Quantum Paradox Regarding the NOT Gates

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Abstract: - With proof by deduction that is relatively corroborating with the formalism of the Euler–Mascheroni constant expression and the more detailed pure mathematical proof given elsewhere, I propose the modal logic semantics in quantum computing. I use the Harmonic Series to demonstrate my idea developed from the astrophysical research previously conducted. As my mathematical intuitions work differently from the formalism, I have placed the results before the proof that $\sum_{n=1}^{\infty}\frac{1}{n}, n \in N \equiv \lim_{\substack{\leftarrow \\ n\in W}} \frac{n^2}{2} - 2n + \frac{1}{n}$.

Key-Words: - Modal logic, Pauli exclusion principle, Quantum semantics, Qubit floats, Commutativity.

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1 Introduction

The current quantum NOT gate follows a linear construct of logics, dissimilar to the quantum indeterminism in its physics construct. The matrix form $X \equiv \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ in the current quantum fundamental logic exactly does not represent the Pauli exclusion principle [1]. Mathematically, it only applied an eigenvector upon the preset eigenvalues for logical representations, as is all binary systems. It was proved by Putnam (1969) [2] with contradiction in bijection that the NOT logics in proof by exhaustion lead to falsehood.

The extended controlled-NOT (CNOT) gate and Hadamard gate inherited the actions of Hilbert space by the electric current [1]. The principle of least action governs the CNOT gate and Hadamard gate, but the least action from the locality of the observer's and the observer's apparatus' perspectives does not necessarily imply the least relativistic action to the traceable cosmic ray sources detected, as what I have been communicating on scientific computation and apparatus rationale [3, 4].

The Hadamard gate utilizes the alternating and direct currents circuits, but the mechanical conformability further undermines the potentials of quantum information on asymptotic analyses [5]. I categorize the problem into a logic fundamentalism problem in quantum computation with the mathematical foundation tracing back to Shannon entropy [6]. To put it in relevant mathematical terms, I'd like to put forth the quiz I've been entertaining with with the Harmonic Series (HS) $\sum_{n=1}^{\infty}\frac{1}{n}$ that is there a "$n+1$" in the HS? The question relates back to the NOT gate logical constructs of quantum computing regarding the processing of Hamiltonian zero.

The research explores the modal logic possibilities in quantum computing, in the stead of the classical logic constructs. It transforms the classical logics in proof by exhaustion in dealing with infinities to the vectorized eigenvalues by probability and necessity [7]. In other words, the research discusses the semantics of quantum computation in optimizing performance and its purposes in quantum physics.

2 Problem Formulation

Theorem 0:

$$\sum_{n=1}^{\infty}\frac{1}{n}, n \in N \equiv \lim_{\substack{\leftarrow \\ n\in W}} \frac{n^2}{2} - 2n + \frac{1}{n}.$$

3 Problem Solution

In order to shift the logics, I change the way of dealing with the questions of infinity in calculus with the logics [8]

$$\forall n \to \infty \textit{ is countable} \Rightarrow n-1 \textit{ is countable} \quad (1)$$

$$\forall n-1 \in n \to \infty \text{ is countable} \Rightarrow \neg n \text{ is countable} \quad (2)$$

$$n, \infty \vDash \text{is countable} \quad (3)$$

Similarly

$$\frac{m}{n}, m \wedge n \text{ is countable} \vDash \text{is repeating decimal} \quad (4)$$

Cantor's diagonal argument is restricted to natural numbers, and in order to establish a bijection between the natural numbers and negative integers, the set of whole numbers has to be assigned with alternative eigenvalues (there's the same thinking in [9]).

Consider the XOR operator to assign eigenvector to real numbers from the whole number line, where the negative whole numbers are operated by assigning the indeterminate in the XOR truth table to natural numbers, the bijective computational basis can be established, with the exceptions for 0 and negative and positive infinities [10, 11]. The remaining three becomes the triangular starting points for the eigenvectors.

With the HS being able to be translated to

$$\frac{2-1}{1}+\frac{3-2}{2}+\frac{4-3}{3}+\ldots+\frac{n-(n-1)}{n-1}+\frac{1}{n} = -(n-1)+\frac{1}{n}+(2-\frac{1}{2})+(2-\frac{2}{3})+(2-\frac{3}{4})+\ldots+(2-\frac{n-2}{n-1}) \quad (5)$$

and

$$(1-\frac{0}{1})+(1-\frac{1}{2})+(1-\frac{2}{3})+(1-\frac{3}{4})+\ldots+(1-\frac{n-2}{n-1})+(1-\frac{n-1}{n}) = n-(\frac{1}{2}+\frac{1}{3}+\ldots+\frac{1}{n}) \quad (6)$$

the mathematical basis exists in computing the HS on the term of non-divergence from the logic of statement (3). By the logic of statement (2), the inverse limit of the HS from equations (5) & (6)

$$\begin{cases} \underset{\leftarrow}{lim}_{n\in N}(\frac{1}{n}-1+\sum_{n=2}^{\infty}\frac{1}{n-1}) \\ \underset{\leftarrow}{lim}_{n\in N}(\frac{1}{n}+n-\sum_{n=2}^{\infty}\frac{1}{n-1}) \end{cases} \quad (7)$$

is equivalent to the Hadamard gate $H=\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}$ on very different ends in terms of Shannon entropy [1, 6]. The semantics of $\neg\diamond\neg equation(5) \wedge \neg\diamond\neg equation(6) \leftrightarrow \Box HS$ optimizes the Hadamard gate, where the paradox on the transfinite number is resolved by the -1 term in the matrix form with the CNOT gate conducting back to the NOT gate [11].

The equivalence to the Hadamard gate can also be derived with the HS from equation (5)

$$\frac{2}{1}-1+(2-\frac{1}{2})-1+(2-\frac{2}{3})-1+(2-\frac{3}{4})-1+\ldots+(2-\frac{n-2}{n-1})-1+\frac{1}{n}, n\in N$$
$$= -(n-1)+\frac{1}{n}+2(n-1)-\frac{1}{2}-\frac{2}{3}-\frac{3}{4}-\ldots-\frac{n-2}{n-1}, n\in N$$
$$= n+\frac{1}{n}-1-\sum_{n=2}^{\infty}\frac{n-2}{n-1}, n\in N \quad (8)$$

and $\sum_{n=2}^{\infty}\frac{n-2}{n-1}, n\in N$ can be expanded to

$$\frac{\frac{\prod_{n=2}^{\infty}n-1}{2}+2\frac{\prod_{n=2}^{\infty}n-1}{3}+3\frac{\prod_{n=2}^{\infty}n-1}{4}\ldots+(n-2)\frac{\prod_{n=2}^{\infty}n-1}{n-1}}{\prod_{n=2}^{\infty}n-1} = [n]-\sum_{n=2}^{\infty}\frac{1}{n-1}, n\in N \quad (9)$$

corroborating with the HS being not necessarily divergent, as far as the concept of infinity adheres with the **value** of *n*.

The completeness can be therefore expressed:

$$\forall n-1 \in n \to \infty \text{ is countable} \Rightarrow \neg[n-(n-1)] \text{ is countable} \quad (10)$$

with the logical derivations from statement (2)

$$\Box n-(n-1) \text{ is countable} \Rightarrow n \text{ is countable} \quad (11)$$

An example for propositions (10) & (11), without restriction to natural numbers, is when $n=i$, whereby the count of $n-1$ can only be defined by $n+1$, or in the other direction, by its infinite product. The example, or case therein, leads to the inference

$$n+1, n\to\infty \text{ is countable} \vDash \text{is countable} \quad (12)$$

Equation (8) can thus be expanded to

$$\underset{\substack{\leftarrow \\ n\in N}}{lim}(2n+\frac{1}{n}-1+\sum_{n=2}^{\infty}\frac{1}{n-1}) \quad (13)$$

$$\lim_{\substack{\leftarrow \\ n\in N}}[2n+\frac{1}{n}-1-4(n-1)+\sum_{n=2}^{\infty}(\frac{1}{n-1}+4)] \quad (14)$$

$$\lim_{\substack{\leftarrow \\ n\in N}}\{-2n+\frac{1}{n}+4+\sum_{n=2}^{\infty}[\frac{3(n-1)+n}{n-1}-\frac{1}{n-1}]\} \quad (15)$$

$$=\lim_{\substack{\leftarrow \\ n\in N}}[\frac{1}{n}-2n-1+\sum_{n=2}^{\infty}(\frac{2n}{n-1}-\frac{2n}{n-1}\times\frac{n-1}{2n})] \quad (16)$$

$$=\lim_{\substack{\leftarrow \\ n\in N}}(\frac{1}{n}-2n-1+\sum_{n=2}^{\infty}\frac{n+1}{n-1}) \quad (17)$$

4 Conclusion

In the previous experiments and research I conducted, I only extended to new layers of expression from the binary logics with astrophysics and cosmology (see in [4, 12]), but possible fallacies are not eliminated with the deductive paths to the epistemology of scientific theories [13, 14]; just as equations (5) & (6) do not necessarily, in mathematical terms, compute back to the HS exactly, which only the critical line of Riemann zero can temporarily solve depending on logarithm approaches [6, 15]. In this case, they are $1+n$ and $0+n$ respectively [4].

In equation (5), 0 does not have eigenvalue and in equation (6) 1 does not, adhering to the logics of Pauli exclusion principle for quantum superposition while offering higher degrees of freedom for input than the Hadamard gate [1]. Another infinite product series on the probabilities of the eigenvalue of the HS can be derived from the sum of equations (5) & (6)

$$2\sum_{n=1}^{\infty}\frac{1}{n}=1+\frac{2}{n}+2(n-1)-(n-1)=n+\frac{2}{n} \quad (18)$$

and the product of equations (5) & (6)

$$(\sum_{n=1}^{\infty}\frac{1}{n})^2=[(n-1)+\frac{1}{n}-(\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+...+\frac{1}{n-1})]\times[(n+1)+\frac{1}{n}-(\frac{1}{2}+\frac{1}{3}+...+\frac{1}{n-1})]$$
$$=[\frac{1}{n}-(\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+...+\frac{1}{n-1})]^2+n^2-2n(\frac{1}{2}+\frac{1}{3}+...+\frac{1}{n-1})+1$$
$$=(n^2-2n+\frac{1}{n})(\frac{1}{2}+\frac{1}{3}+...+\frac{1}{n-1}) \quad (19)$$

in the form [16]

$$\begin{cases}\sum_{n=1}^{\infty}\frac{1}{n}=\frac{n}{2}+\frac{1}{n}\\(\sum_{n=1}^{\infty}\frac{1}{n})^2=[(n-1)^2+(\frac{1}{n}-1)](\frac{1}{2}+\frac{1}{3}+...+\frac{1}{n-1})\end{cases} \quad (20)$$

The theorem proposed, however, is not unprecedented concerning Euler's number $e^{i\pi}+1=0$. It is only when 0 is countable as are the infinities [17], the completeness can be established with

$$\square equation(7)\rightarrow equation(8) \quad (21)$$

$$\lim_{\substack{\leftarrow \\ n\in N}}(\frac{1}{n}+\sum_{n=2}^{\infty}\frac{n-2}{n-1}), n\in N \quad (22)$$

Only then can further discussions with Hamiltonian 0 be taken into the quantum realm. The perceived eigenvalue can be seen in table 1.

Bloch Input	Bloch Output	Bloch I⊕O	Riemann Input	Riemann Output	Riemann I⊕O	Bloch⊕Riemann	Feedback
1	1	F	1	1	F	F	Addible
1	1	F	1	¬1	T	T	Entangled
1	0	F	2	¬2	T	T	Entangled
1	0	F	2	2	F	F	Addible
0	1	T	1	¬2 ∧ ¬1	T	F	Addible
0	1	T	1	2	T	F	Carry
0	1	T	1	1	F	T	Entangled
0	0	F	2	¬2	T	T	Entangled
0	0	F	2	2	F	F	Addible

Table 1. The proposed signal mode from the research's theoretical framework.

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