows:

In the first step, a set of random solutions (chromosomes) with a uniform distribution are produced. The first generation is a $N \times D$ dimensions matrix, in which N and D are identified as the number of chromosomes (solutions) and decision variables (genes), respectively. There are dominated and non-dominated solutions in the population that create different Pareto. In the second step, chromosomes are classified into the aforementioned Pareto using Eq. (14):

$$d_{ij} = \sum_{m=1}^M (f_m^{I_j+1m} - f_m^{I_j-1m}) / (f_m^{Max} - f_m^{Min}) \quad (14)$$

where: d_{ij} = crowding distance of j^{th} solution; M = number of objectives; $f_m^{I_j+1m}$ and $f_m^{I_j-1m}$ = values of m^{th} objective for $(j - 1)^{th}$ and $(j + 1)^{th}$ solution; f_m^{Max} = maximum value of m^{th} objective function among solutions of the current population; and f_m^{Min} = minimum value of m^{th} objective function among solutions of the current population. In the aforementioned equation, index I_j denotes the j^{th} solution in the sorted list and $(j - 1)$ and $(j + 1)$ are the

two nearest neighboring solutions on both sides of I_j . The algorithm then searches mostly near points (solutions) with more value of d_{ij} . This process will cause the more uniform distribution of the resulting Pareto and a vast range of selections for the decision makers. The Pareto is then ranked from the best to the worst solutions, in which the comparison criterion is the distance from ideal Pareto front with the assumed location. In the third step, the selection operator which is the crowded tournament is considered. It is used to select appropriate chromosomes (which are called parents) from the previous generation. The crowded tournament operator compares different solutions using two criteria: (1) a non-dominated rank and (2) a crowding distance in the population. In this process, if a solution dominates the others, it will be selected as the parent. Otherwise, the solution with the higher value of crowding distance will be selected as the winner. The crowding distance is an important concept proposed by Deb et al.(2002) in his algorithm NSGA-II. It serves for getting an estimate of the density of solutions surrounding a particular solution in the population. Deb and Agrawal (1995) developed a simulated binary recombination (crossover) operator, called the SBX operator, to combine two chromosomes and create two new chromosomes (children) [15]. This operator is similar to one-point cut crossover in the binary data. The probability distribution is calculated as:

$$P(\beta_i) = 0.5 (\eta_c + 1) \beta_i^{\eta_c}, \text{ if } \beta_i \leq 1, \text{ and otherwise } P(\beta_i) = 0.5 (\eta_c + 1) \beta_i^{1/\eta_c + 2} \quad (15)$$

$$\beta_i = (2u_i)^{1/\eta_c + 1}, \text{ if } u_i \leq 0.05, \text{ and otherwise } \beta_i = (1/2(1-u_i))^{1/\eta_c + 1} \quad (16)$$

In which: $P(\beta_i)$ = crossover probability; β_i = difference between the objective functions of parents and children; η_c = a constant which shows the difference between the objective functions of parents and children (a large value of η_c gives a higher probability for creating near-parent solutions); and u_i = a random value in $[0, 1]$. The above mentioned difference between parents and children is calculated with Eq. (17) and the children's values are produced by Eq. (18)[16]:

$$\beta_i = \left| x_1^{child} - x_2^{child} / x_1^{parent} - x_2^{parent} \right| \quad (17)$$

$$x_1^{child} = 0.5[(1 + \beta_i)x_1^{parent} + (1 - \beta_i)x_2^{parent}] \quad (18)$$

$$x_2^{child} = 0.5[(1 - \beta_i)x_1^{parent} + (1 + \beta_i)x_2^{parent}]$$

where x_1^{child} , x_2^{child} = value of the first and second child's chromosomes and x_1^{parent} , x_2^{parent} = value of the first and second parent chromosomes,

respectively. The other GA operator is mutation. A polynomial mutation operator that was proposed by Deb and Goyal (1996) has been used in this paper:

$$\mu_i = (2r_i)^{1/(\eta_m + 1)} - 1, \text{ if } r_i < 0.5, \text{ and } \mu_i = 1 - [2(1 - r_i)]^{1/(\eta_m + 1)}, \text{ if } r_i \geq 0.5 \quad (19)$$

in which μ_i = mutation value; r_i = a random value in $[0, 1]$; and η_m = distribution constant of mutation. The μ parameter is added to the parent gene value, as follows:

$$x^{child} = x^{parent} + \mu \quad (20)$$

A new generation which is a combination of the parents' and children's chromosomes is then produced. In the new generation, different chromosomes are ranked and chromosomes of the first rank are selected for the next generation. If the number of these chromosomes is less than the population size, chromosomes with a lower rank will be added to fulfill the new generation. Figure 1 shows the NSGA-II procedure.

4.3 Best compromise solution

Once the Pareto optimal set is obtained, it is possible to choose one solution from all solutions that satisfy different goals to some extent [18]. Due to the imprecise nature of the decision maker's (DM) judgment, it is natural to assume that the DM may have fuzzy or imprecise nature goals of each objective function [19]. Hence, the membership functions are introduced to represent the goals of each objective function; each membership function is defined by the experiences and intuitive knowledge of the decision maker. In this study, a simple linear membership function is considered for each of the objective functions [20]. The membership function is defined as follows:

$$\begin{aligned} M_i &= 1 && \text{if } F_i \geq F_i^{max} \\ M_i &= (F_i^{max} - F_i) / (F_i^{max} - F_i^{min}) && \text{if } F_i^{min} < F_i < F_i^{max} \\ M_i &= 0 && \text{if } F_i \leq F_i^{min} \end{aligned} \quad (21)$$

where F_i^{min} and F_i^{max} are the minimum and the maximum value of the i th objective function among all non-dominated solutions, respectively. The membership function M is varied between 0 and 1, where $M = 0$ indicates the incompatibility of the solution with the set, while $M = 1$ means full compatibility. For each non-dominated solution k , the normalized membership function M_k is calculated as

$$M_k = \sum_{i=1}^{N_{obj}} M_i^k / \sum_{k=1}^m \sum_{i=1}^{N_{obj}} M_i^k \quad (22)$$

Where m is the number of non-dominated solutions and N_{obj} is the number of objective functions. The function M_k can be considered as a membership

function of non-dominated solutions in a fuzzy set, where the solution having the maximum membership in the fuzzy set is considered as the best compromise solution.

4.4 Adaptation of NSGA-II in the proposed model

In this paper, NSGA-II is coded in MATLAB software and tested on a Core 2 Duo/2.66 GHz processor. Eight numerical cases in small scale and eight numerical cases in large scale are provided to demonstrate the

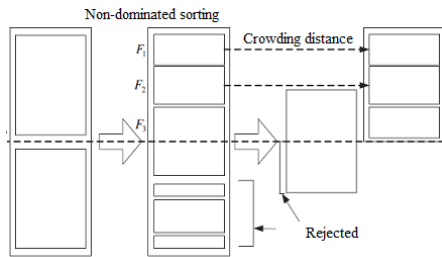


Fig. 1. Schematic of the NSGA-II procedure [17]

application of this method. To obtain the best Crossover probability and Mutation probability, an auto tuning approach is used. First some numbers for example 10 numbers in the range 0.55 to 0.85 are selected randomly for crossover probability and then by observing the best answer, we tried the next random number could be close to the crossover probability of the best answer. Exactly the same procedure in the range 0 to 0.45 is repeated for mutation probability. These initial ranges are considered according to both existing literature in the field of NSGA-II and some tentative running of NSGA-II program. This process is performed by an external NSGA-II program for auto tuning parameters. As shown in Figure 2, crossover probability equals $P_c = 0.73$, mutation probability equals $P_m = 0.37$. These two parameters are tuned in 10th iteration approximately. It means that if we consider 10 random numbers in each range in each iteration, these parameters are tuned with considering 100 times running of algorithm. In this paper, Initial population size $npop$ is assumed 50 and 150 for small and large scales, respectively.

As shown in Figure 3, to illustrate the performance of the used procedure to optimize the proposed model, problem 1 in small scale is considered. To check the quality of solutions obtained by the NSGA-II, four evaluation metrics including: (1) number of Pareto solutions (NOS), (2) maximum spread or

diversity metric [21], (3) mean ideal distance (MID) metric [22] and (4) time of running have been used. Diversity and MID metrics are formulated as follows:

$$\text{Diversity} = \sqrt{\sum_{i=1}^m (\max_n f_n^i - \min_n f_n^i)^2} \quad (23)$$

$$\text{MID} = \sum_{i=1}^n C_i / n \quad (24)$$

where in Eq. (23), m is the number of objectives, n is the number of Pareto solutions and in Eq. (24), n is the number of Pareto solutions and C_i is the distance of i th Pareto solution from ideal point $((0,0)$ in bi-objective minimization).

Figure 4 shows MID metrics for problem 2 in small scale. For better display, MID axis is considered under Logarithmic scale. As shown in Figure 4, in the first iterations, there are more infeasible solutions and they cause adding large penalty functions to objective values, but during the process of algorithm, the infeasible solutions because of great objective values are discarded and objective values are more real and then convergence process goes smoothly. Also, to view the output of the decision variables, one Pareto member of problem 6 in small scale is given in the appendix. Tables 1 and 2 show the computational results of NSGA-II for some small and large scale problems with two iteration numbers 200 and 500. Figure 5 shows the comparison of optimal Pareto front of problem 1 in small scale in 200th and 1000th iteration numbers. For more validation of the proposed method, some small cases are solved with LP-metric by Lingo 13.0. Table 3, shows the results of LP-metric method for problem 2, 4 and 5 in small scale with $p=1, 2$ and 3. The weights of objectives are assumed identical. In table 4 the best solution of Pareto front (best compromise solution according to section 4.3) are shown for these small scale cases.

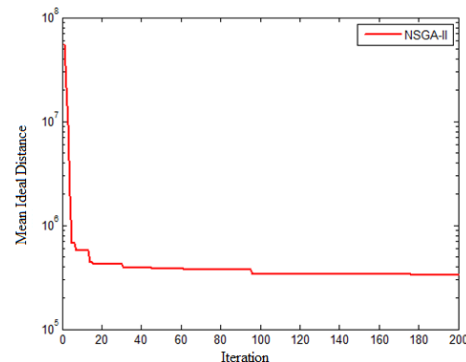


Fig. 4. MID metric for problem 8 in small scale

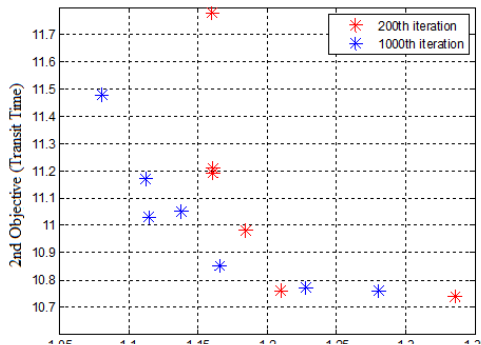


Fig.5. Comparison of Pareto fronts of problem 1 in small scale by NSGA-II (200th and 1000th iteration)

Results show the average of percentage errors of objective functions compared to the LP-metric method are 1.05% and 0.05%, respectively. Furthermore, it can be seen that with increasing size of the problems, while the time of problemsolving increases exponentially by using the LP-metric method, the running time of NSGA-II is more stable.

Table3. Computational results of LP-metric method for some small scales

Some problems in small cases	<i>p</i>	First objective	Second objective
Problem 2	1	58740	1.894
	2	59791	1.879
	3	59791	1.879
Problem 4	1	297399.63	1.850
	2	299489.48	1.825
	3	302801.48	1.817
Problem 5	1	237972.92	1.439
	2	252391.48	1.390
	3	252391.48	1.390

5 Conclusion

In this paper, a bi-objective optimization model for capacitated multi-vehicle allocation of customers to distribution centers is proposed. The optimization objectives are to minimize transit time and total cost including opening cost, assumed for opening a potential DC and shipping cost per unit from DC to the customers. Considering different types of vehicles leads to a more realistic model and causes more conflicting in these two objectives. An evolutionary algorithm named non-dominated sorting genetic algorithm (NSGA-II) is presented as the optimization tool to solve the model. The crowding distance technique is used to ensure the best distribution of the non-dominated solutions. For ensuring the robustness of the proposed method, the computational results in some small cases are compared with those obtained by LP-metric method. Results show the percentage errors of objective functions compared to the LP-

metric method are less than 2%. Furthermore, it can be seen that the time of problem solving increases exponentially by using the LP-metric method, so these show the advantages and effectiveness of NSGA-II in reporting the Pareto optimal solutions in large scale. Future research may use other multi-objective meta-heuristic algorithms for validation and comparison with NSGA-II. Additionally, may be modeled location allocation for non-deterministic condition, such as stochastic demand. Furthermore, given the successful application of a NSGA-II to the bi-objective warehouse allocation problem, the used algorithm can be modified to obtain non-dominated solutions for warehouse allocation problems with more than two objectives.

Appendix

One Pareto member for problem 6 in small scale in 200th iteration by NSGA-II approach is as follows (where number of customers =14, number of DCs =6, types of vehicles = 3):

Number of Pareto front Members = 3

For 1st element of Pareto front, depot vector is:

$$y = 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1$$

For 1st element of Pareto front, Allocation matrix is:

X=	DC1	DC2	DC3	DC4	DC5	DC6	Types of vehicles
customer1	0	0	0	0	0	1	3
customer2	0	0	1	0	0	0	3
customer3	0	0	0	1	0	0	3
customer4	0	0	0	1	0	0	2
customer5	0	0	0	0	1	0	2
customer6	0	1	0	0	0	0	3
customer7	1	0	0	0	0	0	1
customer8	0	1	0	0	0	0	3
customer9	0	0	0	1	0	0	1
customer10	0	0	1	0	0	0	3
customer11	0	0	0	0	1	0	2
customer12	0	0	0	0	1	0	2
customer13	0	0	1	0	0	0	2
customer14	1	0	0	0	0	0	1

For 1st element of Pareto front, objective values are:

Final objective values: Total Cost = 3.1670e+005

Transit Time = 2.8700

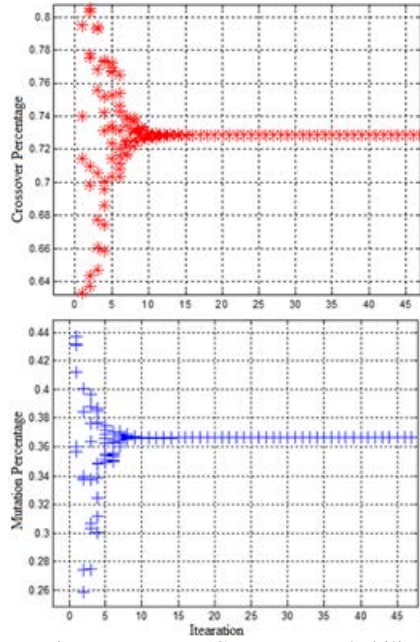


Fig.2. Auto tuning parameters (Crossover Probability, Mutation Probability)

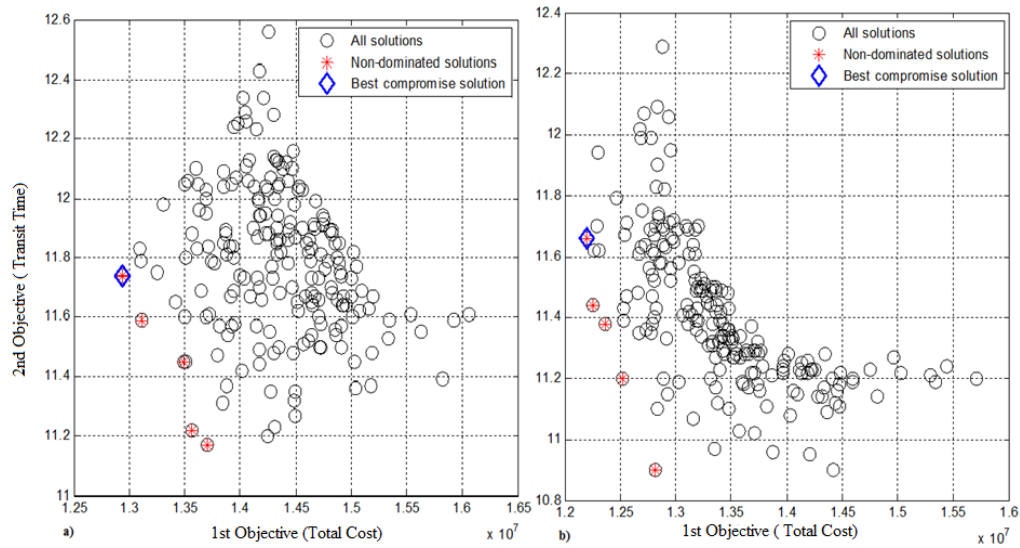


Fig.3. Pareto front of problem 1 in small scale by NSGA-II
 a) 3rd and b) 100th iteration with $n_{pop}=200$

Table1. Computational results of NSGA-II for small scale cases

Problems	Number of customers	Number of depots	Types of Vehicles	NSGA-II with 200 iterations			NSGA-II with 500 iterations		
				NOS	Diversity	Time (min)	NOS	Diversity	Time (min)
problem 1	21	7	3	6	9754	2.65	9	9948	5.85
problem 2	8	3	2	3	6350	2.36	7	7660	5.25
problem 3	15	3	2	4	4614	2.54	4	3713	5.79
problem 4	10	4	2	4	1792	2.40	3	3272	5.53
problem 5	12	5	2	2	2045	2.50	4	6572	5.76
problem 6	14	6	3	3	3250	2.68	4	7910	5.98
problem 7	20	5	2	2	3800	2.52	3	3809	5.53
problem 8	26	4	2	6	8155	2.45	5	9543.9	5.80
Average	-	-	-	3.75	4970	2.51	4.875	6553.49	5.69

Table2. Computational results of NSGA-II for large scale cases

problems	Number of customers	Number of depots	Types of Vehicles	NSGA-II with 200 iterations			NSGA-II with 500 iterations		
				NOS	diversity	Time (min)	NOS	diversity	Time (min)
problem 1	32	7	3	6	7987	19.45	5	8984	41.44
problem 2	40	11	3	5	1954	19.52	6	4778	43.12
problem 3	24	6	2	4	5955	18.85	5	6389	45.32
problem 4	70	9	3	6	6533	20.66	6	8348	49.24
problem 5	62	9	3	3	1706	18.60	5	1986	50.02
problem 6	80	7	3	2	3786	20.68	5	4961	46.53
problem 7	68	11	3	4	1825	19.79	4	1950	41.58
problem 8	60	10	3	6	2519	18.68	6	3792	39.98
Average	-	-	-	4.5	4033.12	19.53	5.25	5148.5	44.65

Table4. Comparison of NSGA-II and LP-metric method for solving some small scale cases

Some problems in small cases	Number of variables	LP-metric method			Best compromise solution by NSGA-II (500 iterations)			Error of NSGA-II	
		First objective	Second objective	Time (min)	First objective	Second objective	Time (min)	First objective	Second objective
Problem 2	51	58740.00	1.894	5.25	59147.00	1.882	8.65	0.69%	0%
Problem 4	84	297399.63	1.850	5.53	300419.52	1.850	14.08	1.01%	0%
Problem 5	125	237972.92	1.439	5.76	241403.43	1.441	21.37	1.44%	0.14%
Average	-	198037.52	1.73	5.51	200323.32	1.72	14.70	1.05%	0.05%

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