Application of linear and parabolic regression methods to determine the customs value of foreign trade goods

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Abstract: - This article is devoted to the issues of determining the customs value of goods using linear and parabolic regression models. An analysis of scholars' scientific works on this issue has been conducted, and the relationship between the value of goods and customs payments has been modeled. A mathematical model for forecasting customs payments has been developed and the essence of this model has been revealed based on possible values of regression coefficients. The article pays special attention to the practical implementation of the developed algorithm, presents generalized results on exemption from customs escort of foreign trade cargo and an increase in the amounts of insurance policies

Key-Words: - customs control, customs value, linear and parabolic regression, mathematical and statistical modeling, customs duties, forecasting

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1 Introduction

In the context of the accelerated development of global trade relations and the deepening of digitalization processes, the reliable calculation of customs duties is becoming an important factor in the stability of foreign economic activity and the guarantee of revenues to the state budget. Determining customs value is not only the main link in calculating customs duties, but also plays an important role in protecting the domestic market, ensuring fair competition and automating customs risk analysis.

Automated systems, mathematical models, and forecasting mechanisms are widely used in the customs sector today. In particular, statistical regression methods are widely used in the analysis and forecasting of state budgets of countries.

At the same time, today's analysis shows that international e-commerce is developing with unprecedented patterns and occupies a strong place in global trade. In particular, e-commerce retail sales in 2020 amounted to $3,932.4 billion, which is 2.1 times more than in 2016. By 2024, this figure amounted to 6065.9 billion US dollars.

The largest e-commerce enterprises in the world are Amazon.com , Alibaba.com , Ebay and others. In 2019, Amazon.com held 50% of the total US e-commerce market. In China, 30% of food is sold through e-commerce, with the share of e-commerce Alibaba.com It is about 80% [1].

The development of e-commerce in Uzbekistan has also risen to the level of state policy, with separate legislative and government decrees devoted to this issue. [2].

This objective reality is increasingly putting on the agenda the issue of establishing effective customs control over transported goods based on external e-commerce and ensuring proper, fair and complete collection of customs duties. This requires a correct and strict determination of the customs value of the specified goods based on the norms of national legislation. Because revenues to the state budget in the form of customs payments are determined in inseparable dependence on the customs value of foreign trade goods.

In this regard, this article discusses the application of linear and parabolic regression models to the task of determining the customs value of goods in the process of organizing foreign trade by means of electronic commerce, as well as the implementation of its algorithms in practice

2 Statement of the problem of determining the customs value of goods in conditions of uncertainty of the contract price of the goods

As you know, the customs value of a foreign trade product is the main element in calculating customs payments or an insurance policy. However, in the context of e-commerce, customs authorities are provided with only accompanying documents as an appendix to the declaration, and a foreign trade contract is often missing. Therefore, due to the lack of a contractual value of the goods, it is necessary to determine the customs value of the goods based on other sources in order to calculate customs duties or an insurance policy.

The main norms of national legislation on customs payments and establishing the customs value of goods are based on the Customs Code [3] and the Tax Code [4] of the Republic of Uzbekistan, which determine the types of customs payments, the procedure for their calculation and payment.

Also adopted are the normative acts of international organizations responsible for this issue, which include the "Agreement on Customs Valuation" of the World Trade Organization [5] and the recommendations of the World Customs Organization on the standardization and simplification of customs procedures [6], which are key international documents in determining customs value.

The issue of calculating customs payments and determining the customs value of goods is of great scientific and practical importance. Therefore, scientific research in this area is carried out continuously, based on the current situation in international trade. For example, the issue of simplification of customs procedures was studied by Brown and his colleagues, who note that the introduction of electronic databases gives the expected results in this direction [7].

The problem of using mathematical models and databases for calculating and forecasting customs duties was reflected in the research of Liu and Wang [8]. The Smith and Johnson study analyzed whether the use of automated systems for calculating customs duties could promote the promotion and transparency of foreign trade [9]. The use of statistical methods in forecasting customs duties and planning foreign trade is central to the article by Kim and Lee. It showed that linear regression helps businesses predict future costs and make financial plans [10].

Although the above principles of procedures, as well as the results of the above scientific studies, allow us to estimate the customs value of goods and predict customs payments, the application of regression methods to this process allows us to speed up the identification of goods and automate the computational process.

3 Solving the problem of determining the customs value of goods in conditions of uncertainty of the contract price of goods

In connection with the above conclusions, a number of models for assessing the customs value of foreign trade goods are considered below.

3.1 Linear Regression

Linear Regression is applied based on the data of shipping documents (invoice). In this case, the formula for calculating customs duty is determined as follows:

$$Y = \beta_0 + \beta_1 x + \epsilon \qquad (1)$$

For example $if\ \beta_0 = 500,\ \beta_1 = 0{,}25, x = 5\,000\ \$$ then $Y = 500 + 0.25 \cdot 5000 = 1\,750\ \$$

Here:

Y - customs duty amount (projected amount);

x - cost of goods (independent variable);

β_0- permanent part of the payment (customs expenses);

customs duty is also present when the cost of the goods x=0, β_0- reflects customs services, documentation, and other minimum costs.

in the real case x=0 cannot be, and β_0 is the point of intersection of the regression line with the Y axis.

β_1- tariff coefficient (duty rate);

shows how much customs duties increase when the value of the goods (x) increases by 1 unit (for example: 1 US dollar).

β_1- it can be equal to or close to the customs payment rate (for example:0.3 →30%).

ϵ – method inaccuracies.

a difference that arises because the regression model cannot fully capture all influencing factors. It is estimated as the relative error between the cost of the product and the cost of an identical product, which is registered in the database of the customs service - $o(\Delta x)^2$.

3.2 Parabolic Regression

Parabolic Regression (d=2) applies to similar or similar goods for which shipping document (invoice) data is not available and which previously had similar or similar goods registered in the database.

In this case, the formula for calculating customs duty is determined as follows 11]:

$$Y = \beta_0 + \beta_1 x + \beta_2 x^2 + \epsilon \qquad (2)$$

For example: if β_0=500, β_1=0,25, β_2=0.00001, x=5000 $ then,

$$Y = 500 + 0.25 \cdot 5000 + 0.00001 \cdot (5000)^2 = 500 + 1250 + 250 = 2\,000\ \$$$

3.3 Multivariate linear regression

Multivariate linear regression is applied to similar or similar products for which shipping document (invoice) data is not available and which previously had numerous similar or similar products registered in the database.

In this case, the formula for calculating customs duty is determined as follows:

$$Y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_n x_n + \epsilon \qquad (3)$$

Here $x_1, x_2 \ldots x_n$ are factors such as weight, size, country of origin of the goods

For example: the calculation of the total amount of customs duties, taking into account several factors for 1 product, is shown in Table 1.

Table 1.

Factor	x_n	value	β_n	Sum
Costs of customs services	-	-	*500*	*500*
Cost of goods	x_1	*5000*	*0.25*	*1 250*
weight of goods	x_2	*20*	*-10*	*-200*
Number of Operations	x_3	*6*	*-50*	*-300*
Y				**1 250 $**

In the example given in Table 1, the following factors affecting the amount of customs duties are taken into account: the cost of customs services is 500$, the cost of the goods is 5000$, weight of goods – 20 tons, number of operations related to transportation, loading and unloading operations - 6 operations. From the above regression coefficients, it can be seen that factors related to the costs of the cargo owner, such as the weight of the goods, the number of operations related to transportation, and loading and unloading operations negatively affect the amount of customs duties. This leads to significant support for participants in foreign trade and eases their tax burden.

3.4 Ridge and Lasso regression

Ridge and Lasso regression (regularization models) are used in cases where there are a sufficient number of characteristics and correlations for a product.

The **Ridge Regression** formula is defined as follows:

$$\hat{\beta} = argmin\{\|y - X\beta\|^2 + \tau\|\beta\|^2\} \qquad (4)$$

τ- regulation coefficient;

$\|\beta\|^2$- sum of the squares of all coefficients;

if τ>0, then the model forces the coefficients to decrease.

The **Lasso Regression** formula is defined as follows:

$$\hat{\beta} = argmin\{\|y - X\beta\|^2 + \tau\|\beta\|_1\} \qquad (5)$$

$\|\beta\|_1$- sum of absolute values of coefficients;

some β_i may take the value 0 (model factors are not taken into account).

For example: in the above case, if there is only one factor *x*=5000$, then the impact of regulation will be very small, because:

$\beta_1 = 0.25$ the meaning is small,

if this coefficient is very small (<0.001), then

Y ≈1 750 $ (there may be a difference± 1-2 $)

3.5 Random Forest and Decision Tree Regressor

Random Forest and Decision Tree Regressor (artificial intelligence model) is used when there is a sufficiently large database (big data) about a product brand.

If we denote the accuracy coefficient for the aforementioned regression models as R^2, it is a statistical indicator that predicts customs payments. This coefficient is determined as follows [12].

$$R^2 = 1 - \frac{\sum(y_i - \hat{y}_i)^2}{\sum(y_i - \bar{y}_i)^2} \qquad (6)$$

Here:

y_i- the volume of actual customs payments;

$\hat{y}_i$- model forecast;

$\bar{y}_i$- average value of payments

Using the values of this coefficient, the efficiency of the model can be determined as follows:

$$\begin{cases} R^2 = 1, \ perfect\ match \\ R^2 \geq 0.95, \ high\ reliability \\ 0.7 \leq R^2 \leq 0.9, \ average\ reliability\ level \\ R^2 < 0.7, \ unreliable\ level \end{cases} \qquad (7)$$

4 Conclusion

The above methods and algorithms for calculating the customs value of goods and the amount of the guarantee for customs payments are now implemented in practice as part of customs information systems. Information systems implemented in practice show their effectiveness.

In particular, until 2023, the customs escort procedure was applied to all imported goods transported by road in the Republic of Uzbekistan. The introduction of algorithms for calculating the customs value of goods and the amount of the customs payment guarantee since 2023 has made it possible to automatically exempt these goods from customs escort procedures. As a result, in the first half of 2025, 83.5% of the total volume of imported goods transported by road in the Republic of Uzbekistan were exempt from customs escort procedures. This figure was 81.9% in 2023 and 87.1% in 2024. And the amounts of insurance policies calculated automatically amounted to 15.4 billion soums in the first half of 2023, 20.4 billion soums in 2024 and 45.9 billion soums in 2024, respectively.

The use of automated systems, mathematical models and linear regression improves the accuracy of calculations and allows for the forecasting of future customs payments.

References:

[1] Cafarova D. F., Karshiboev Kh. Q. Analysis of e-commerce and its problems in Uzbekistan // Scientific journal "Academic Research in Educational Sciences," 2023, Issue 6. Volume 4. Pages 41-51.

[2] [2]. Resolution of the Cabinet of Ministers of the Republic of Uzbekistan dated December 26, 2024 No. 885 "On measures for the further development of the sphere of electronic commerce in the Republic of Uzbekistan" (// https://lex.uz/ru/docs/7284840).

[3] Customs Code of the Republic of Uzbekistan // adopted by the Oliy Majlis of the Republic of Uzbekistan on January 20, 2016. (https://lex.uz/ru/docs/2876354)

[4] Tax Code of the Republic of Uzbekistan // adopted by the Oliy Majlis of the Republic of Uzbekistan on December 30, 2019. (https://www.lex.uz/acts/4674902)

[5] World Trade Organization. Agreement on Implementation of Article VII of the GATT 1994 (Customs Valuation Agreement). WTO, 1994. (https://www.wto.org/english/docs_e/legal_e/20-val.pdf)

[6] World Customs Organization. International Convention on the Simplification and Harmonization of Customs Procedures (Revised Kyoto Convention). WCO, 1999 (https://www.wcoomd.org/en/topics/facilitation/instrument-and-tools/conventions/pf_revised_kyoto_conv.aspx)

[7] Liu, X., & Wang, Y. Mathematical Modeling for Optimal Customs Duty Calculation. // International Journal of Appliyed Mathematics, 2019, 8(1), pp.23-37.

[8] Smith, J., & Johnson, L. Automation in Customs Procedures and Its Impact on International Trade. // Journal of International Trade Studiyes, 2020, 15(2), pp.112-130.

[9] Brown, T., et al. Enhancing Efficiyency in Customs Procedures through Automation. // Global Trade Reviyew, 2021, 9(4), pp.78-95.
[10] Kim, S., & Lee, J. Forecasting Customs Dutiyes Using Statistical Methods. International // Journal of Trade and Commerce, 2020, 12(3), pp.65-80.
[11] Seber, G.A.F., & Wild, C.J. (2003) Nonlinear Regression // Springer Seriyes in Statistics. 2003, ISBN: 978-0-471-18394-2
[12] Montgomery, D.C., Peck, E.A., & Vining, G. GIntroduction to Linear Regression Analysis (5th Edition). 2021, ISBN: 978-1-118-37865-0

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