A reliable and computationally fast model to simulate non-linear soil consolidation in saturated clays

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Abstract: - The 1D non-linear soil consolidation processes is ruled by diffusive equations in which the coefficients of the addends are complex functions of the dependent variable, the excess pore water pressure. In addition, since the expulsion of water leads to the contraction of the soil, the assumption of this contraction in the model causes it to become a problem classified within the type of moving-boundary. The numerical model proposed in this paper, based on the network method, makes use of the powerful computational algorithms implemented in the circuit simulations codes, capable to solve strong non-linearities, couplings and any kind of boundary conditions, reducing errors to negligible values. The model, for whose design basic rules of circuit theory is required, is described in detail illustrating its application with two problems in which comparison with linear cases are carried out.

Key-Words: - Numerical model, network method, soil consolidation

1 Introduction

Many problems in engineering, particularly most of those that are non-linear, can only be solved by numerical techniques. Non-linearities can arise due to different causes as, among other, the existence of not constant parameters whose relation with the dependent variable is approached by mathematical functions.

The advanced soil consolidation problem belongs to this class of problems since the unitary dependences of the parameters involved, void ratio and hydraulic conductivity, are proportional to the unitary changes in the effective stress, directly related with the excess pore water pressure, the dependent variable [1,2]. This means that these parameters are potential functions of the dependent variable. In addition, the expulsion of water from the porous domain involves the contraction of the soil and, as a consequence, a decrease in the size of the volume element adopted for the domain discretization. This makes the consolidation process to fit within the so-called moving-boundary problems, whose simulation is more complex [3].

In this paper a numerical model for the reliable simulation of non-linear consolidation problems is proposed. Its design, based on the network method [4,5], allows the implementation of a circuit whose mathematical model is equivalent to the finite-difference differential equations that result from the spatial discretization of the partial-differential equations of the consolidation problem; the time remains as a continuous variable in the network model.

The main advantage of using an electric model is that its simulation is carried out by means of the powerful mathematical algorithms implemented in the circuit solution codes such as Ngspice [6], necessary for the type of complex signals and very high frequencies involved in the electronic and communications devices. These codes generally provide the exact solution of the circuit, relegating errors to the mesh size imposed to the domain.

On the other hand, the design of the model requires the application of very few programming rules based on both the constitutive relations of linear electrical devices, such as resistors, capacitors and constant generators, and non-linear devices called controlled voltage or current generators. The latter, versatile enough to implement any type of non-linearity or coupling existing in the governing equations.

Each term or addend of the finite-difference differential equation is implemented in the network of the volume element or cell by a suitable device whose electric current is balanced with the currents of the other devices in a common node, according to the topology of the equation. Cells, in turn, are also coupled together by ideal electrical connections to reproduce the network of the whole domain. Finally, the boundary conditions are added by using the same kind of devices. Once the complete model is ready, the code provides its solution without further mathematical manipulation than the processing of the tabulated data from the simulation.

To illustrate the performance of the proposed model, two application scenarios are described.

2 Nomenclature

C_i	electric capacitor
c_v	coefficient of consolidation (m^2/s)
$c_{v,1}$	initial coefficient of consolidation (m^2/s)
e	void ratio (dimensionless)
e_o	initial void ratio (dimensionless)
G_i	controlled current source
H_1	initial thickness (m)
H_2	final thickness (m)
H_i	instant thickness (m)
j_C	electric current of a capacitor (A)
j_G	electric current of a source (A)
k	hydraulic conductivity (m/s)
k_1	initial hydraulic conductivity (m/s)
m_v	coefficient of volumetric compressibility (m^2/N)
$m_{v,1}$	initial coefficient of volumetric compressibility (m^2/N)
q_o	surface applied load (N/m^2)
S_∞	final settlement (m)
S_i	instant settlement (m)
t	time (s)
u	excess pore water pressure (N/m^2)
$\bar{U}_s$	average degree of consolidation (dimensionless)
V	volume (m^3)
z	vertical spatial coordinate (m)
γ_k	nonlinear coefficient of change of permeability with effective pressure (dimensionless)
γ_v	non-linear coefficient of compressibility (dimensionless)
γ_w	specific weight of water (N/m^3)
σ'	effective pressure (N/m^2)
σ'_1	initial effective pressure (N/m^2)

3 Governing equations

The physical scheme of the 1D soil consolidation process is shown in Fig.1. Due to the application of loads on the soil surface, the saturated porous of the soil gradually expel the water contained in its interstices reducing its thickness until reaching a stationary situation.

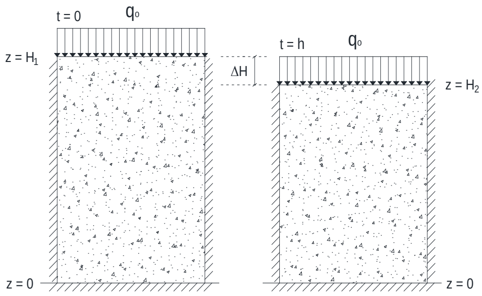

Figure 1. Physical scheme of 1D consolidation

The mathematical model of the non-linear soil consolidation problem, under the hypotheses of incompressibility in water and soil particles, is deduced from the water conservation balance in combination with the Darcy's flow equation

$$\frac{d\sigma'}{dt} = -\frac{1}{\gamma_w m_v}\frac{\partial}{\partial z}\left(k\frac{\partial u}{\partial z}\right) \qquad (1)$$

Assuming, on the one hand, negligible change in thickness and weight of sample water, as well as the hypotheses of the oedometer test

$$\frac{\partial u}{\partial z} = -\frac{\partial \sigma'}{\partial z} \quad \text{and} \quad \frac{\partial u}{\partial t} = -\frac{\partial \sigma'}{\partial t}$$

and, on the other hand, the most common dependences between soil volume, hydraulic conductivity and effective soil pressure, given by Juárez-Badillo [7]

$$\frac{dV}{V} = -\gamma_v \frac{d\sigma'}{\sigma'}$$

$$\frac{dk}{k} = -\gamma_k \frac{d\sigma'}{\sigma'}$$

and introducing the coefficients and relations

$$\lambda = 1 - \gamma_k$$

$$\frac{k_1\sigma'_1}{\gamma_w\gamma_v} = \frac{k_1}{\gamma_w m_{v,1}} = c_{v,1}$$

$$c_v = \frac{k}{\gamma_w m_v} = \frac{k_1\sigma'_1}{\gamma_w\gamma_v}\left(\frac{\sigma'}{\sigma'_1}\right)^{1-\gamma_k} = c_{v,1}\left(\frac{\sigma'}{\sigma'_1}\right)^{\lambda}$$

equation (1) writes in the form

$$\frac{\partial\sigma'}{\partial t} = \sigma' c_{v,1}\frac{\partial}{\partial z}\left[\left(\frac{\sigma'}{\sigma_1'}\right)^{-\gamma_k}\frac{1}{\sigma_1'}\frac{\partial\sigma'}{\partial z}\right] \quad (2)$$

According to Fig.1, and assuming a negligible change in thickness, the following boundary and initial conditions complete the mathematical model:

$$\frac{\partial u}{\partial z}_{(z=0,t)} = 0 \quad \text{(Impervious bottom edge)}$$

$$u_{\,(z=H_1,t)} = 0 \quad \text{(Upper free drainage)}$$

The model has no analytical solutions except for the case λ=0 [7].

A less restrictive variant that assumes the initial value of the void ratio no negligible leads [8] to

$$\frac{\partial\sigma'}{\partial t} = \frac{(1+e_o)\sigma'_1 k_1}{\gamma_w\gamma_v}\left(\frac{\sigma'}{\sigma'_1}\right)^{1-\gamma_k}\left[-\frac{\gamma_k}{\sigma'}\left(\frac{\partial\sigma'}{\partial z}\right)^2+\frac{\partial^2\sigma'}{\partial z^2}\right] \quad (3)$$

an equation nearly the same that (2) except for the factor $(1+e_o)$.

4 The network model

The equivalence between the physical and network model is established by introducing a formal analogy between the dependent and independent variables of the problem and the different elements or variables of an electric circuit: the excess pore pressure (or the effective pressure) is assumed to be the voltage quantity of the model while the flow of water is the electric current.

Developing the equation (2), we have

$$\frac{1}{\sigma'}\frac{\partial\sigma'}{\partial t} = -\frac{c_{v,1}\gamma_k}{(\sigma_1')^2}\left(\frac{\sigma'}{\sigma_1'}\right)^{-\gamma_k-1}\left(\frac{\partial\sigma'}{\partial z}\right)^2 + \frac{c_{v,1}}{\sigma_1'}\left(\frac{\sigma'}{\sigma_1'}\right)^{-\gamma_k}\frac{\partial^2\sigma'}{\partial z^2} \quad (4)$$

an equation that, following the nomenclature of Fig.2, can be expressed in finite-difference form as

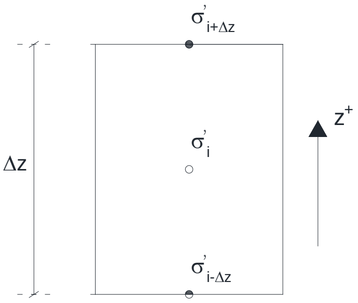

Figure 2. Nomenclature of the 1D element cell

$$\frac{\partial\sigma'}{\partial t} = \left(\frac{\sigma'_i}{\sigma_1'}\right)^{-\gamma_k+1}\left[\frac{\sigma'_{i+\Delta z}-\sigma'_i}{\frac{(\Delta z)^2}{2c_{v,1}}}\right] - \left(\frac{\sigma'_i}{\sigma_1'}\right)^{-\gamma_k+1}\left[\frac{\sigma'_i-\sigma'_{i-\Delta z}}{\frac{(\Delta z)^2}{2c_{v,1}}}\right] - \frac{c_{v,1}\gamma_k}{\sigma_1'}\left(\frac{\sigma'_i}{\sigma_1'}\right)^{-\gamma_k}\left[\frac{(\sigma'_{i+\Delta z}-\sigma'_{i-\Delta z})^2}{(\Delta z)^2}\right] \quad (5)$$

In the network model the four terms of the above equation define the following electric currents

$$\left.\begin{array}{l} j_C = \frac{\partial\sigma'}{\partial t}, \quad j_{G+\Delta z} = \left(\frac{\sigma'_i}{\sigma_1'}\right)^{1-\gamma_k}\left[\frac{\sigma'_{i+\Delta z}-\sigma'_i}{\frac{(\Delta z)^2}{2c_{v,1}}}\right] \\ j_{G-\Delta z} = \left(\frac{\sigma'_i}{\sigma_1'}\right)^{1-\gamma_k}\left[\frac{\sigma'_i-\sigma'_{i-\Delta z}}{\frac{(\Delta z)^2}{2c_{v,1}}}\right] \\ j_G = \frac{c_{v,1}\gamma_k}{\sigma_1'}\left(\frac{\sigma'_i}{\sigma_1'}\right)^{-\gamma_k}\left[\frac{(\sigma'_{i+\Delta z}-\sigma'_{i-\Delta z})^2}{(\Delta z)^2}\right] \end{array}\right\} \quad (6)$$

that are balanced in a common node since $j_C = j_{G+\Delta z} - j_{G-\Delta z} + j_G$.

The only linear term of (5) is implemented by a capacitor of capacitance unity (C_i) while the rest terms are by current-sources controlled by voltage. The outputs of these sources are given by (6) and the values of dependent variables in each cell are read at the common nodes of the cell, Fig.2. Thus, the network model (Fig.3) has three sources, G_i, $G_{i-\Delta}$ and $G_{i+\Delta}$, related to the currents j_G, $j_{G-\Delta z}$ y $j_{G+\Delta z}$, respectively. The control nodes of each source are indicated in Fig.3 next to the sources. The resistors in parallel with each source do not play any roll but their implementation is generally required to check the behavior of the model under steady condition.

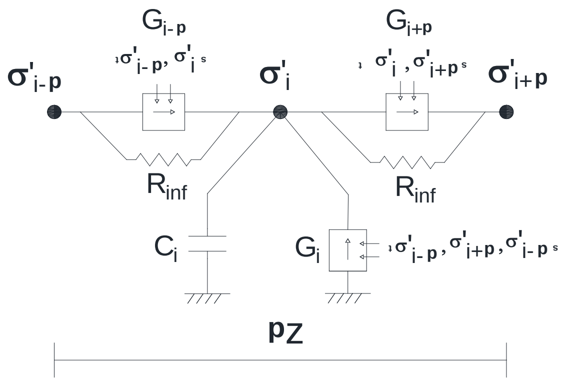

Figure 3. Network model of the volume element

The network model for the equation (3) is nearly the same except for the specification of the output sources G_i, $G_{i-\Delta}$ and $G_{i+\Delta}$; since these currents are implemented by software, the only change is easily made. The currents needed to reproduce to equation (3) are

$$\left.\begin{aligned} j_C &= \frac{\partial \sigma'}{\partial t}, \quad j_{G+\Delta z} = \left(\frac{\sigma'_i}{\sigma'_1}\right)^{1-\gamma_k} \left[\frac{\sigma'_{i+\Delta z}-\sigma'_i}{\left(\frac{(\Delta z)^2}{2c_{v,1}(1+e_o)}\right)}\right] \\ j_{G-\Delta z} &= \left(\frac{\sigma'_i}{\sigma'_1}\right)^{1-\gamma_k} \left[\frac{\sigma'_i-\sigma'_{i-\Delta z}}{\left(\frac{(\Delta z)^2}{2c_{v,1}(1+e_o)}\right)}\right] \\ j_G &= \frac{c_{v,1}\gamma_k}{\sigma'_1}\left(\frac{\sigma'_i}{\sigma'_1}\right)^{-\gamma_k} \left[\frac{(\sigma'_{i+\Delta z}-\sigma'_{i-\Delta z})^2}{\frac{(\Delta z)^2}{(1+e_o)}}\right] \end{aligned}\right\} \quad (7)$$

whereas the scheme of the network model coincides with that of the model of equation (2), Fig.3 .

5 Applications

In this section, two consolidation scenarios are illustrated, with the aim of comparing and analyzing the results obtained with the different network models designed. For each of the two examples, different values of the parameters have been given to the model: γ_v, γ_k, e_o, H_1 (m), σ'_1 (N/m^2), σ'_2 (N/m^2) and k_1 (m/year) for the non-linear models, equations (2) and (3), JB model and JB variant model respectively, and $c_{v,1}$ (m^2/year) for the well-known linear case [9], Table 1.

set	γ_v	γ_k	e_o	H_1	σ'_1	σ'_2	k_1	$c_{v,1}$
1	0.1	0.5	1	1	30000	60000	0.02	0.61
2	0.1	0.5	3	2	30000	35000	0.04	1.22

Table 1. Physical and geometrical soil parameters

Fig.4 and Fig.5 show the evolution of the average degree of consolidation ($\bar{U}_s$) over time. This important function represents the percentage of settlement achieved in a given time, as the ratio between the instant settlement and the final settlement reached at the end of the consolidation process:

$$\bar{U}_s = \frac{S_i}{S_\infty} = \frac{H_i - H_1}{H_2 - H_1} \quad (8)$$

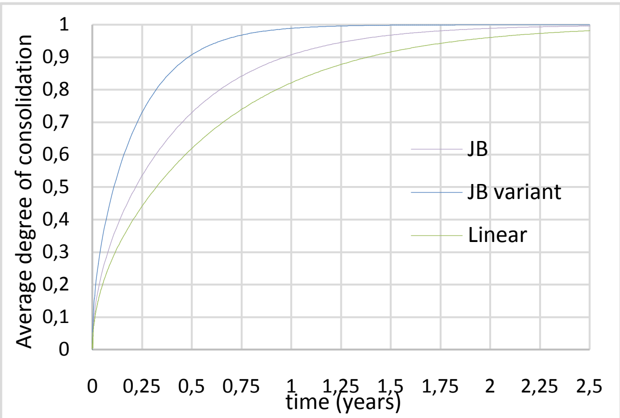

Figure 4. $\bar{U}_s$ as a function of time. Set 1

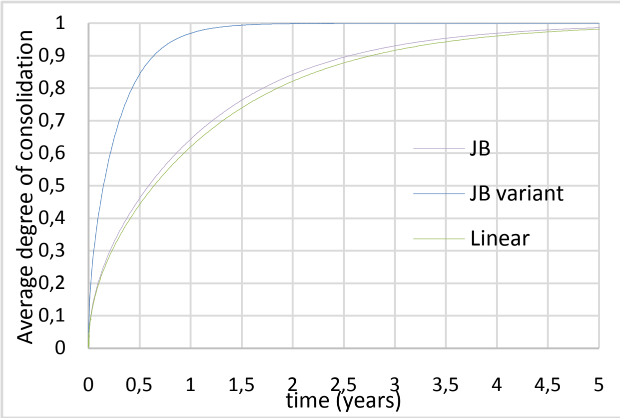

Figure 5. $\bar{U}_s$ as a function of time. Set 2

In both figures it can be observed that the non-linear models report smaller consolidation times in comparison with the solution of the linear model. On the other hand, the JB variant model, which takes into account the value of the initial void ratio (e_o), reports smaller times than the JB model, the smaller the higher e_o, Fig.4 and Fig.5. This fact becomes even more evident in the set 2 (Fig.5), in which when a small loading step is applied, the JB model and the linear one give almost the same solution, whereas the JB variant model does not. This is because, by its formulation, the JB model tends to be a linear model as the load step is smaller.

All this means, therefore, that the JB variant model is much more precise in its solution, giving a value, sometimes, much lower of the consolidation time, which can imply important advantages and cost savings in the execution of an engineering work.

6 Conclusion

The network method has proven to be a versatile tool for the design of network models in non-linear problems of soil consolidation. Each term of the equation is assumed to be an electric current that balances with the others in a common node in such a way that the solution – given by a circuit simulation code – provides the currents and voltages that satisfy this condition. The non-linear terms of the equation are implemented, regardless of the type of functions involved within them, by a special kind of component called controlled current source. The output of this source – or the current to which the term is related – is specified by software and may depend on functions whose arguments are the currents at other components or the voltage at other nodes of the circuit. Thus, non-linear or coupled terms may be implement in the model in a direct way by a same type of component.

The complete model extends the volume element between adjacent cells by ideal electrical contacts, to cover the complete geometry of the domain. Finally, the boundary conditions, whatever the functions that define them, are also implemented by controlled sources or other simple components. In short, very few rules are required for the design of the soil consolidation non-linear problem.

The reliability of the model is shown by two illustrative applications using the free version of the Ngspice code simulation.

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