On visco-elastic biological materials

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Abstract: In this paper we discuss different approaches to modeling of the main properties of visco-elastic biological materials. The main attention is paid to those methods which involve (at least partly) the fractional technique.

Key-Words: biological materials, visco-elasticity, memory effect, power-law relaxation (temporal and spatial), fractional derivative

1 Introduction

Biological materials attract an interest of scientists of different background from practicing doctors till pure physicists and mathematicians. Many of these materials can be classify as visco-elastic whose properties occupy certain position between pure elasticity and inelastic/viscous fluidity. Thus, these substances are not real solids and not real fluids. They are called rheological materials and their study forms a special branch of sciences called rheology (see, e.g., classical books on subject [1], [2], [3]). Several biological materials possess such a behavior, e.g. periodontal ligament as a part of tooth system, cartilage tissue joining parts of the skeleton and cervical spine ligament.

The most characteristic properties of the rheological biomaterials are highlighted. Among them we can point out hereditary or memory effect (see, e.g., [4], [5]), power-law relaxation (see, e.g., [6], [7] and the classical paper by Nutting [8]).

This paper is devoted to the discussion of different approaches to modeling of the above mentioned and some other properties of the biological materials. Since this subject is very large we restrict ourselves only on certain specific results which can have perspectives from our point of view.

The paper is organized as follows. We start in Sec. 2 with discussion few historical facts which led to discovering of some specific properties of biological materials. Section 3 is devoted to the description of the nowadays understanding of these properties. At last, in Sec. 4 we outline mathematical approaches which can be most useful (from our point of view) to provide a proper modeling of these properties.

2 Rheological properties of the biological materials

Rheology as a science deals with those materials which possess deformation and flow as respond on the applied forces or stresses. Thus, this science occupies the intermediate position between elasticity theory and hydrodynamics. The notion “rheology” as official title for the branch of sciences was proposed by E.Bingham at the inaugural meeting of Society of Rheology in December 1929 in Washington, DC. The rheological property was initially related to the discovered by him sigma-phenomenon [1]. The word “rheology” inspired by the Greek aphorism “panta reo” (“all is flowing”). So, rheological materials (solids or liquid) are those which can be as deformed as flow.

2.1 Power-law of relaxation

First evidence of existence of materials with these specific properties was due to Nutting who reported [8] (see also [9-10]) on his experiments concerning temporal dynamics of relaxation. He discovered that the relaxation of shear strains $\sigma(t)$ at instantaneous deformation/stress ε of visco-elastic materials satisfy power-law

$$\sigma(t) \approx const \cdot \Delta\varepsilon \cdot t^{-\alpha}, \qquad (1)$$

where the parameter $\alpha \in (0,1)$. This observation was in contradiction to the classical understanding of the exponential character of relaxation. Later the Nutting's observation was justified by Gemant who studied the properties of visco-elastic materials under harmonic load. He shown that the memory function $\eta(t)$ can have power-type relaxation behavior proportional to $t^{-3/2}$. In 1950 Gemant published a series of 16 articles entitled "Frictional Phenomena" in Journal of Applied Physics since 1941 to 1943, which were collected in a book of the same title [11]. In his eighth chapter-paper [12, p. 220], he referred to his previous articles [13], [14] for justifying the necessity of fractional differential operators to compute the shape of relaxation curves for some elasto-viscous fluids. Nowadays an existence of a long-time power-type relaxation is supposed to be related to the large-scale processes in materials (see, e.g. [5, p. 821]).

Next step in understanding of a special behavior of visco-elastic materials was done by Scott Blair (an extended description of the work by Scott Blair is presented in [15]). He studied properties of food products such as cheese, flour dough and clay. In fact, Scott Blair (together with Coppen) also came to the form of Nutting's equation, but from another consideration. They argued that the material properties are determined by various states between an elastic solid and a viscous fluid, rather than a combination of an elastic and a viscous element as proposed by Maxwell. In [16] (see also [17]) it was pointed out that, since for Hookian solids, strain is proportional to stress and to unit power of time, for intermediate materials, it might be expected to be proportional to stress and to some fractional power of time with exponent $\alpha, 0 < \alpha < 1$, and described this relation in the form

$$\psi = \varepsilon^{\beta}\sigma^{-1}t^{\alpha}, \qquad (2)$$

where proportionality coefficient ψ is a constant. The equation (2) looks entirely empirical, tough the fundamental significance of α (which is called the *dissipation coefficient*) is shown in experiments provided by Scott Blair and Coppen (see, e.g. [18]).

2.2 Memory effect

Power-law of relaxation was represented by Scott Blair in [19, 20] in the following differential form

$$\frac{\partial^{\mu}\sigma}{\partial t^{\mu}} = \frac{\Gamma(\alpha+1)}{\Gamma(\alpha-\mu+1)} t^{\alpha-\mu}\psi^{-1}\varepsilon. \qquad (3)$$

Such equation belonging to the class of differential equations of fractional order [21] deals also with another characteristic property of the visco-elastic materials, namely, "memory effect" or "aftereffect". Gerasimov [22] was probably the first who described this relation in the direct and clear form. He wrote "…It is of interest the case when the stress $\sigma(t)$ depends not on the strain itself but of its speed. For these processes hereditary part of the relationship linking σ and ε has the following form

$$\sigma(t) = \int_0^{\infty} K(\tau)\dot{\varepsilon}(t-\tau)d\tau. \qquad (4)$$

Memory (hereditary) function for certain materials has the following form

$$K(\tau) = \frac{A}{\tau^{\alpha}}, 0 < \alpha < 1." \qquad (5)$$

Hence, the right hand-side of Gerasimov's equation (4)-(5) has the form of fractional derivative of order α, which is called nowadays Caputo derivative or Dzherbashain-Caputo derivative or Gerasimov-Caputo derivative (see, e.g., [23], [24]):

$$\left({}^{C}D_{a+}^{\alpha}\varphi\right)(t) := \frac{1}{\Gamma(n-\alpha)}\int_a^t \frac{\varphi(\tau)d\tau}{(t-\tau)^{\alpha-n+1}}, \qquad (6)$$
$$n-1 < \alpha \le n.$$

The behavior of the materials with memory (or hereditary mechanics) is one of the important directions which allows us to be more close in understanding of the nature of biological materials.

3 Modern approaches to the study of visco-elastic biological materials

3.1 Fractional modeling

The theory of fractional calculus is a well-adapted tool to the modelling of many phenomena in Physics, Mechanics, Chemistry, Economics etc., allowing to take into account some peculiarities that classical integer-order model simply neglect (see multivolume encyclopedia of Fractional Calculus [25] and references therein). The importance of fractional order mathematical models is that they can be used to produce a more accurate description, and so give a deeper insight into the physical processes underlying long range memory behaviors. In the study of visco-elastic biological materials the Fractional Calculus is one of the leading approaches. The most extended development is made for the linear case of visco-elasticity [26]. Several books and survey papers describing fractional modeling for visco-elastic materials have been published during last two decades. We have to mention here two-volumes book by Uchaikin [27-28], where different physical processes are described from fractional point of view.

3.2 Another analytic methods for identifying properties of visco-elastic biological materials

There are only few common features of biological materials. Therefore, in order to describe specifics of precise materials it is necessary to develop different approaches. The book [29] by Nakhushev presents a number of approaches to the study biological materials although they partially based on fractional modeling (see also a survey paper [30]).

The same is true also for the encyclopedic paper [31] as well as for survey paper [32] on the development of the theory of visco-elastic solids. The latter deals also with dynamical properties of visco-elastic (or soft) solids. Both articles [31], [32] are the good sources for the recent citation on subject. Nonlinear character of certain processes in biological materials still needs to be handled more carefully. One of recent approaches of this kind is presented in [33]. An overview of the different fractional order circuit models are presented in [34]. The methods used to extract the impedance parameters from collected datasets are presented too. Applications of fractional order circuit models for study human tissue, plant physiology, respiratory systems are presented to highlight the significance of these models and their perspectives for further research.

The book [35] presents a comprehensive and unifying approach to analytical identification of material properties of biological materials. Focusing on depth-sensing indentation testing, pipette aspiration testing, and torsion of soft tissues, it discusses the following important aspects in detail: damping, adhesion, thickness effect, substrate effect, elastic inhomogeneity effect, and biphasic effect.

The indentation test is designed to characterize a material’s behaviour under complex loading. The test is performed by pressing a stiff steel sphere into a softer test specimen. Applied load and displacement are measured during the test, while the residual impression can be measured after the test has been completed. The results from this test are typically used to validate a material model developed solely with uniaxial data and to determine the dent resistance of a material.

Different techniques are used to quantify material characteristics at smaller scales. Measuring mechanical properties for materials, for instance, of thin films, cannot be done using conventional uniaxial tensile testing. As a result, techniques testing material "hardness" by indenting a material with a very small impression have been developed to attempt to estimate these properties.

3.3 Asymptotic analysis

Biological materials have a complex structure. Therefore the proposed models describe their behavior only approximately. Among the approximate method we can pointed out the method of small parameter. As an example of the articles where this method is realized to describe the behavior of the cartilage layer we can mention the paper [36] (see also the references listed in this article).

A comprehensive and unifying approach to articular contact mechanics with an emphasis on frictionless contact interaction of thin cartilage layers is presented in the book [37]. The asymptotic method is described in the book in a complete form. It allows us to have a deep insight in the behaviour of such a specific biological material, as well as to study the materials with similar properties.

It should be noted that the above mentioned properties can be modeling not only using fractional approach (see, e.g., [38]). Thus memory effect can be discussed on the base of certain integral equations with delay, spatial relaxation needs to utilize partial differential equations. It should be noted that complex character of biological materials leads to necessity to apply combination of different methods in order to better understanding the nature of such materials (see, e.g., [39]).

3.4 Numerical analysis

Much more attention is paid to numerical analysis of the properties of visco-elastic biological materials. The developed mechanical tools allow to perform simulation of different situations and, in particular, to provide non-invasive analysis of the above said properties. Specific approaches are presented in many papers. We mention here only few of them, e.g., [40], [41], [42].

3.5 Non-material properties of visco-elastic materials

Different behavior of similar materials at the same temperature allows us to presume that there is something determined their diversity. This can be represented in form of collection of internal variables. Usually (see, e.g., [5]) it is supposed that these variables are of two types, namely, physical parameters and mathematical parameters, which guarantee a necessary interpolation.

Anyway, already in 1940s it was supposed that not all parameters can be of such a type (see, e.g., [17, 19, 20]). Instructive is to cite some words by Scott-Blair quoted by Stiassnie in their correspondence, see [43] (see also [44]): *I was working on the assessing of firmness of various materials (e.g. cheese and clay by experts handling them) these systems are of course both elastic and viscous but I felt sure that judgements were made not on an addition of elastic and viscous parts but on something in between the two so I introduced fractional differentials of strain with respect to time*.

Different quasi-properties (i.e. non-material parameters) are discussed, e.g., in [45] in relation to food products. Such quasi-properties appear to compactly describe textural parameters such as the ‘firmness’ and ‘tackiness’ of real-world material, in particular, of many biological materials.

4 Conclusion

In this paper we discuss different approaches which allow us to have a better understanding the behavior of the visco-elastic biological materials starting from the pioneering works on the subject. Such a behavior has not only common features but also is rather specific for different type of biological materials. Therefore, our analysis can be considered only preliminary one and cannot pretend to be close to completeness. This kind of study has to be continued.

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