Physics of the Pendulum-Driven Oscillator

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Abstract: This paper presents a theoretical analysis of the working dynamics of the pendulum-lever oscillatory system, which its inventor called a two-stage mechanical oscillator [1]. The input of the system is a pendulum, while the output of the system is a lever that can idle like a hammer or deliver power to a water pump. It is shown that the transfer of energy from the input to the output of the system represents the damped operating mode of the parametric oscillator [2]. Namely, the oscillation of the lever moves the pivot point of the pendulum up and down in such a way that it takes away energy from the pendulum. This way of working is the opposite of the way a child swings a swing [3]. In this article, the forces acting on the driving pendulum are presented, and the focus of the paper is on the analysis of the change in centrifugal force as a reaction to the main component of the tension force in the pendulum rod. Since the tension force of the pendulum transfers its energy to the lever, it is necessary to prevent the rapid transfer of energy so that the pendulum does not stop, and on the other hand to ensure that the transferred energy has sufficient power for the lever to perform work.

Key-words: pendulum, lever, pivot point, centrifugal force, parametric oscillator, energy transfer.

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1 Introduction

The pendulum-lever system, known as a two-stage mechanical oscillator, was invented by the Serbian inventor Veljko Milkovic in 1999 [1]. The pendulum is the input part of the system that drives the lever. It is attached to a shorter part of the lever that can oscillate around its axis. The longer part of the lever is the output part of the system on which there is a counterweight or spring. A water pump piston can be attached to the longer part of the lever. The frequency of the lever is twice the frequency of the pendulum. When the pendulum is raised to the initial position and allowed to fall down, it will develop a sufficiently strong tension force near the bottom position, the moment of which will outweigh the moment of the weight of the counterweight on the opposite side of the lever. Then the opposite side of the lever will go up and the pendulum will go down. When the pendulum bob goes to the opposite side, the counterweight will return its side of the lever down, and the pivot point of the pendulum will return to the upper, starting, position. It means that the lever made a full cycle of oscillation while the pendulum completed only one-half period. Due to the movement of the pivot point of the pendulum up and down this system works as a parametric oscillator [2]. Since the pendulum loses energy, as it transfers it to the lever, it is a damped parametric oscillator. Since the speed of the pendulum is variable, as well as the tension force in the pendulum rod, the lever does not oscillate harmonically and therefore neither does the pendulum's pivot point. This is why this system is inharmonious. The mathematical description of such a system is very complicated, so a description of a pendulum with a fixed pivot point will be given here first. Then the influence of moving the pivot point on the tension force of the pendulum and its main component, the normal force, will be analyzed.

After the appearance of this invention, it turned out that it was very easy to pump water with it if a piston water pump was attached to the output part of the system. Namely, with the piston pump it was very difficult to pump water while it was working alone, due to the high friction between the piston and the cylinder. Once installed on the oscillator, pumping water was easy. Once the pendulum had swung, it could be maintained with the finger of one hand. The other hand was free to accept the water. Therefore, there was great enthusiasm for the use of this system. An additional argument to such enthusiasm was the opinion that the lever does not affect the operation of the pendulum, because if the lever is braked the pendulum continues to swing freely. This claim has generated much discussions, questions and doubts [4]. Several international scientists and scholars have also devoted some of their time to making and analyzing mathematical models of oscillators.

It has been shown, however, that the lever affects the operation of the pendulum, although this was not obvious to users of the system at first glance. A lever can speed up the swing of a pendulum, if it is set in motion at a precise time, at a precisely determined frequency. Likewise, the lever takes away energy from the pendulum when performing work. While the oscillator worked like a hammer, without consuming energy, the lever hit the stop and bounced. Due to the elastic collision of the lever and the limiter, most of the energy remained in the system and gave a false impression of a large output work.

2 Two-Stage Mechanical Oscillator

Fig. 1 shows a two-stage mechanical oscillator without energy consumption, which works like a hammer. The pendulum in Fig. 1 differs from the mathematical pendulum in that its pivot point O moves along the semicircular path of the lever on which it is suspended. The vertical path of the pivot point is much longer than the horizontal displacement, because the lever has a small angle of displacement in a circular path around point F, so the horizontal displacement of the pendulum pivot point will be neglected here.

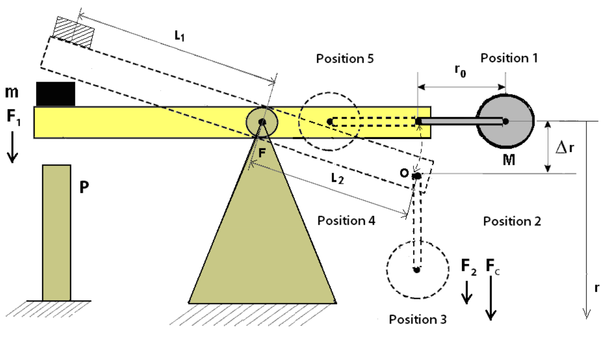

Fig. 1

Centripetal force and weight force act on the pendulum bob in motion. Centripetal force occurs in curved movements and is directed towards the center of the curvature of the path of the pendulum. That is why it is often called the normal force. The centripetal force and part of the weight force make up the tension force in the pendulum rod. The force of weight due to gravity and the force of tension in the pendulum rod are the only external forces that are taken into account in the calculation of the movement of the pendulum [5]. The strength of the tension force in the rod of the pendulum is equal to the centripetal force increased by the influence of the weight force in that direction. At the bottom point of the pendulum, in position 3, the strength of the tension force is exactly equal to the sum of the centripetal force and weight. The mathematical calculation of the pendulum can be found in the book on dynamics [5] or pendulums [6][7], so it will not be repeated here. In this paper, however, the term centrifugal force will be used more often, bearing in mind that it is a reaction to the centripetal force, which acts on the pivot point of the pendulum and not on the pendulum bob as a centripetal force.

The formula for centripetal and centrifugal force is:

$$F_c = \frac{M v^2}{r} \quad (1)$$

where r is the radius of the curvature of the path, which is equal to the distance of the body from the center only in the case of circular paths. The radius of curvature increases when the pendulum's pivot point is allowed to move downwards, and this weakens the centrifugal force. Centrifugal force weakens not only due to the increase in the radius of curvature r, but also due to the decrease in the speed of the pendulum v.

The pendulum with a fixed pivot point will be analyzed first in order to clarify the behavior of the pendulum when the point of pivot is allowed to move and do work with the help of a tension force in the pendulum rod.

2.1 Operation of the Pendulum Drive

Fig. 2 shows a pendulum with a fixed pivot point. It is designed to be launched from an initial angle of 90 degrees from the vertical line, position 1 or position 5. In these positions, the pendulum is raised from the lower position 3 to a height equal to the length of its rod r_0. It then has a potential energy equal to:

$$E_p = M g r_o \quad (2)$$

When the pendulum is allowed to fall downwards it loses potential energy but gains speed, thus converting potential energy into kinetic energy. In Fig. 2, the tension force is denoted by T, while its reaction acting on the pivot point is denoted by T'.

In practice, these are equal forces, so onwards, only force T will be used.

The formula for the tension force in the rod of a pendulum with a fixed point of pivot is [5]:

$$T = Mg\ (3\cos(\varphi) - 2\cos(\varphi_o)) \quad (3)$$

where φ_o is the angle of the initial position 1, and φ is the angle from the vertical line.

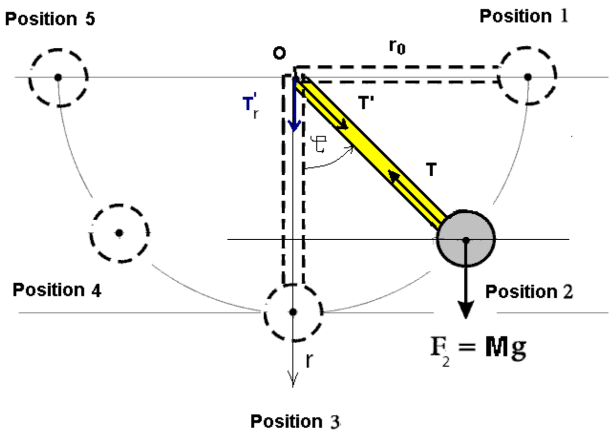

Fig. 2

It can be seen that the strength of the tension force T, and its centripetal force component, and therefore its centrifugal force reaction, do not depend on the length of the pendulum rod r_0, although this length exists in formula (1). The reason is that the speed of the pendulum depends on the transformed potential energy into kinetic energy, and the potential energy depends on the length of the pendulum rod, so that quantity is shortened in formula (1). It will be seen later that moving the pivot point increases the radius of curvature of the pendulum bob and nevertheless weakens the centrifugal force.

Since the pivot point can only move vertically, only the vertical component of the tension force T_r does work. That component also weakens with increasing angle φ, as well as the tension force T itself. The formula for the vertical component of the tension force is:

$$T_r = T\cos(\varphi) = Mg(3\cos(\varphi) - 2\cos(\varphi o))\cos(\varphi) \quad (4)$$

Below is a table for forces T and T_r depending on the starting angle φ_o. The forces were calculated for angles of zero degrees, 30 degrees and 45 degrees. For zero degrees (position 3) the forces T and T_r are the same. At zero degrees, one Mg in the tensile force belongs to gravity, and the rest to the centrifugal force.

Table 1

φ_o	60	90	120	180
T (0), T_r(0)	2.0 Mg	3.0 Mg	4.0 Mg	5.0 Mg
T (30)	1.6 Mg	2.6 Mg	3.6 Mg	4.6 Mg
T_r(30)	1.4 Mg	2.3 Mg	3.1 Mg	4.0 Mg
T (45)	1.1 Mg	2.1 Mg	3.1 Mg	4.1 Mg
T_r(45)	0.8 Mg	1.5 Mg	2.2 Mg	2.9 Mg

Fig. 2 shows the pendulum in position 2. Position 2 represents the angle when force T_r becomes strong enough to overcome the mass m on the left side of the lever from Fig. 1 and starts to lift it up, that is, when the pivot point of the pendulum moves downwards. We called that angle the critical angle.

The force T_r rapidly weakens with the increase of the critical angle, i.e., angle of position 2. If the pendulum is allowed to overcome the mass m at a large critical angle, then a very weak tension force T_r will do work in moving the pivot point downwards. The weak tension force T_r will give a small acceleration to the mass m on the left side of the lever, so the movement of the pivot point will be small at first. Therefore, the output work will be small in the beginning. That work could be useless if the load, attached to the left side of the lever, requires more force for its work.

2.2 Centrifugal Force and Angular Momentum

When a body with circular motion is not acted upon by a moment of force about the axis of rotation, as in the case of central forces during the rotations of the planets around the Sun, then the law of conservation of angular momentum applies. This law states that if the distance of the body from the center of rotation increases, then the speed of the body decreases proportionally, and vice versa.

$$M\, v_0\, r_0 = M\, v\ r \quad (5)$$

When the pivot point of the pendulum is lowered, the radius of the path of the pendulum bob is extended, and when the pivot point is raised, the radius decreases. It means that the law of conservation of angular momentum applies to the pendulum of the oscillator in the vicinity of position 3. The pendulum then behaves like a parametric oscillator. Note that the reverse rules apply to the case when a child swings the swing by rising in the lower position and lowering in the end positions [3]. It means that the pendulum is slowing down, unlike the child who is speeding up the swing.

The law of conservation of angular momentum does not apply to the left and right of the lower position 3. On the right side of position 3, the moment of the force of gravity acts, which adds speed to the pendulum because it converts potential energy into kinetic energy. To the left of position 3, the pendulum climbs up and loses speed as the force of gravity slows it down by converting kinetic energy into potential energy.

It can be concluded that there are three different zones of operation of centrifugal force. One is in the lower position 3 where the law of conservation of angular momentum applies. The second zone is between position 2 and position 3 where the positive moment of gravity adds speed and reduces the weakening of centrifugal force. The third zone is from position 3 to position 4 where the moment of the weight force subtracts the speed and where the height of the pendulum bob is kept constant due to the lowering of the pivot point. There, the centrifugal force weakens a lot due to the reduced speed and the flattened path of the pendulum bob.

It is necessary to minimize the weakening of the centrifugal force due to the lowering of the pivot point and the extension of the radius of curvature of the pendulum bob to prevent the pendulum not to suddenly lose its energy and stop swinging. Then the energy needed to keep the pendulum swinging will be easily added to the pendulum.

When the pendulum leaves position 4 and moves towards position 5 then its speed becomes low, as does the tension force in the pendulum rod, so the mass *m* on the left side of the lever can easily return the lever to its initial state.

2.3 Minimization of Centrifugal Force Change

As the pivot point moves downwards, from position 2, the path of the pendulum weight is no longer circular, but tends to straighten the circle until position 4, when the weight of the mass on the left side of the lever overcomes the tension force on the right side of the lever. It is the same as if the radius of curvature r_0 has been extended by Δr, see Fig. 3. In Fig. 3, the path of the pendulum bob from position 2 to position 4 is shown by a thick line.

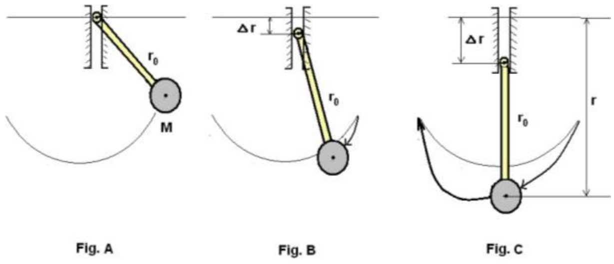

Fig. 3

Mathematically, the centrifugal force can be approximately determined in the vicinity of the lower position 3.

The new formula for the centrifugal force at the bottom point is:

$$F_c = M v^2 / (r_0 + \Delta r) \tag{6}$$

The decrease in centrifugal force is also due to the decrease in the speed of the pendulum. It has already been said that lowering the pendulum bob at the bottom point reduces its speed due to the law of conservation of angular momentum (5). That law is valid only in the vicinity of the lower point, in position 3, because there is no moment of force of the weight *Mg* in relation to the pivot point *O*.

The speed in position 3 changes due to the movement of the pivot point according to the law of conservation of momentum (5). It is given by the formula:

$$v = \frac{r_0}{r_0 + \Delta r} v_0 \tag{7}$$

where v_0 is the speed of the pendulum in the lower point (position 3) of a pendulum with a fixed pivot point, that is, before the change in the radius of the curve of the pendulum bob.

By changing the formula (7) to (6), we get the formula for the centrifugal force in the lower position 3, which includes the influence of the movement of the pivot point in that position.

$$F_c = M \frac{v_0^2 \, r_0^2}{(r_0 + \Delta r)^3} \tag{8}$$

The maximum speed v_0 of a pendulum with a fixed pivot point can be easily found if the initial angle of the pendulum φ_o, i.e., the height of position 1 is known. If that position had 90 degrees, then the initial height of the pendulum was r_0. In the lower position 3, all the potential energy of the pendulum has turned into kinetic energy, so the expression applies:

$$M g r_0 = \tfrac{1}{2} M v_0^2 \qquad (9)$$

and from here velocity v_0 can be found:

$$v_0^2 = 2 g r_0 \qquad (10)$$

By changing (10) to (8) the final formula for the centrifugal force is:

$$F_c = 2 M g \frac{r_0^3}{(r_0 + \Delta r)^3} \qquad (11)$$

The above formula is derived for an initial angle φ_o of 90 degrees. For another starting angle φ_o, only the constant '2' after the equal sign would change, so the analysis of the formula and the conclusion do not change.

Since the centrifugal force performs work due to the movement of the pivot point, and it is equal to the product of the force and the distance traveled Δr, in order to make this work as large as possible, either the centrifugal force or the movement of the pivot point should be increased. The problem is that the movement of the pivot point reduces the centrifugal force with the third degree, but also the speed of the pendulum and its kinetic energy. It means that the path of the pivot point should not be extended lightly.

In a pendulum with a large length r_0, the movement of the pivot point Δr is proportionally small compared to the length of the pendulum. This means that the movement of the pivot point will have a small effect on the reduction of the centrifugal force due to the increase in the radius of curvature and the decrease in speed.

Below is a table for pivot point movements Δr of 1 cm, 2 cm, 3 cm, 5 cm and 10 cm. The term from formula (11) is calculated for various values of the pendulum rod length r_0.

$$\rho = \frac{r_0^{\,3}}{(r_0 + \Delta r)^3} \qquad (12)$$

Table 2

r_0	0.25m	0.5m	0.75m	1m	2m	3m
ρ(1cm)	0.889	0.942	0.961	0.971	0.985	0.990
ρ(2cm)	0.794	0.889	0.924	0.942	0.971	0.980
ρ(3cm)	0.712	0.839	0.889	0.915	0.956	0.971
ρ(5cm)	0.578	0.751	0.824	0.864	0.929	0.951
ρ(10cm)	0.364	0.578	0.687	0.751	0.864	0.906

Table 2 clearly shows the improvement of the parameter ρ at larger lengths of the pendulum rod r_0, and thus the reduction of the negative influence on the centrifugal force. In order for the weakening of the centrifugal force to be less than 10%, the length of the pendulum should be taken where the value of ρ is greater than 0.9.

The important thing to make clear is that the centrifugal force takes the pendulum's kinetic energy to do its work. In order for this not to happen too quickly, the movement of the pivot point must be small, but then the transmitted energy at the output of the oscillator will also be small. Therefore, the output energy should be increased by increasing the mass of the pendulum.

3 Initial Power of the Pendulum

By increasing the length of the pendulum rod, the oscillation period of the pendulum (15) is reduced, and thus the lever. Since power is equal to the product of the tension force and the speed of the lever, one would think that the power of the oscillator would weaken if the pendulum rod were lengthened. It will be proven below that increasing the length of the pendulum rod increases the initial power of the pendulum itself, but increasing the length of the pendulum rod has no effect on the maximum output energy of the oscillator because the tension force (3) does not depend on the length of the pendulum r_0 for a pendulum with a fixed pivot point.

By looking at formula (2) again, it can be seen that the initial energy of the pendulum E_p is proportional to the height of the initial position, and in the case of a deflection of the pendulum of 90 degrees, it is the same as the length of the pendulum rod r_0.

$$E_p = M g r_0 \qquad (13)$$

The power will be calculated in the lower position 3 when all the potential energy is converted into kinetic energy and when the tension force of the pendulum is at maximum. The time required for the pendulum to reach the bottom position is one quarter of the oscillation period i.e., $P/4$.

Power is defined as the quotient of energy and time:

$$W = E_p / (P/4) \qquad (14)$$

Time P is the oscillation period of the pendulum and for initial angle φ_o of 90 degrees it is:

$$P = 2\pi\sqrt{\frac{r_0}{g}}(1+\frac{1}{4}\sin^2(90)) \quad (15)$$

Substituting equations (13) and (15) into (14), the power in position 3 is equal to:

$$W = (8Mg/5\pi)\sqrt{g\,r_0} \quad (16)$$

This means that although the time of oscillation increases with the increase of the pendulum length, the power also increases. The reason is that the initial potential energy increases proportionally with the length of the pendulum while the period of oscillation increases with the square root of the length of the pendulum. It means that the potential energy grows faster than the oscillation time.

4 Conclusion

In this paper, it was shown that increasing the length of the pendulum slows down the oscillation time of the pendulum-lever system, but still increases the power of the pendulum itself. However, the power of the pendulum can be directly transferred to the output of the oscillator only by increasing the mass of the pendulum or the initial angle of the pendulum, because this increases the tension force in the rod of the pendulum, and it does work on the lever. Increasing the length of the pendulum rod does not affect the tension force (3) or the centripetal force (1) of a pendulum with a fixed pivot point. However, the movement of the pivot point still weakens the centripetal force due to the increase in the radius of the curvature of the trajectory of the pendulum bob. Therefore, a longer pendulum length will reduce the impact of that movement and reduce the sudden loss of pendulum energy.

The reason for the easy pumping of water using the pendulum-lever system is twofold. One of the reasons is the accumulated energy in the pendulum, which must first be raised to the initial position 1. However, there is another important reason for the advantage of oscillators in piston pumps. These pumps use a vacuum to lift water, so there is a lot of friction between the piston and the cylinder. It is known that there is static and dynamic friction. Static friction is up to two times higher than dynamic one [8][9]. The pendulum develops a centrifugal force so that the total tension force is several times greater than the weight of the pendulum. That force will easily overcome the static friction and move the pump piston. When the piston moves, the pendulum will lose energy, but further movement of the piston will be easy because the dynamic friction is weaker.

References:

[1] Milkovic, Veljko, "Rucna pumpa za vodu sa klatnom", patent no: YU 49002 B – P-577/99, Zavod za intelektualnu svojinu Beograd, 1999. www.pendulum-lever.com/images/patents/patent1a.jpg

[2] Slavkovic, Aleksandar B, "Mr. Milkovic's two-stage oscillator as a parametric oscillator", VEMIRC, 2009. www.veljkomilkovic.com/Images/Aleksandar_Slavkovic_Milkovic's_Two_Stage_Oscillator_As_a_Parametric_Oscillator.pdf

[3] Mokhiemer, Tareq Ahmed, "How to Pump a Swing", King Fahd University of Petroleum and Minerals, 2008. https://web.archive.org/web/20111003161817/http://staff.kfupm.edu.sa/phys/tahmed/How%20to%20pump%20a%20swing.pdf

[4] "Scientific and Expert Opinions on the Two-Stage Mechanical Oscillator", VEMIRC, 2005. www.veljkomilkovic.com/MisljenjeEng.html www.vemirc.com/en/references-citation

[5] Rusov, dr Lazar, *Mehanika III: Dinamika*, Naucna Knjiga, 1994.

[6] Pook, L.P., *Understanding Pendulums: A Brief Introduction*, Springer, 2011.

[7] Baker, Gregory L; Blackburn, James A., *The Pendulum: A Case Study in Physics*, Oxford University Press, 2005.

[8] "Friction", *Wikipedia, The Free Encyclopedia*, Wikimedia Foundation, April 11, 2026. https://en.wikipedia.org/wiki/Friction

[9] "Static and Kinetic Friction", NSF – GK12 Program of Division of Graduate Education, *RAISE – Revitalizing Achievement by using Instrumentation in Science Education 2004-2007*, Polytechnic Institute of New York University, 2020. https://mechatronics.engineering.nyu.edu/pdf/raise-static-and-kinetic-friction.pdf