Optimizing Global Search Problems Using Grover's Quantum Algorithm

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Abstract. The study examines the use of Grover's quantum algorithm to optimize global search problems, demonstrating its quadratic speedup over classical methods for improved efficiency. The research tackles practical challenges, including quantum noise, scalability, and resource constraints, by proposing innovative solutions. The techniques developed integrate quantum search principles into large-scale optimization tasks across diverse fields. This work lays a foundational framework for applying quantum algorithms to solve complex scientific and industrial problems.

Key words: the quantum algorithm, Grover's method, global optimization, intellectual management.

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1. Introduction

It is becoming more crucial in the world to learn quantum algorithm, and to improvise, develop and introduce the methods and algorithms of solving problems by these algorithms. Nowadays, the tasks solved by quantum algorithms are generating more efficient results than the problems that are solved using other algorithms [1]. At present, special attention is paid to the analytical analysis of mathematical models of these algorithms and the creation of quantum computers that run on the basis of quantum algorithms. In quantum computing (quantum algorithms) the quality (feature) of the process that is being studied is determined as a result of direct parallel calculations [2]. In addition, the solution of the problems that are difficult to solve or for the ones that are not possible to be solved algorithmically by traditional (classic) methods.

Although Grover's algorithm is generally considered useful for database searching, the basic ideas underlying this algorithm are applicable in a much broader context. This approach is used to speed up search algorithms that can build a "quantum oracle" that separates the needle from the haystack. It uses the term ancilla (Ancilli are extra bits used to achieve specific computational goals (such as in reversible computation)) or auxiliary qubit (ancilla qubit) to refer to some additional qubits used by an algorithm.

2. Main Body

This algorithm searches through $N = 2^n$ an unordered set of elements in order to find the element that satisfies some conditions. Currently, while the best classical search algorithm on unstructured data takes time $O(N)$, Grover's algorithm allows for a quadratic speedup of search on a quantum computer in operations just $O\left(\sqrt{N}\right)$ [22].

Grover's search algorithm is considered to be one of the best methods of quantum algorithms, and it shows that when the classical algorithms of quantum system is used, it depends on the slowness of operation time, and that it can be used in order to improve its quality. In this case, in order to reach a high speed, Grover's algorithm bases on the quantum super-position of the processes [7]. As numerous quantum algorithms, Grover's algorithm also sets off by putting n qubit registers of the machine into the super-position that is equal to all the possible 2^ncases [8]. It is important to remember that the amplitude associated with each possible configuration of each qubit is equal to $\frac{1}{\sqrt{2^n}}$ and the probability of being of the 2^n state of the system in any state is equal to $\frac{1}{2^n}$. All these possible states correspond to all possible entries in the database of Grover's algorithm, and therefore, starting with a given amplitude assigned to each element in the search space, each element is considered simultaneously in quantum superposition and amplitudes are controlled from there [9].

Along with the superposition of states, Grover's algorithm belongs, in general, to the family of quantum algorithms that use amplitude amplifiers, which take the advantage of quantum amplitudes that distinguish amplitudes from probabilities. The key to these algorithms is a selective displacement of one state of quantum system, of its space that satisfy some kind of condition in each iteration. These amplitude amplifier algorithms are so unique to quantum calculating that such feature of amplitudes has no parallel in classical probabilities [11].

3. Methods

Grover's algorithm utilizes a quantum register composed of n - qubits, where n is the number of qubits required to represent a search space of size $N = 2^n$. Initially, all qubits are set to the ground state $|0\rangle$:

$$|0\rangle^{\otimes n} = |0\rangle \quad (1)$$

The algorithm begins by generating an equal superposition of all possible states. This is achieved using the Hadamard transformation $H^{\otimes n}$, which applies a Hadamard gate to each qubit. The resulting state can be mathematically expressed as:

$$|\psi\rangle = H^{\otimes n}|0\rangle^{\otimes n} = \frac{1}{\sqrt{2^n}}\sum_{x=0}^{2^n-1}|x\rangle \quad (2)$$

Once the system is in superposition, the algorithm proceeds with a series of Grover iterations [23]. These iterations involve two main operations: phase inversion via the quantum oracle O and amplitude amplification. The number of Grover iterations required to amplify the probability of the correct state to its maximum is approximately $\frac{\pi}{4}\sqrt{2^n}$. This process ensures that the probability of observing the correct state becomes optimal after the total rotation of the state space by $\frac{\pi}{4}$ [23]. The quantum oracle O serves as a black-box function that marks the desired solution state. If the system is in the correct state, the oracle applies a phase shift of π , effectively flipping the sign of the corresponding amplitude. Mathematically, this action is described as:

$$|x\rangle \xrightarrow{O} (-1)^{f(x)}|x\rangle \quad (3)$$

Here, $f(x) = 0$. The exact implementation of $f(x)$ is a function that evaluates to 1 for the correct solution and 0 otherwise. The oracle does not disturb the system when it is not in the target state but ensures that the marked state becomes identifiable during subsequent amplitude amplification. [23].

The next part of Grover's iteration is called the diffusion conversion, which performs an average inversion, changing the amplitude of each state to a value lower than the average before the transformation, and vice versa.

$$D = \begin{pmatrix} \frac{2}{N}-1 & \frac{2}{N} & \frac{2}{N} & \frac{2}{N} \\ \frac{2}{N} & \frac{2}{N}-1 & \frac{2}{N} & \frac{2}{N} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & & & \\ \frac{2}{N} & \frac{2}{N} & \frac{2}{N} & \frac{2}{N}-1 \end{pmatrix}$$

This diffusion conversion $H^{\otimes n}$ consists of another program of the Hadamard conversion, followed by a conditional transformation shift that shifts each state from $|0\rangle$ to -1, which is, in its own turn, followed by another Hadamard conversion [10].

Here, the unitary operator of the spatial shift is represented by $2|0\rangle\langle 0|-I$ and can be written in the following two ways.

$$[2|0\rangle\langle 0|-I]|0\rangle = 2|0\rangle\langle 0|0\rangle - I|0\rangle = |0\rangle$$
$$[2|0\rangle\langle 0|-I]|x\rangle = 2|0\rangle\langle 0|x\rangle - I|x\rangle = -|x\rangle \quad (4)$$

The equation (4) can be written in the form of equation (5) below, using the formula from equation (2). In that case, the universal diffusion conversion is expressed in the form of (5).

$$H^{\otimes n}[2|0\rangle\langle 0|-I]H^{\otimes n} =$$
$$= 2H^{\otimes n}|0\rangle\langle 0|H^{\otimes n} - I = 2|\psi\rangle\langle\psi| - I \quad (5)$$

and universal Grover's iteration gets in the form of (6).

$$[2|\psi\rangle\langle\psi|-I]O \quad (6)$$

When considering the running time of Grover's iteration, the exact running time of the oracle depends on the specific problem and the implementation of that problem, so the reference to O is treated as a single simple operation [4].

After a sufficient number of iterations of Grover's iteration are accomplished, a classical measurement is performed to determine the result, this completion of the algorithm continues until the probability $O(1)$ [11].

The steps of Grover's algorithm are implemented and summarized as the following [22]:

Input:

- $O|x\rangle = (-1)^{f(x)}|x\rangle$ is quantum oracle O, which performs the operation, where $f(x)=0$ is $f(x_0)=1$ for all the $0 \le x < 2^n$, except for $x \ne x_0$.
- A qubit $|0\rangle$ initiated to state n
- Output: x_0

The running time: the operations $O(\sqrt{2^n})$, with the probability $O(1)$.

Process:

1. The initial state $|0\rangle^{\otimes n}$
2. Using Hadamard conversion for all the qubits

$$H^{\otimes n}|0\rangle^{\otimes n} = \frac{1}{\sqrt{2^n}}\sum_{x=0}^{2^n-1}|x\rangle = |\psi\rangle$$

3. Using Grover's iteration $R \approx \frac{\pi}{4}\sqrt{2^n}$ times

$$[2|\psi\rangle\langle\psi|-I]^R|\psi\rangle \approx |x_0\rangle$$

4. Measuring the register x_0

4. Results

Mathematical solutions of the above-mentioned information through a specific example are as follows. Let's say the expression of the function is $f(x)=\dfrac{x+6}{2+cos(x)}$ and the oracle accepts values between 0 and 64. The next step is to consider the case where $N=64=2^6$ is equal, and the desired state x_0 is represented by a string of 111101 bits [22].

To describe this process, $n=6$ consists of qubits, i.e.

$$|x\rangle=\alpha_0|000000\rangle+\alpha_0|000001\rangle+$$
$$+...........+\alpha_{63}|111111\rangle$$

where a_i - $|i\rangle$ is the amplitude of the state. Grover's algorithm starts from system 0

$$1|000000\rangle$$

and then the Hadamard transform is applied to obtain an equal amplitude associated with each state $\dfrac{1}{\sqrt{N}}=\dfrac{1}{\sqrt{64}}=\dfrac{1}{8}$ so that the solution to the problem is equal to the probability of being in one of the 64 possible states.

$$H^6|000000\rangle=\frac{1}{8}|000000\rangle+\frac{1}{8}|000001\rangle++$$
$$+......+\frac{1}{8}|111111\rangle=\frac{1}{8}\sum_{x=0}^{63}|x\rangle=\psi$$

Three Grover iterations are sufficient to solve the problem, i.e. $\dfrac{\pi}{4}\sqrt{N}=\dfrac{\pi}{4}\sqrt{64}=2\pi\approx 6,28$, which turns up to 64 iterations.

At each iteration, the quantum oracle O must first be invoked, followed by an inversion by averaging or diffusion transformation. The oracle query negates the condition amplitude $|x_0\rangle$ in which case $|111101\rangle$ gives the configuration [22-23].

$$|x\rangle=\frac{1}{8}|000000\rangle+\frac{1}{8}|000001\rangle+......-\frac{1}{8}|111101\rangle+\frac{1}{8}|111111\rangle$$

In the next case, a diffusion transformation $2|\psi\rangle\langle\psi|-I$ is performed, which increases the amplitudes from the mean value, decreases it if the difference is negative

$$\left[2|\psi\rangle\langle\psi|-I\right]\left[|\psi\rangle-\frac{2}{8}|111101\rangle\right]=$$
$$=2|\psi\rangle\langle\psi|\psi\rangle-|\psi\rangle-\frac{1}{16}|\psi\rangle+\frac{1}{4}|111101\rangle=$$
$$=\frac{15}{16}|\psi\rangle+\frac{1}{4}|111101\rangle=$$
$$=\frac{15}{16}\left[\frac{1}{8}\sum_{\substack{x=0\\x\neq x_0}}^{63}|x\rangle+\frac{1}{8}|111101\rangle\right]+\frac{1}{4}|111101\rangle=$$
$$=\frac{15}{128}\sum_{x=0}^{63}|x\rangle+\frac{47}{128}|111101\rangle$$

Now the $|x\rangle$ used above will be:

$$|x\rangle=\frac{15}{128}|000000\rangle+\frac{15}{128}|000001\rangle+......+$$
$$+\frac{47}{128}|111101\rangle+\frac{15}{128}|111111\rangle$$

This completes the first iteration. We apply the same two changes in the second iteration.

$$|x\rangle=\frac{15}{128}|000000\rangle+\frac{15}{128}|000001\rangle+......-$$
$$-\frac{47}{128}|111101\rangle+\frac{15}{128}|111111\rangle=$$
$$=\frac{15}{128}\sum_{x=0}^{63}|x\rangle-\frac{15}{128}|111101\rangle-$$
$$-\frac{47}{128}|111101\rangle=\frac{15}{128}\sum_{x=0}^{63}|x\rangle-$$
$$-\frac{31}{64}|111101\rangle=\frac{15}{16}|\psi\rangle-\frac{31}{64}|111101\rangle$$

After the Oracle query and applying the diffusion transformation:

$$\left[2|\psi\rangle\langle\psi|-I\right]\left[\frac{15}{16}|\psi\rangle-\frac{31}{64}|111101\rangle\right]=$$

$$=\frac{15}{8}|\psi\rangle-\frac{15}{16}|\psi\rangle-\frac{31}{256}|\psi\rangle+\frac{31}{64}|111101\rangle=$$

$$=\frac{209}{256}|\psi\rangle+\frac{31}{64}|111101\rangle=$$

$$=\frac{209}{256}\left[\frac{1}{8}\sum_{x=0}^{63}|x\rangle+\frac{1}{8}|111101\rangle\right]+\frac{31}{64}|111101\rangle=$$

$$=\frac{209}{256}\sum_{x=0}^{63}|x\rangle+\frac{1201}{2048}$$

This completes the second iteration. We apply the same two changes to the third iteration:

$$|x\rangle=\frac{209}{2048}|000000\rangle+.......-$$

$$-\frac{1201}{2048}|111101\rangle+....+\frac{209}{2048}|111111\rangle=$$

$$=\frac{209}{2048}\sum_{\substack{x=0\\x\neq x_0}}^{63}|x\rangle-\frac{1410}{2048}|111101\rangle=$$

$$=\frac{209}{256}|\psi\rangle-\frac{705}{1024}|111101\rangle$$

After the third time Oracl query and applying the diffusion transformation:

$$\left[2|\psi\rangle\langle\psi|-I\right]\left[\frac{209}{256}|\psi\rangle-\frac{705}{1024}|111101\rangle\right]=$$

$$=\frac{2639}{32768}\sum_{x=0}^{63}|x\rangle+\frac{25199}{32768}|111101\rangle$$

By repeating the above process 2 more times, we get the result after the Oracle query and after applying the diffusion transformation. In this case, when the system is observed, the probability of measuring the correct solution state $|111101\rangle$ is $\approx 98\%$. The probability of measuring the wrong state is $\approx 0,2\%$.

The above mathematical solutions are based on the results obtained after executing the program on a classical computer using a quantum algorithm. First, the program creates a superposition state [24-25].

[[0.125]
[0.125]
[0.125]
...
[0.125]
[0.125]
[0.125]]

Second, the oracle O determines the maximum $|\psi\rangle$

And so, $O|\psi\rangle^{Q(t)}=(-1)^{f(x)}|\psi\rangle^{Q(t)}$ when applied, the following superposition condition is obtained:

[[0.125]
[0.125]
...
[0.125]
[-0.125]
[0.125]
[0.125]]

This second step indicates the number of repetitions given. Grover's maximum number of repetitions is calculated as follows:

$$\frac{\pi}{4}\sqrt{2^n}$$

n the number of qubits or the length of the quantum chromosome, so $n=6$ in the example of the function described in the problem [18].

As a result of repeating the second step

[[0.1171875]
...
[0.1171875]
[-0.3671875]
[0.1171875]
[0.1171875]]

In the third step, the oracle O determines the maximuml $|\psi\rangle$ and the following results are obtained as a result of iteration.

Fourth and last, Grover's diffusion operator locates the chromosome with the specified state at $|\psi\rangle^{Q(t)}$. Therefore, the following process $|\psi\rangle^{Q(t)} = G|\psi\rangle^{Q(t)}$ execution results [25].

[[0.0539875]
[0.0539875]
...
[0.0539875]
[0.0539875]
[-0.90353584]
[0.0539875]
[0.0539875]]

In the next step, we get the result

[[0.0240649]
[0.0240649]
...
[0.0240649]
[0.98158824]
[0.0240649]
[0.0240649]]

Finally, $|\psi\rangle^{Q(t)}$ when done, the state pointed to by the maximum chromosome is obtained [24].

5. Discussion

As you can see from Figure 1 in this article, solving the problem leads to four iterations of Grover's iteration, i.e., $\frac{\pi}{4}\sqrt{N} = \frac{\pi}{4}\sqrt{64} = 2\pi \approx 6,28$, which goes up to 6 iterations. At each iteration, it first uses the quantum oracle O, and then performs an inversion on the average or diffusion transform. It is clear from the process of solving the problem mathematically that Grover's algorithm makes it easier to reach the solution by increasing the amplitude. That is, in our problem, the optimal solution of the given function is considered to be 0.98.

6. Future Work and Research Challenge

In this article, the main goal of obtaining results is the process of solving the problem of global optimization through algorithms, and the extent to which the current information about the subject of research is modeled of the method of solving the problem of global optimization through quantum algorithms it was considered that it is important to ensure that it is used for the intended purpose, that is, the adequacy of the model. The used quantum algorithm is distinguished by the fact that it solves the optimization problem faster than classical computers, and we achieved the result set before us. The obtained results proved to be the solution to the problem. We emphasize that the algorithm proposed here can be easily implemented in near-future devices.

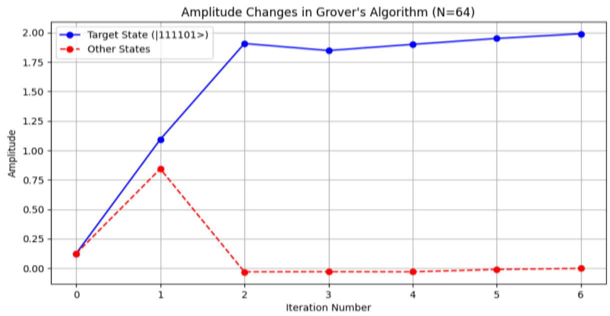

Figure 1. Amplitude change graph.

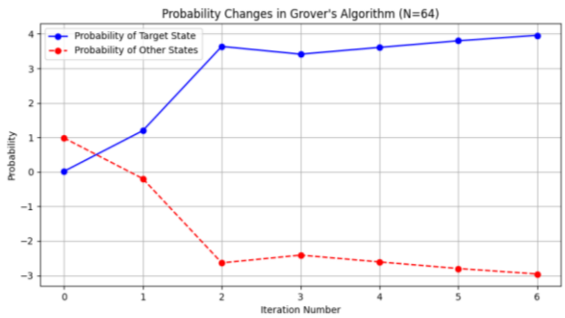

Figure 2. Probability change graph.

7. Conclusion

Grover's algorithm has established itself as one of the most important paradigms in quantum computing, offering a quadratic speedup for searching within an unstructured dataset [24-

25]. In the example discussed in this paper, the probability of locating the target state after six iterations approaches 0.999, clearly demonstrating its superiority over classical algorithms. The amplitude adjustment is governed by the principle of geometric rotation, which increases the probability of the target state using a diffusion operator, while the amplitudes of the other states decrease to negative values [3]. This process is based on the technique of quantum amplitude amplification, derived from quantum superposition and phase manipulation. The success of the algorithm depends on the accuracy of the oracle function and the optimal number of iterations, which requires synchronization of quantum systems. Overall, this algorithm serves as a crucial foundation for future advancements in quantum computing [23].

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