International Journal of Education and Learning Systems
http://iaras.org/iaras/journals/ijels

Modelling of a Heat Exchange Process for Teaching the Laplace Transform in Engineering

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Abstract: - Based on the educational innovation perspective implemented in engineering programmes, new teaching paradigms are being used to achieve greater intellectual capacity development, skills acquisition, the replacement of obsolete techniques with more efficient and faster ones, and better integration of knowledge in the teaching-learning process. With this objective in mind, and thanks to the inclusion of information technology, we have implemented new pedagogical methodologies and various teaching strategies for transdisciplinary work in the development of the topic Laplace Transform corresponding to the subject Advanced Calculus.
The teaching proposal presented corresponds to a system related to the analysis of a heat transfer device, in which the concept of transfer function and the application of a step disturbance when the temperature of a fluid is raised are addressed analytically and graphically, giving meaning to the curriculum content and facilitating the interpretation and conceptualisation of the theory.

Key-Words: - Laplace Transform, energy, modelling, transdiscipline, simulation, technology, visualization.

Received: July 14, 2025. Revised: October 29, 2025. Accepted: November 24, 2025. Published: March 30, 2026.

1 Introduction

The content of academic programmes in engineering degrees is highly abstract and generalised, a situation that is particularly evident in subjects in the area of advanced mathematics. Teaching mathematics through the modelling of dynamic systems helps students to engage responsibly with the content being studied, promoting the development of independent learning and facilitating the meaningful incorporation of concepts, while preparing them for their future professional careers.

In order to train versatile professionals, it is essential to coordinate the degree programme's curriculum in order to achieve the required professional competencies, as well as to select content and activities that are in line with these competencies. Within this framework, institutional policies and academic management processes accompany curriculum evaluation and development.

A relevant aspect is the design of curricular activities based on learning approaches oriented towards concept building, aimed at promoting student leadership in the teaching and learning process and reinforcing the transdisciplinary nature of curricular content.

The selection of alternative mathematical models has a positive influence on students' interest and motivation. If the proposed work is attractive and relevant, it enables the analysis and visualisation of complex systems, which promotes a better understanding of the content covered.

This proposal is aimed at students of the Advanced Calculus course in the Mechanical Engineering degree programme. It takes as its starting point knowledge of the Laplace transform, incorporating its application through the use of computational tools, specifically the software Mathematica. The aim of the proposal is to introduce and develop fundamental conceptual and methodological guidelines that will later be applied in the areas of Fluid Mechanics and Automatic Process Control.

To this end, a simplified version of a water heating system is analysed, mathematically modelled and subjected to excitation by a temperature-dependent step function, which allows the dynamic response of the system to be studied and facilitates understanding of the concepts involved.

2 Methodology

The engineering analysis is based on the use of computer-aided design and planning systems. It is essential to foster an environment in which various activities and forms of communication can be developed, as these constitute a central axis of the teaching and learning process.

Within this framework, theoretical, practical and technological activities are organised, in which students carry out applications related to the content covered in class, establishing relationships between the topics of the subject, other subjects at the same level or at different levels in the area, as well as with other disciplines, by completing projects whose complexity is determined by the basic knowledge previously acquired.

Until not many years ago, it was common for research on the teaching-learning of any area of knowledge, including mathematics, to focus on cognitive processes, or how a student is able to capture, encode, store and work with the information that is normally transferred to him or her by the teacher [1]

Likewise, teaching strategies are defined based on the development of multiple practical activities, without neglecting the corresponding theoretical foundation. In coordination with the areas of the professional cycle and in line with a transdisciplinary education, activities are proposed with a strong focus on professional practice, aimed at identifying and analysing industrial applications.

These activities take the form of projects with clearly established objectives and goals, which promote ongoing updating and continuous coordination between the different areas of knowledge that make up the degree programme.

The aim is to create a working environment where students can be reflective and critical, make decisions and also identify the mistakes made in the process. This creates a need for students to understand the subject matter and create the internal conditions for actively and independently assimilating new knowledge.

In this type of experience, the teacher takes on the role of guiding the process and acts as a facilitator of learning until a thorough understanding of the subject under study is achieved, including questions that activate a mechanism of continuity in the expansion and subsequent investigation of different lines of inquiry on the proposed problem.

It is essential that students incorporate and use computational methods from the beginning to the end of their degree programme in their chosen specialisation, as this helps to strengthen an integrated view of mathematics and its applications, as well as providing them with the essential tools for their professional performance. Lack of competence in scientific solving problems in various scientific spheres can be viewed as a function of inadequate understanding of basic mathematical principles [2].

In this sense, when the learning process reproduces the forms of action typical of the professional practice of engineering, it can be said that truly meaningful learning is achieved.

3 Motivation and Objectives

The transdisciplinary approach to teaching aims to relate the content and concepts taught in class to real engineering projects, in order to equip future professionals with the necessary tools to identify the relevant variables of a problem, interpret them and propose solutions from among different alternatives, thereby increasing their capacity for analysis, rational selection of proposals and decision-making based on the solutions found.

The proposed teaching activities consist of emphasising theoretical research by students with the guidance of teachers, then drawing an analogy with the physical parameters of the subject under study, and finally using the information gathered to solve the proposed problem using technological resources.

The necessary change in paradigm in the teaching profession requires training and innovation, as is recognised by international organisations such as the UNESCO (1998), which makes explicit reference to the need for teachers to become involved in the development of student-focussed programs. This will require a renovation of existing contents, methods, practices and learning experiences. However, innovation and training cannot be separated from one's convictions, and their interpretation will be determined by the theoretical position of each teacher [3].

The methodological approach for presenting mathematical content is based on finding models that simulate the situation to be formulated or the technical situation in mathematical terms. To do this, a simplified situation is presented, translated into mathematical terminology, and worked on using the resulting model. This methodology stimulates interest in discovery and builds confidence in using the educational aspects of mathematics in relation to other areas of knowledge, such as the analysis of the dynamics of a given fluid.

Mathematical competence formation in university students is a pedagogical process that takes place in several stages. According to the formation logic in the Mathematics academic discipline, the formation of mathematical competence in university students will form in the following stages: Motivational target, content-activity and productive evaluative [4].

The pedagogical proposal is developed in clearly defined phases, organised in sequence. Firstly, the systems that will be analysed are presented in order to contextualise the issue to be addressed. Then, students move on to a stage of theoretical research, aimed at reviewing and understanding the fundamental concepts related to these systems. Next, modelling of these parameters and analysis of the relevant parameters is carried out, allowing for a deeper understanding of their behaviour and characteristics. Finally, the results obtained are interpreted from a technical perspective, which facilitates the articulation between the theoretical framework and the conclusions derived from the analysis carried out [5].

4 Didactic and Pedagogical Foundations of the Class

A theoretical-practical technology class is designed in two sessions of three hours each, in which the aim is to guide students through a meaningful learning process on the proposed topic.

The stages designed to carry out the class are as follows: forming groups of students, presenting a problematic situation, reviewing and subsequently selecting bibliographic material on the topic, reviewing the content developed in the theoretical class, modelling the situation and resolving the proposed model, and drawing conclusions.

In the laboratory class, students work in groups of up to three members. To carry out the planned task, they are presented with a simple engineering application case, with the aim of encouraging them to use computer tools. If lecturers know in advance the students, they could propose the pairs in order to guarantee that they will cooperate and collaborate during the experiment [6].

Students are free to form groups according to their own criteria and choose which one they want to participate in, but teamwork and fostering a collaborative environment are essential requirements. Each group has at least one desktop computer, but they are also allowed to bring their personal computers to work.

It is essential for the educational concept introduced in this paper that the degree of difficulty at the start of the projects seems to be beyond the present capabilities of the students. The knowledge and skills necessary to complete the tasks successfully will be taught during the course of the semester, thus producing an increased interest on the part of the students in the subjects they are studying. In this way we can compensate for one of the weak points in the educational system, namely the lack of time for reflection on knowledge gained and the interconnection of the different disciplines taught [7].

Since one of the mathematical competences is system modelling, students must be able to analyse and interpret different models. The purpose of the class is to connect the knowledge acquired with different tools that enable the resolution and validation of situations in changing scenarios.

5 Modelling of the System Under Study

In any research based on mathematical modelling, the validity of the conceptual model must be carefully considered to ensure that the simplifying assumptions, selected variables, and established relationships adequately represent the real phenomenon that is intended to be described, analysed, or predicted.

Actual experimental results generally suggest changes in the conceptual model, changes that in turn can provide a better mathematical model. In other words, there must be a sequential relationship between observation, theory, models and results, with a subsequent return to observations to validate the results found.

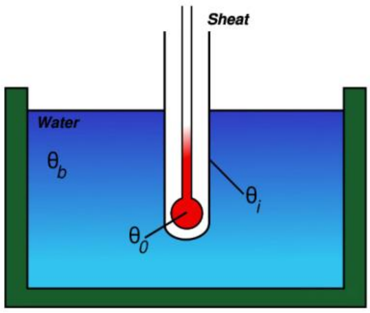

Fig.1: Diagram of the system under study

This methodology is applied in this study with the aim of finding a conceptual mathematical model that represents the physical system shown in Fig. 1, in order to then analyse the changes that occur in said system and proceed with the analysis to establish an approximate mathematical model to study the behaviour of the system under investigation.

In this experiment, students design a device consisting of a beaker containing water at 23 °C, into which a thermometer covered by a sheath containing 2 ml of pure glycerine at room temperature is inserted and adjusted using a perforated rubber stopper.

A fluid, in this case water, is heated in the container until it reaches its boiling point. At that moment, the thermometer and test tube are placed in the water bath, causing a sudden change in the thermometer reading. Fig. 1 presents a representative diagram of the device used in the study.

In this laboratory experiment, students first try to determine how the temperature on the thermometer changes when a step disturbance is applied that raises the temperature to 100 °C, using a data series of 74 values at constant time intervals.

The step-type disturbance is a mathematical function called the Heaviside step function [8], which is defined as:

$$\mu(t) = \begin{cases} 0 & si \quad t < 0 \\ 1 & si \quad t \geq 0 \end{cases}$$

In this experiment, the analysis of the dynamics of the heat exchange process is carried out in four working sessions, taking into account the following stages:

- 1st. session: Qualitative and numerical analysis of data.
- 2nd. session: Development of a conceptual model.
- 3rd. session: Changes to the developed model.
- 4th. session: Comparison of results: Validity of the theoretical model.

5.1 Qualitative and Numerical Analysis of Data

The students record the temperature measured by the thermometer at 5-second intervals. Table 1 is an extract from the dataset obtained by the students for their corresponding analysis.

$t\ (seg.)$	$\theta(t)$
0	23
5	23,5
10	24
15	25
20	26
25	29
30	33
35	37
40	41
45	45
50	50
… … … …	… … … …
315	93,5
320	94
325	94
330	94
335	94
340	94
345	94
350	94
355	94
360	94
365	94

Table 1. Extract of the data

The values obtained experimentally are plotted and shown in Fig. 2, using the comprehensive technical computing platform MATHEMATICA, which combines numerical and symbolic calculation, visualisation, programming, and data management.

Visualization of mathematical concepts and hands-on-manipulations with these abstract models in environment of dynamic maths software represent new forms of active learning methods [9].

Once the empirical data has been obtained, they are asked to find a function that fits these values as closely as possible. This task is not easy for students, as they encountered difficulties in finding a mathematical model that determines an appropriate fit, and proposed to do so using a function whose law is given by sections for two different intervals of the sample under study [10].

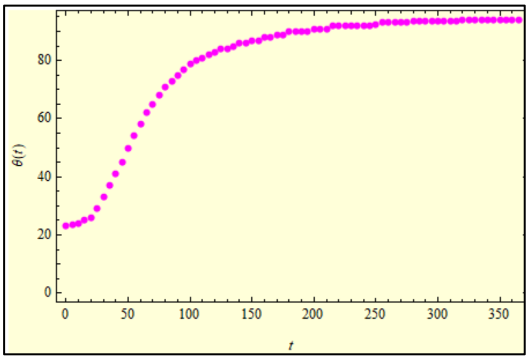

Fig. 2 Data series

The law of the function obtained in the adjustment is as follows [10]:

$$f(t) = \begin{cases} a + be^{-t} + ct^2 - dt^3 & si \quad 0 < t < 120 \\ gt^3 + ht^2 - je^{-t} + k & si \quad 120 \leq t < 365 \end{cases}$$

Being: $a = 22{,}77$, $b = 0{,}23$, $c = 0{,}015$, $d = 8{,}88 \times 10^{-5}$, $g = 8{,}25 \times 10^{-7}$, $h = 3{,}93 \times 10^{-4}$, $j = 2{,}96 \times 10^{54}$ y $k = 80{,}99$.

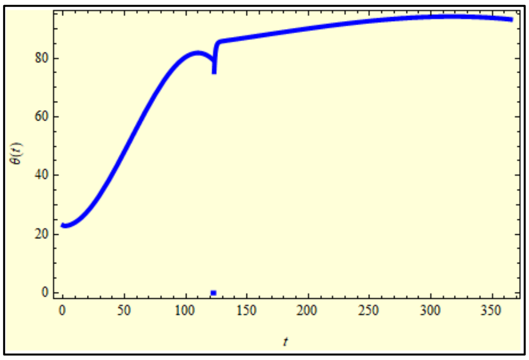

Fig. 3 Graph of f

The graphical representation of the function of f is observed in Fig. 3.

Once the graphs in Fig. 2 and Fig. 3 have been created, students generate Fig. 4, which is a new graphical representation that shows both the empirical data and the graph of the function f [10].

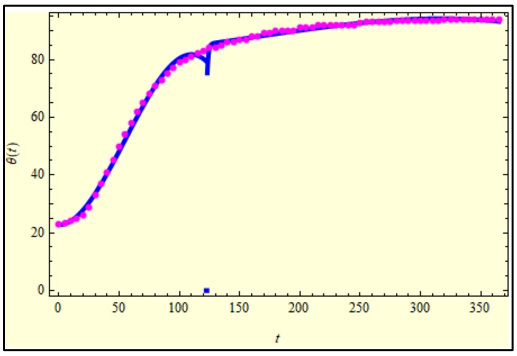

Fig. 4 Data comparison

5.2 Development of a Conceptual Model

In this workspace, students are asked to research the theoretical foundations of the experience. The students aim to analyse the changes that occur in this system with the aim of finding a conceptual mathematical model that represents the physical system shown in Fig. 1. The aim is to establish an approximate mathematical model that will enable them to analyse its behaviour.

The teachers in charge guide the research with the following questions:

a) What are the equations corresponding to the energy balance of the system under study?
b) Does the physical system correspond to a first or second order model? Why?
c) What kind of function is used to excite the system?
d) Are there applications in the chemical industry where the process under analysis takes place, or any equipment whose operation responds to the behaviour of the system under study?

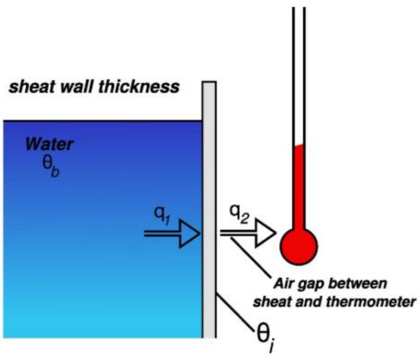

Fig. 5: Heat flow in the system

From the research carried out, the following conclusions are inferred [5]:

- Heating a thermometer in a water bath corresponds to a first-order system.
- If we insert a test tube –sheath– between the fluid and the thermometer, the system changes its behaviour, resulting in a hyper-damped second-order system.
- The heat delivered by the bath q_1 (Fig. 5) is transmitted through the wall of the sheath, where heat accumulates in the air chamber between it and the thermometer, and finally to the thermometer, where heat accumulates again.
- Considering the heat resistance of the sheath material to be negligible, the resulting heat flow consists of the sum of the heat accumulated in the sheath plus the heat flowing to the thermometer [11].

They then consider the energy balance and manage to model the system for the data provided by the actual laboratory [5].

The following equations were obtained from the energy balance:

$$q_1 = h_1 A_v(\theta_b - \theta_i) = m_v C_{pv} \frac{d\theta_i}{dt} + q_2 \qquad (1)$$

Where, h_1: film transmission coefficient of the sheath, A_v: sheath area, m_v: sheath mass. C_{pv}: specific heat of the sheath material, , θ_b: water temperature,: sheath temperature, θ_i and q_2: quantity of heat delivered to the thermometer .

If the capacity of the air inside the sheath and the resistance to heat transfer of the glass are considered negligible, the heat flowing to the thermometer is the same as the heat accumulated in the thermometer. Under these considerations, equation (2) was obtained [5]:

$$q_2 = h_2 A_t(\theta_i - \theta_o) = M_t C_{pt} \frac{d\theta_o}{dt} \qquad (2)$$

Where, h_2: film transmission coefficient of the thermometer, A_t: area of the sensitive thermometer, M_t: mass of the thermometer, C_{pt}: specific heat of the sensitive element of the thermometer and θ_o: temperature of the thermometer.

The equation R is defined as resistance to the passage of heat, which is inversely proportional to the transmission coefficient, being [12]:

$$R = \frac{1}{hA} \qquad (3)$$

From this definition, R_v is resistance to the passage of heat in the sheath and R_t is the resistance to the passage of heat of the thermometer, being [12]:

$$R_v = \frac{1}{h_1 A_v} \quad (4)$$

$$R_t = \frac{1}{h_2 A_t} \quad (5)$$

Además, se define a la capacidad térmica C como el producto de la masa por el calor específico de una sustancia [11].

$$C = m\, C_p \quad (6)$$

In addition, the thermal capacity is defined C as the product of mass by the specific heat of a substance [12].

$$C_v = m_v C_{pv} \quad (7)$$

$$C_t = M_t C_{pt} \quad (8)$$

If T is defined as the time constant, then T_v is the time constant of transmission of the heat of the sheath, and similarly T_t is the time constant of the constituent material of the thermometer, being:

$$T_v = C_v R_v \quad (9)$$

$$T_t = C_t R_t \quad (10)$$

If in equation (1) the students replace R_v, C_v and T_v from equations (4), (7) and (9), and solve for θ_b, equation (14) is obtained:

$$q_1 = \frac{1}{R_v}(\theta_b - \theta_i) = C_v \frac{d\theta_i}{dt} + q_2 \quad (11)$$

$$\frac{1}{R_v}(\theta_b - \theta_i) = C_v \frac{d\theta_i}{dt} + q_2 \quad (12)$$

$$(\theta_b - \theta_i) = R_v C_v \frac{d\theta_i}{dt} + R_v q_2 \quad (13)$$

$$\theta_b = \theta_i + T_v \frac{d\theta_i}{dt} + R_v q_2 \quad (14)$$

If students apply Laplace Transform to both expressions of equation (14) and considering zero initial conditions, equation (17) is obtained: [8], [13]:

$$\mathcal{L}[\theta_b] = \mathcal{L}[\theta_i] + T_v\, \mathcal{L}\left[\frac{d\theta_i}{dt}\right] + \mathcal{L}[R_v q_2] \quad (15)$$

$$\theta_b(s) = \theta_i(s) + sT_v \theta_i(s) + R_v \mathcal{L}[q_2] \quad (16)$$

$$\theta_b(s) = \theta_i(s)(1 + sT_v) + R_v \mathcal{L}[q_2] \quad (17)$$

If in equation (2) is replaced by equation (6) and then Laplace Transform is applied, equation (19) is obtained [8], [13]:

$$q_2 = C_t \frac{d\theta_o}{dt} \quad (18)$$

$$\mathcal{L}[q_2] = C_t\, \theta_o(s)\, s \quad (19)$$

Also, starting from the equation (2), and replacing previously by the parameters R_t, C_t and T_t, of the equations (5), (8) and (10), students get θ_i as seen in equation (23).

$$h_2 A_t (\theta_i - \theta_o) = M_t C_{pt} \frac{d\theta_o}{dt} \quad (20)$$

$$\frac{1}{R_t}(\theta_i - \theta_o) = C_t \frac{d\theta_o}{dt} \quad (21)$$

$$\theta_i = \theta_o + R_t\, C_t \frac{d\theta_o}{dt} \quad (22)$$

$$\theta_i = \theta_o + T_t \frac{d\theta_o}{dt} \quad (23)$$

If they then apply Laplace Transform to equation (23), they obtain:

$$\theta_i(s) = \theta_o(s) + T_t\, \theta_o(s) s \quad (24)$$

$$\theta_i(s) = \theta_o(s)(1 + T_t\, s) \quad (25)$$

Students replace in equation (17), from equations (19) and (25), and clear the ratio of temperatures, equation (28) is obtained, which is the sought transfer function, given by:

$$\theta_b(s) = \theta_o(s)\, \gamma\, \delta + \theta_o(s) R_v C_t\ s \quad (26)$$

$$\theta_b(s) = \theta_o(s)\, [\gamma\, \delta + R_v C_t\ s] \quad (27)$$

Being: $\gamma = 1 + T_t\, s$ and $\delta = 1 + T_v\, s$

$$\frac{\theta_o(s)}{\theta_b(s)} = \frac{1}{(1 + T_t\, s)(1 + sT_v) + R_v C_t\ s} \quad (28)$$

Applying the reverse Laplace Transform to equation (28), the temperature ratio is found in function of time, which results in equation (29) [8], [13]:

$$\frac{\theta_o(t)}{\theta_b(t)} = \frac{e^{-t\frac{\alpha - \sqrt{\beta}}{2T_t T_v}} - e^{-t\frac{\alpha + \sqrt{\beta}}{2T_t T_v}}}{\sqrt{\beta}} \quad (29)$$

Being: $\alpha = C_t R_v + T_t + T_v$ $\beta = -4T_t T_v + \alpha^2$

5.3 Development of a Conceptual Model

From the analysis of the energy balance and the transfer function obtained in equation (28), the students deduce that the mathematical model describing the experiment is complex and that, although the function adjusted to the data from the first session presents a good experimental fit, it is not consistent with the theoretical model derived subsequently. Consequently, the investigation continues, delving deeper into the behaviour of heat exchangers and their analogy with the prototype studied, which allows new considerations to be introduced that facilitate the determination of the time constant and the delay time.

By continuing with the search for information, a new working hypothesis arises: The use of a sheath causes a delay in reading temperature measurements. The students investigate and conclude that this peculiarity occurs in a heat exchanger when a temperature measuring element is located downstream and it takes some time before the hot fluid leaving the equipment reaches the point where the temperature is measured.

As an individual repeats and reflects an action it may be interiorized to a mental process. A process performs the same operation as the action, but wholly in the mind of the individual, enabling her/him to imagine performing the transformation without having to execute each step explicitly. For example, an individual with a process understanding of a function thinks about it in terms of inputs, possibly unspecified, and transformations of those inputs to produce outputs [14].

In a heat exchanger, both the location of the sensor element and its installation influence the quality of the system control. This refers to the time it takes for the sensor to detect a change in temperature in the equipment.

If this sensor is located at a certain distance from the fluid outlet of the exchanger, leaving an intermediate pipe distance, a delay is added to the temperature detection.

Valves of the type known as equal percentage are used in heat exchangers. This type of valve is best suited for wide working ranges. This fact, which is of great importance for the correct design of the heat transfer equation, was taken into account when making changes to the model and also considering that the system was excited by a step function.

The students consider that at the point where the temperature is taken, the same thing happens as at the point where the fluid exits (A), after L seconds, or what is similar to saying that at the point where the temperature is taken, what happens at A occurs, but L seconds later.

Then a new parameter is estimated in the experience called dead time or system delay, therefore multiplied to the transfer function by the factor e^{-LS}.

The dead time element, commonly called distance versus speed delay, is a phenomenon found in the vast majority of process systems.

From the research carried out, the students found that, in order to simplify the study of the control system, a good approximation could be obtained using a second-order system with the same time constant plus the dead time element.

When a second-order system is represented by two first-order elements, and each of these elements introduces its own delay, the total response of the system becomes proportionally slower with each added delay or dead time.

Therefore, the transfer equation in simplified form, taking into account the delay element symbolised by L and unifying the time constants, when the system is excited with a step function, the resulting transfer function is described in equation (29):

$$F(s) = \frac{Ke^{-Ls}}{s(1+Ts)^2} \quad (29)$$

Where: $F(s)$ is the simplified transfer function of the second-order system, K constant and T time constant. The inverse transfor obtained leads to equation (30):

$$f(t) = 1 - \left(1 + \frac{t-L}{T}\right) e^{-\frac{(t-L)}{T}} \quad (30)$$

Resulting the equation of the final temperature described by equation (31):

$$\theta(t) = 23 + 71\left[1 - \left(1 + \frac{t-L}{T}\right) e^{-\frac{(t-L)}{T}}\right] \quad (31)$$

5.4 Results and Validity of the Model

After studying the theoretical foundations of the experiment, students revisit the data from the laboratory experiment, which relates time to temperature measurements taken every 5 seconds. Using the approximate theoretical equation that fits the behaviour of the physical system and the data from the temperature-time table, the parameters associated with equation (31) are adjusted using the least squares approximation method.

This allows to obtain the delay time L, which is one of the parameters of the transfer equation, and the other parameter which is the time constant T. The values obtained are: $L = 5{,}47\ seg.$ and $T = 34\ seg.$

Therefore, the equation for the final temperature is equation (32):

$$\theta(t) = 23 + 71\left[1 - \left(1 + \frac{t-5{,}47}{34}\right) e^{-\frac{t-5{,}47}{34}}\right] \quad (32)$$

Use of technology make it possible to make more explicit the role of modes of representation. In particular, the way in which the complementarity between graphic, numerical and symbolic representation, produce best comprehension using technology and help develop coordination processes.

Fig. 6 shows the graph of equation (32)

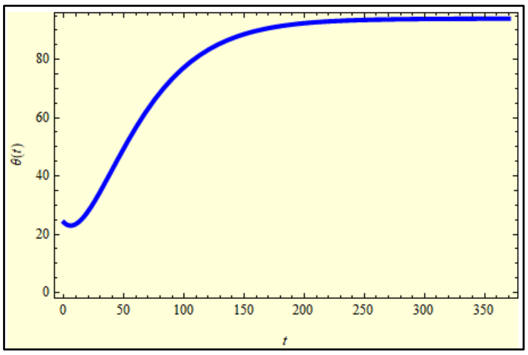

Fig. 6: Temperature vs. time

Fig. 7 shows the model graph contrasting with the graph of empirical data obtained from the experiment [10].

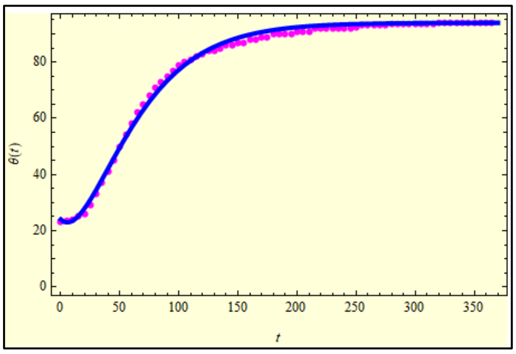

Fig. 7: Experimental data and its adjustment

6 Future Work

It is proposed that third-year students consolidate and validate the experience acquired through its application in Higher Cycle subjects, particularly in Control Theory and Fluid Mechanics, incorporating the use of a specific programming environment for this purpose. In this context, the aim is for students to continue developing their experience by implementing the LabVIEW programming platform, which is characterised by its flexibility and graphical approach.

Student groups are expected to identify their learning needs adn find learning resources [15]. By taking advantage of the potential offered by available technological resources, a methodological change is introduced in the way the previously analysed system is approached. This modification falls within the guidelines of Process Control Theory, with the aim of promoting the integration of knowledge from different disciplinary areas and incorporating a graphical programming environment that facilitates the analysis and understanding of the system's dynamics.

Focusing on Mathematics related subjects in Higher Education, their teaching has sometimes been done through algorithmic procedures often decontextualized from real applications and their relationship with other subjects [1].

7. Conclusion

Students find that researching the theoretical framework of the experience is important in order to design the conceptual model. However, they also consider data processing and the study of theoretical energy balances using computational tools to be equally important, since otherwise they would have to perform extensive and complex calculations to determine the fundamental variables involved in the analysis of the system.

The different work sessions increase the degree of difficulty of the calculations, while at the same time deepening the analysis until the appropriate theoretical model and its corresponding validation are achieved. This process is enriching in that it promotes transdisciplinarity in the study of differential equations, encouraging familiarisation with these equations, identification of their areas of application, and knowledge of the appropriate methods for their analysis and resolution.

As a continuation of this work, students are asked to carry out a statistical study of the laboratory data, with the aim of analysing its reliability and formulating hypotheses that lead to the validation of this data and the obtaining of further information from it.

ACKNOWLEDGEMENTS

The authors of this paper express their sincere gratitude to the distinguished professor and researcher Mg. Eng. Alicia María Tinnirello, whose ongoing dedication and commitment were fundamental in training the authors in the field of mathematics education research.

The authors would also like to thank Eng. Franco Ruggero for his valuable and selfless collaboration in reviewing and correcting the English language in this paper.

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Contribution of individual authors to the creation of a scientific article (ghostwriting policy)

Eduardo Gago: Implementation and development of the classroom experience. Application of mathematical, computational, or other formal techniques to analyze or synthesize study data. Conducting a research and investigation process, specifically performing the experiments, or data/evidence collection. Preparation, creation and/or presentation of the published work, specifically writing the initial draft (including substantive translation). Management and coordination responsibility for the research activity planning and execution. Management and coordination responsibility for the research activity planning and execution.
Paola Szekieta: Development of the classroom experience. Development or design of methodology; creation of models. Verification, whether as a part of the activity or separate, of the overall replication/reproducibility of results/experiments and other research outputs.
Lucas D'Alessandro: Development of the classroom experience. Verification, whether as a part of the

activity or separate, of the overall replication/reproducibility of results/experiments.

Sources of Funding for Research Presented in a Scientific Article or Scientific Article Itself
No funding was received for conducting this study.
Conflict of Interest
The authors have no conflicts of interest to declare.