A Contribution On Observer Design For A Class Of Discrete Bilinear Singular Systems With VariableTime Delay

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Abstract: - In this paper, authors propose a new design of an unknown input observer, in discrete time-domain, for a class of bilinear singular systems. In this contribution, a variable delay is considered on the state vector and a constant one is represented in both known input vector and bilinear form present in the system dynamic equation. Authors propose to apply the Lyapunov Krasovskii stability theory to compute the observer gain independently from the unknown input. Then, an unbiasedness dynamic of the observer is given to reconstruct both a state functional and a part of the unknown input. A numerical example is presented to prove the effectiveness of the proposed approach.

Key-Words: - Bilinear, Delay, Singular, Observer, Time-domain, Discrete

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1 Introduction

Numerous researches have considered bilinear models when studying control, filtering and stability subjects [6, 11, 17, 19]. In fact, bilinear models describe well engineering systems, especially, when non-linear representations generate complicated mathematical constraints and when linear descriptions fail to reproduce real evolutions of the considered systems. Moreover, bilinear models are widely used when representing real systems in various domains such economic field, chemical processes, automotive district, nuclear researches [20].

Furthermore, delays consideration has been the focus of many studies [3, 4, 7, 15]. In fact, such parameter describes the time delay of the input propagation through the actuators dynamics to the measurement provided by sensors. Thus, state delay can affect stability and system performances. Then, many results are developed to be used for stability analysis and control loop synthesis purposes for delayed models [12, 18].

In addition, singular representations are of great utility. Such models describe well engineering systems rather than regular configurations [5]. They combine dynamic evolution with state algebraic constraints. In this context, numerous results have been developed to revise stability, controllability and observability theories [13].

In one hand, in the last decade, many researchers have been interested in the state estimation by developing a suitable observer for a class of linear singular delayed systems in the continuous time-domain [14]. A wide attention has been given, also, when considering non-linear models. Then linear results have been improved and applied when adopting a linear submodels approach [16]. Besides, a great focus has been dedicated to the state estimation and partial or full reconstruction of the unknown input for fault detections purposes [9].

In the other hand, few results have been made in the discrete time-domain to reconstruct a functional state of a delayed singular bilinear model [1, 2, 8, 10].

Motivated by these facts, authors propose in this contribution, an observer design algorithm for a class of singular delayed bilinear systems in the discrete time-domain. The considered state-space model, in this paper, is affected by two types of delay present in the dynamic equation. A variable delay is considered in the state vector and a constant one present in the known input vector. The delayed input vector is, also, represented in the bilinear form. The authors aim to reconstruct both a functional state and a part of the unknown input. The proposed observer gain is computed by imposing unbiasedness conditions leading to a stable dynamic. Observer stability is ensured by considering a Lyapunov functional condition which is transformed to a set of linear matrix inequalities.

The present paper is organized as follows. Paragraph 2 is dedicated to compare the present approach to a set of similar results and to define the context of some used results. Section 3 defines the proposed bilinear model and formulates the problem to solve. The fourth section presents the observer scheme and the synthesis steps. Then, authors propose a numerical example to prove the effectiveness of the observer design approach and section 6 concludes the paper.

2 Related Works

This contribution is based on [9, 17, 19] which deal with observers for a class of bilinear models. In fact, comparing to [17], the present contribution proposes delays in both state and input vectors. Besides, the proposed approach uses decoupling scheme introduced in [17, 19] to isolate the unknown input vector and results have been applied for descriptor systems as shown in [9]. In this paper and comparing to [9], discrete time case is discussed for singular bilinear systems and variable time delay is considered in the state vector. Thus, constant delay is present in both regular input vector and bilinear form described in the dynamic equation of the presented state space bilinear model. In order to develop a stable observer dynamic, the proposed approach is based on the Lyapunov Krasovskii stability theory as done in [12, 13, 15], so the observer gain is calculated using a set of LMI conditions.

3 Problem Formulation

Let us consider a system described by the next bilinear model:

$$\begin{aligned} Ex(k+1) &= Ax(k) + A_d x(k-h_1(k)) \quad (1)\\ &\quad +Fv(k) + Bu(k) + B_d u(k-h_2)\\ &\quad +\sum_{i=1}^{m} D_i u_i(k) x(k)\\ &\quad +\sum_{i=1}^{m} D_{id} u_i(k-h_2) x(k)\\ y(k) &= Cx(k) + Gv(k) \quad (2)\\ z(k) &= Lx(k) \quad (3) \end{aligned}$$

$x(k)$ is the state vector, $y(k)$ is the output vector, $u(k) = [u_1(k) \quad u_2(k) \quad \dots \quad u_m(k)]^T$ is the known input vector, $z(k)$ is the functional state vector and $v(k)$ is the considered unknown input vector.
E, A, A_d, B, B_d, $D_{i_{1\leq i\leq m}}$, $D_{di_{1\leq i\leq m}}$, C, F, G and L are known and invariant matrices of appropriate dimensions.
$h_1(k)$ is the variable state delay such h_1 the upper bound: $0 < h_1(k) < h_1$.
h_2 is the considered constant known input delay.

Assumptions: [17]
The proposed approach is based on the next assumptions:

$$rang \begin{bmatrix} F \\ G \end{bmatrix} = q \quad (4)$$

$$rang\,(G) = \bar{q} < q \quad (5)$$

Then, according to [17] there exists a non-singular matrix W and an orthogonal matrix V as follows:

$$V^T G W = \begin{bmatrix} I_{\bar{q}} & 0 \\ 0 & 0 \end{bmatrix} \quad (6)$$

When multiplying $y(k)$ explicated in (2) by V^T and according to (6), we obtain:

$$\begin{aligned} y_1(k) &= C_1 x(k) + v_1(k) \quad (7)\\ y_2(k) &= C_2 x(k) \quad (8) \end{aligned}$$

with

$$\begin{bmatrix} C_1 \\ C_2 \end{bmatrix} = V^T C, \quad \begin{bmatrix} y_1(k) \\ y_2(k) \end{bmatrix} = V^T y(k),$$

$$y_1(k) \in R^{\bar{q}}, \quad y_2(k) \in R^{p-\bar{q}} \quad (9)$$

When deducing $v_1(k)$ from (7), the system (1-3) was transformed into the following form:

$$\begin{aligned} Ex(k+1) &= A_1 x(k) + A_d x(k-h_1(k)) \quad (10)\\ &\quad +F_1 y_1(k) + F_2 v_2(k) + Bu(k)\\ &\quad +B_d u(k-h_2) + \sum_{i=1}^{m} D_i u_i(k) x(k)\\ &\quad +\sum_{i=1}^{m} D_{id} u_i(k-h_2) x(k)\\ y_1(k) &= C_1 x(k) + v_1(k) \quad (11)\\ y_2(k) &= C_2 x(k) \quad (12)\\ \bar{z}(k) &= \bar{L} x(k) + \bar{I} y_1(k) \quad (13) \end{aligned}$$

Where

$$\begin{bmatrix} v_1(k) \\ v_2(k) \end{bmatrix} = W^{-1} v(k), v_1(k) \in R^{\bar{q}}, v_2(k) \in R^{q-\bar{q}} \quad (14)$$

$$A_1 = A - F_1 C_1 \quad (15)$$

$$[F_1 \quad F_2] = FW \quad (16)$$

$$\bar{L} \begin{pmatrix} L \\ -C_1 \end{pmatrix} \quad (17)$$

$$\bar{I} \begin{pmatrix} 0 \\ I_{n\times\bar{q}} \end{pmatrix} \quad (18)$$

$y(k)$ is decomposed into $y_1(k)$ and $y_2(k)$. According to (7), $y_1(k)$ is totally affected by the unknown input $v_1(k)$ and as shown in (8), $v_2(k)$ is disturbance free.

4 The Observer Design

By this contribution, authors aim to reconstruct the state functional given by (3) and, then, deduce $v_1(k)$. There exists a non-singular matrix S:

$$S = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \tag{19}$$

such that

$$\begin{aligned} aE + bC_2 &= \bar{L} & (20)\\ cE + dC_2 &= 0 & (21) \end{aligned}$$

The proposed observer scheme is given as follow:

$$\begin{aligned} \eta(k+1) &= H\eta(k) + H_d\eta(k-h_1(k)) & (22)\\ &+L_1y_1(k) + L_2y_2(k) \\ &+L_dy_2(k-h_1(k)) + Ju(k) \\ &+J_du(k-h_2) \\ &+\sum_{i=1}^{m} N_iu_i(k)y_2(k) \\ &+\sum_{i=1}^{m} N_{id}u_i(k-h_2)y_2(k) \\ \hat{\bar{z}}(k) &= \eta(k) + P_1y_1(k) + P_2y_2(k) & (23) \end{aligned}$$

where $\eta(k)$ is the state vector of the proposed observer and $\hat{\bar{z}}$ is the estimation of the functional state $\bar{z}(k)$. H, H_d, L_1, L_2, L_d, J, J_d, N_i, N_{id}, P_1 and P_2 are matrices of appropriate dimensions.

Objectif:
The observer given by (22) and (23) is determined when H, H_d, L_1, L_2, L_d, J, J_d, N_i, N_{id}, P_1 and P_2 are computed such as $\hat{\bar{z}}$ converges asymptotically to $\bar{z}(k)$.

$$\lim_{t->\infty} (\hat{\bar{z}}(t) - \bar{z}(t)) = 0 \tag{24}$$

4.1 Conditions of the unknown input observer synthesis

The estimation error is given by:

$$e(k) = \hat{\bar{z}}(k) - \bar{z}(k) \tag{25}$$

When replacing $\bar{z}(k)$ and $\hat{\bar{z}}(k)$ by their expressions given, respectively, by (13) and (23), we have:

$$e(k) = \eta(k)+(P_1-\bar{I})y_1(k)+(P_2C_2-\bar{L})x(k) \tag{26}$$

Let us suppose that:

$$\bar{I} = P_1 \tag{27}$$

According to (20)-(21), we have:

$$e(k) = \eta(k)-(a+\beta c)Ex(k)+(P_2-b-\beta d)C_2x(k) \tag{28}$$

we suppose that:

$$P_2 = b + \beta d \tag{29}$$

then, we can write:

$$e(k) = \eta(k) - \varphi Ex(k) \tag{30}$$

with

$$\varphi = a + \beta c \tag{31}$$

Theorem 1 *The system (22-23) is a functional observer for the system (1-3) if and only if the following conditions are reached:*

1. $e(k+1) = He(k) + H_de(k-h_1)$ *is asymptotically stable*
2. $H\varphi E + L_2C_2 - \varphi A_1 = 0$
3. $H_d\varphi E + L_dC_2 - \varphi A_d = 0$
4. $J = \varphi B$
5. $J_d = \varphi B_d$
6. $L_1 = \varphi F_1$
7. $\varphi F_2 = 0$
8. $N_a = \varphi D_aC_a^+ = 0$

with

$$\begin{aligned} \varphi &= [F_2 \quad D_a\kappa C_a] & (32)\\ C_a &= \begin{bmatrix} C_2 & \dots & 0 & 0 & \dots & 0 \\ 0 & \dots & C_2 & 0 & \dots & 0 \\ 0 & \dots & 0 & C_2 & \dots & 0 \\ 0 & \dots & 0 & 0 & \dots & C_2 \end{bmatrix} & (33)\\ N_a &= [N_1 \quad \dots \quad N_m \quad N_{d1} \quad \dots \quad N_{dm}] & (34)\\ D_a &= [D_1 \quad \dots \quad D_m \quad D_{d1} \quad \dots \quad D_{dm}] & (35) \end{aligned}$$

Proof 1 *When computing the quantity $\Delta e(k)$ by replacing $\eta(k+1)$ by its expression in (22) and $Ex(k+1)$ by its expression in (10) and by ensuring an unbiasedness dynamic Theorem 1 becomes obvious.*

4.2 Computing of observer matrices

When replacing relation (31) in conditions $b)$, $c)$, $g)$ and $h)$ from the Theorem 1, we have:

$$\begin{aligned} aA_1 &= HaE + KC_2 - \beta cA_1 & (36)\\ a_dA_d &= H_daE + K_dC_2 - \beta cA_d & (37)\\ aF_2 &= -\beta cF_2 & (38)\\ aD_aC_a^+ &= N_a - \beta cD_aC_a^+ & (39)\\ K &= L_2 - H\beta d & (40)\\ K_d &= L_d - H_d\beta d & (41) \end{aligned}$$

Equations (36)-(41) are transformed in a matrix form as follows:

$$X\Sigma = \Theta \tag{42}$$

where

$$X = [H \quad H_d \quad K \quad K_d \quad \beta \quad N_a] \tag{43}$$

$$\Theta = [\bar{L}A_1 \quad \bar{L}A_d \quad \bar{L}D_i \quad \bar{L}F_2]_{1\leq i\leq m} \tag{44}$$

$$\Sigma = \begin{bmatrix} aE & 0 & 0 & 0 \\ 0 & aE & 0 & 0 \\ C_2 & 0 & 0 & 0 \\ 0 & C_2 & 0 & 0 \\ -cA_1 & -cA_d & -cF_2 & -cD_a \\ 0 & 0 & 0 & C_a \end{bmatrix} \tag{45}$$

There exists a solution of (42) if and only if:

$$rang \begin{bmatrix} \Sigma \\ \Theta \end{bmatrix}_{1\leq i\leq m} = rang(\Sigma) \tag{46}$$

The solution of equation (42) is:

$$X = \Theta\Sigma^+_{1\leq i\leq m} - Z(I - \Sigma\Sigma^+) \tag{47}$$

With $\Sigma^+$is the inverse of the matrix Σ whereas Z is an arbitrary matrix of appropriate dimension which will be determined by the LMI approach.
The matrix H to be computed, is given by:

$$H = \begin{pmatrix} I \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{48}$$

By replacing X given by (42) in (43), we obtain the following expression:

$$H = \Theta\Sigma^+ \begin{pmatrix} I \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} - Z(I - \Sigma\Sigma^+) \begin{pmatrix} I \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{49}$$

We consider that:

$$H_1 = \Theta\Sigma^+ \begin{pmatrix} I \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \; and \; H_2 = (I - \Sigma\Sigma^+) \begin{pmatrix} I \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{50}$$

Then

$$H = H_1 - ZH_2 \tag{51}$$

By using the same algorithm given by equation (48)-(51), we obtain the matrix H_d:

$$H_d = H_{d1} - ZH_{d2} \tag{52}$$

where

$$H_{d1} = \Theta\Sigma^+ \begin{pmatrix} 0 \\ I \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \; and \;\; H_{d2} = (I - \Sigma\Sigma^+) \begin{pmatrix} 0 \\ I \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{53}$$

At this stage, we propose the following theorem to compute the observer gain Z.

Theorem 2 *The functional observer is an unknown input functional observer for the system (1-3) if there exist matrices $P = P^T > 0$, $Q = Q^T > 0$ and Y which satisfy the following matrix inequality*

$$\begin{pmatrix} -P & \sqrt{h_1}(PH_1 - YH_2) & \sqrt{h_1}(PH_{d1} - YH_{d2}) \\ * & Q + R - h_1 P & 0 \\ * & * & R - Q \end{pmatrix} < 0 \tag{54}$$

Where the arbitrary matrix Z is given by the following equation:

$$Z = P^{-1}Y \tag{55}$$

Proof 2 *It's obtained by considering the following Lyapunov functional [18].*

$$V(k) = V_1(k) + V_2(k) \tag{56}$$

$$V_1(k) = h_1 e^T(k)Pe(k) + \sum_{i=k-h_1}^{k-1} e^T(i)Qe(i) \tag{57}$$

$$V_2(k) = \sum_{i=1}^{h_1} e^T(k+1-i)Re(k+1-i) \tag{58}$$

Using condition $a)$ of Theorem 1 and equations (56)-(58), $\Delta V(k)$ is calculated as follow:

$$\begin{aligned} \Delta V_1(k) &= V_1(k+1) - V_1(k) \\ &= e^T(k)[h_1 H^T PH - h_1 P + Q]e(k) \\ &\quad + h_1 e^T(k)H^T PH_d e(k - h_1(k)) \\ &\quad + h_1 e^T(k - h_1(k))H_d^T PHe(k) \\ &\quad + e^T(k - h_1)[h_1 H_d^T PH_d - Q]e(k - h_1(k)) \end{aligned} \tag{59}$$

$$\Delta V_2(k) = e^T(k)Re(k) - e^T(k - h_1(k))Re(k - h_1(k)) \tag{60}$$

Based on equation (59) and (60), we can write:

$$\Delta V(k) = \varepsilon^T(k)\Psi\varepsilon(k) \quad (61)$$

with

$$\varepsilon(k) = \begin{pmatrix} e(k) \\ e(k-h_1(k)) \end{pmatrix} \quad (62)$$

$$\Psi = \begin{pmatrix} \chi & h_1 H^T P H_d \\ & h_1 H_d^T P H_d - Q - R \end{pmatrix} < 0 \quad (63)$$

where $\chi = h_1 H^T P H - h_1 P + Q + R$

So $\Delta V(k) < 0$ *is equivalent to* $\Psi < 0$.

To avoid the quadratic from present in equations (63), we propose to rewrite this equation as following form:

$$\Psi = U - \Omega^T \Xi^{-1} \Omega \quad (64)$$

where:

$$U = \begin{bmatrix} Q - R - h_1 P & 0 \\ 0 & -Q - R \end{bmatrix} \quad (65)$$

$$\Omega = \begin{bmatrix} \sqrt{h_1} H & \sqrt{h_1} H_d \end{bmatrix} \quad (66)$$

$$\Xi = -P^{-1} \quad (67)$$

According to the Schur lemma [14], $\Psi < 0$ *and* $\Xi < 0$ *if and only if:*

$$\Lambda = \begin{bmatrix} \Xi & \Omega \\ \Omega^T & U \end{bmatrix} < 0 \quad (68)$$

By applying a congruence transformation [14] to Λ *such as:*

$$\Pi^T \Lambda \Pi < 0 \quad (69)$$

with

$$\Pi = \begin{bmatrix} P & 0 & 0 \\ * & I & 0 \\ * & * & I \end{bmatrix} < 0 \quad (70)$$

Replacing Λ *and* Π *by their expressions (68) and (69) and using (48)-(53), Theorem 2 holds.*

5 Numerical Example

5.1 The model matrices

$$E = \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}, A = \begin{pmatrix} -5 & 0 \\ 0 & -3 \end{pmatrix}, A_d = \begin{pmatrix} -1 & 0 \\ 0 & -0.5 \end{pmatrix},$$

$$B = \begin{pmatrix} 2 & 0 \\ -1 & 3.2 \end{pmatrix}, B_d = \begin{pmatrix} 2 & -1 \\ 2 & 0 \end{pmatrix}, C = \begin{pmatrix} 1 & -1 \\ 0.5 & 3 \end{pmatrix},$$

$$F = \begin{pmatrix} 4 & 1 \\ 0 & -2 \end{pmatrix}, D_1 = \begin{pmatrix} -1 \\ -2 \end{pmatrix}, D_2 = D_{d1} = \begin{pmatrix} 0 \\ 0 \end{pmatrix},$$

$$D_{d2} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \quad G = \begin{pmatrix} 2 & 1 \\ 0 & 0 \end{pmatrix}, \quad L = \begin{pmatrix} 2 & -1 \end{pmatrix}$$

The state delay has a sinusoid form such $h_1(k) = 3sin(0.5k)$, then we can easily deduce that $h_1 = 3s$. The input delay h_2 is such $h_2 = 1s$.

5.2 Observer matrices

$$H = \begin{pmatrix} -3.22 & 22.17 \\ 1.73 & -11.94 \end{pmatrix}, \quad H_d = \begin{pmatrix} -0.57 & 2.8 \\ 0.3 & -1.51 \end{pmatrix},$$

$$L_1 = \begin{pmatrix} 8.66 \\ -4.66 \end{pmatrix}, P_1 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, J_d = \begin{pmatrix} 4.33 & -4.33 \\ -2.33 & 2.33 \end{pmatrix},$$

$$L_2 = \begin{pmatrix} 5.05 \\ -2.72 \end{pmatrix}, J = \begin{pmatrix} 10.83 & -6.93 \\ -5.83 & 3.73 \end{pmatrix},$$

$$P_2 = \begin{pmatrix} -0.33 \\ 0.33 \end{pmatrix}, L_d = \begin{pmatrix} 0.36 \\ -1.19 \end{pmatrix}, N_2 = N_{d1} = \begin{pmatrix} 0 \\ 0 \end{pmatrix},$$

$$N_1 = 10^{-15} \begin{pmatrix} -1.7 \\ -0.97 \end{pmatrix}, N_{d2} = 10^{-15} \begin{pmatrix} 5.66 \\ -3.4 \end{pmatrix}$$

5.3 Figures and interpretations

Fig.1 and Fig.2 explicit, respectively, the used known and unknown input vectors in the numerical example. As shown both inputs have two components.

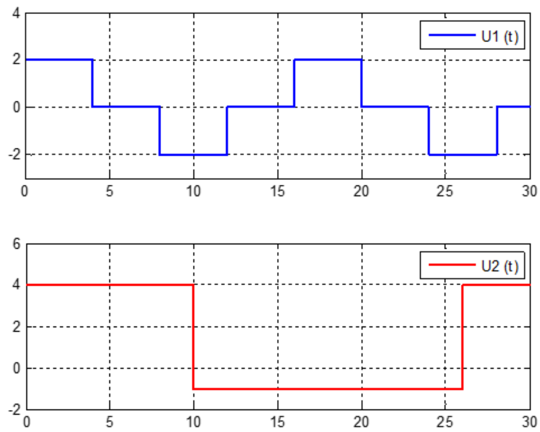

Figure 1: The known input vector

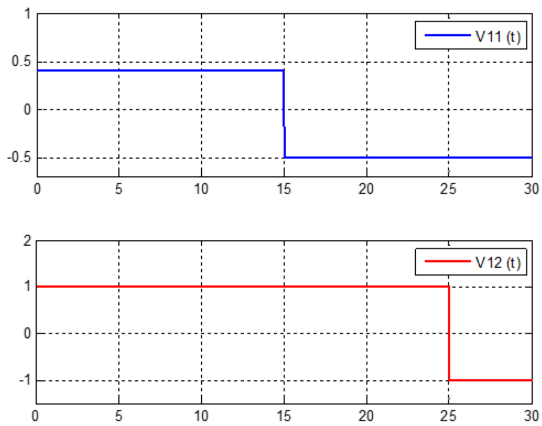

Figure 2: The used unknown input

Fig.3 draws both real and estimated functional state vectors. We can remark a quick convergence during the permanent phase and a short transitory phase $t \in [0, 5]$.

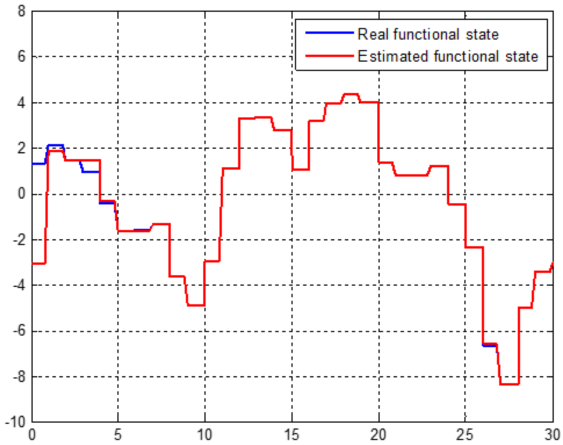

Figure 3: The real and estimated functional state vectors

Fig.4 draws both real and estimated functional unknown input vectors. we can check the quick convergence dynamic of the proposed observer.

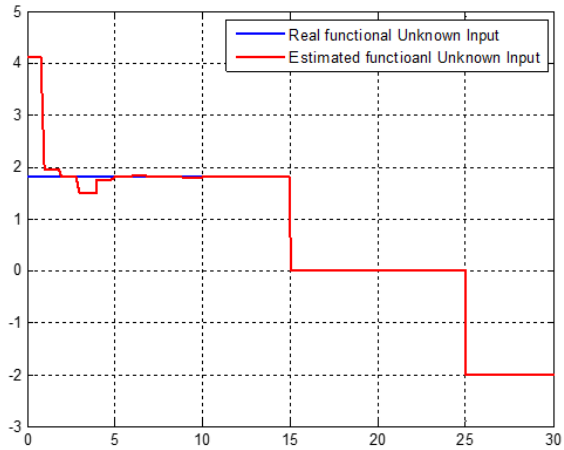

Figure 4: The real and estimated functional state vectors

Fig.5 and Fig.6 show the estimation error of the functional state vector and the unknown input which converge asymptotically to 0 and confirm the effectiveness of the proposed approach.

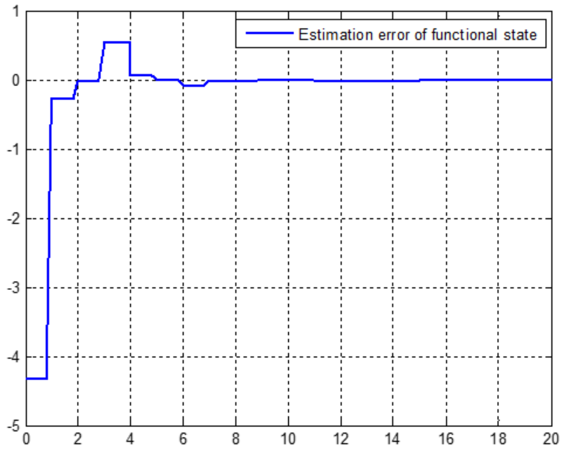

Figure 5: The state estimation error

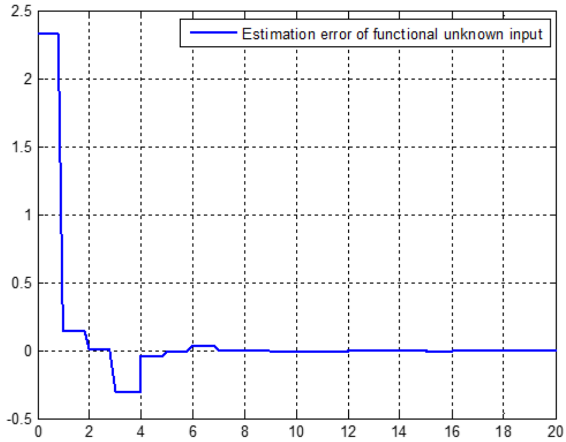

Figure 6: The unknown input estimation error

6 Conclusion

In this paper, authors have presented an observer scheme for singular bilinear systems with variable state delay. A constant delay has been also considered in the input vector in both regular and bilinear form. The observer proves its effectiveness tested on a numerical example and reconstruct both functional state vector and a part of the unknown input vector. The proposed approach is based on the Lyapunov Krasovskii stability theory so the observer gain is calculated by using a Lyapunov functional and ensures an unbiasedness dynamic. The estimation error is independent from the presented unknown input and the considered input delay.

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