The Deep Orbit Parallax of Gravitational Lensing in the Context of Space-based Telescopes

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Abstract: - This paper proposes anchoring the isospin axis within astrophysical data reduction processes using multi-wavelength and multispectral observations. Drawing from the foundations of Zermelo–Fraenkel and Von Neumann–Bernays–Gödel set theories, the time series of black hole dynamics can be reconstructed based on local nuclear particle behaviors—an alternative to conventional atomic clock-based relativistic timekeeping. This framework enables the derivation of more objective models of celestial dynamics, including possible dark energy evidence inferred from black hole and white hole oscillations. The meta-theoretical nature of this work is complemented by white hole data analyses, enhancing cosmological theory with particle-based representations of mechanical sources in an open universe—contrasting the closed Big Bang model. Built upon the measurable hypothesis from Conformal Cyclic Cosmology, this study utilizes analytical geometry and orbital frequency modulations in space-based telescopes to link parallax phenomena with gravitational lensing effects.

Key-Words: - Negative space; frequency of time; qubit oscillation; measurement and mathematics; gravitation; measurement bias; randomness; data and theory; electron deep orbit.

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1 Introduction

The scale of the universe is beyond measurement, and the Big Bang theory hypothesized the scales that are measurable. The measurable hypothesis is inherited in the Conformal Cyclic Cosmology (CCC) with the combination of parallax and gravitational lensing [1]. The measurement of space and time themselves, however, can only depend on the relativistic cosmic rays that are collected in the quantum systems to be sorted out [2]. The astronomical definition of time experienced the paradigm shift from geometric and geological measurements on the solar objects to atomic frequencies [3]. Therefore, in the quantum realm of astrophysical measurements, the dilation of particle sequences and patterns becomes the dilation of time.

It is entailed then that the concept of time in the quantum apparatus can be multidimensional. Time being a dimension itself is core to relativistic physics (Greene [4] has detailedly elaborated on relativistic physics and string theory), but electron deep orbit (EDO) was not discussed in Einstein's legacies. Instead, time crystals in quantum systems and their photonic interactions are boosting the discussions in the potentials for cosmology with light-matter interactions based on the Λcold matter model [5, 6]. The most constructive element in these discussions is the transformation of the concept of time to the concept of the continuum of space [7].

2 Problem Formulation

The methodology being descried in the literature advances the parallax and gravitational lensing methods. As portrayed in Fig. 1, the interaction of light and matter sampled in the quantum systems in observations constitutes a cyclic continuum of a fraction of celestial motions. The cosmic rays collected within the duration does not necessarily describe time linearly as a whole, and for gravitationally lensed objects, the decayed cosmic rays describe, mathematically, the objects' spatial trajectories in the given duration, with some retrogressive analytic transformation [2, 8]. Therefore, the analytic geometry can be applied in the model implied by Fig. 1.

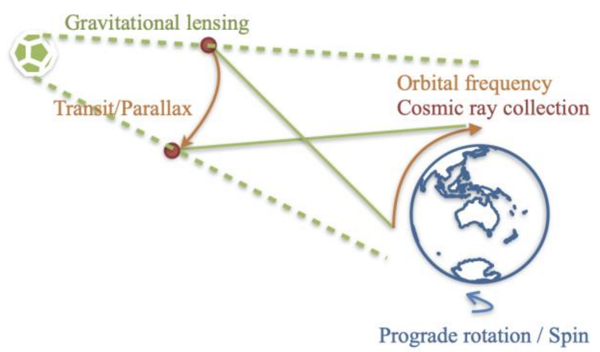

Figure 1 The instrumentation objectification in relation to gravitational lensing.

In terms of quantum mechanics, the direct extrapolations of the quantum information gathered from observations constitute the space of probabilities. The measurement of three-dimension probability space is well-document in [9], and the holographic principle set the degree of freedom to the possible spacetimes in a given set of data [10]. The positive results, however, become non-probable as the uncertainty becomes certain by the quantum indeterminacy, or philosophically spoken as superdeterminism [11].

Local hidden variable theories are, interpretively, adherent to the research framework explicated in Fig. 1. The Hardy–Unruh chains introduced by Janas and Janssen [9] describe the dissection of measurement logics in the quantum systems from the cosmological logics of data collection, which can be simplified to the logics of the mirror image and the logics of cosmology on the relativistic bases. The article focuses on the former, i.e., the mirror image space, in relation to the geometries of the data collection space. The process for mathematical physics application is guaranteed by the infinity category theory [12].

Drawing from the data obtained by Fukushima and Hatsuda [8] and other sources extracted by the large language model artificial intelligence (AI) ChatGTP, a heatmap is generated in Fig. 2 so as to better elucidate the problem.

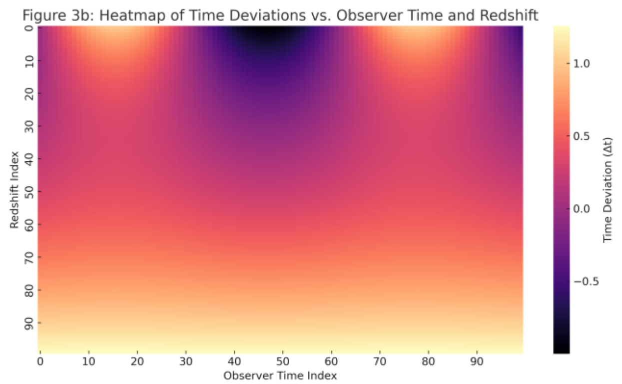

Figure 2 The time deviation in observational cosmology presented as a heatmap of the observable universe with the thermodynamic anomalies.

3 Problem Solution

Here θ is used to denote the angle between the axis of isospin of the target sample of the incident particles collected and the collection plate, α is used to denote the angle between the particle isospin axis and the collection plate's movement direction, and β is used to denote the angle formed by the plane of the isospin axis and the point where α is located, tangential to the direction of the isospin. All angles are right-handed and the spatial relations are illustrated in Fig. 3. It can be seen that α can be simplified to the angle of the incident particle tangential to the orbital plane of the imaging plate in relation and relative to the earth, and β the angle of the isospin axis and the earth's self-rotation axis. The method uses the three important angles to reassemble in the quantum system the mirrored space of the gravitationally lensed object being detected.

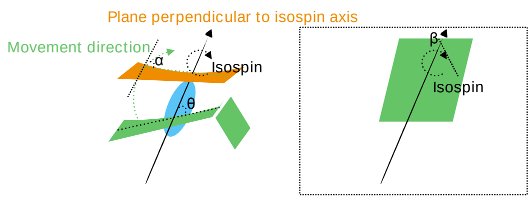

Figure 3 Illustration on the spatial relations of the three angles.

The momentum sequence is defined to the n^{th}, therefore, the future sequence $n+1$ is not defined. The dimensionless unification of the strong nuclear and the electromagnetic interactions $e^{n+1-\pi}\left(e^{\frac{i}{a}}+e^{-\frac{i}{a}}\right)=2$ is adopted from Pellis [13], where a represents the fine-structure constant and the number 2 also happens to be the hypothesized quantum number of the graviton [14]. Since the sequence is adopted for geometric analyses, no meaning in the definition of time is attached to n. Thus, a fifth-dimension m is introduced to supplement the fourth dimension of momenta in the n sequence.

The five-dimension coordinate is introduced to incorporate the real world into the mirrored geometries in the quantum system. Whereas space can be calculated according to estimation in scales, time cannot, in principle. In preservation of current laws of physics in the application for calculations in the research with the four dimensions, the fifth dimension adjusts the time vector unidirectional in four-dimension spacetime for the analytic geometry process; namely, the fifth dimension breaks the linear causal perceptions of time in relativistic physics and forms a meta-causality. Further elaborations will be dissected into a dedicated article in either pure or applied mathematics.

3.1 Meta-Causality

The physical properties of the radiuses of curvature of the angles can be determined depending on the orbital speed and height of the plate, and isospin frequency relative to the self-rotation and electromagnetic field of earth. Therefore, from the moment $n-1$ to n, the plate's orbital speed v_p is the arc-length, with its coordinate at n being $(0, 0, 0, n, n)$, illustrated in Fig. 4.

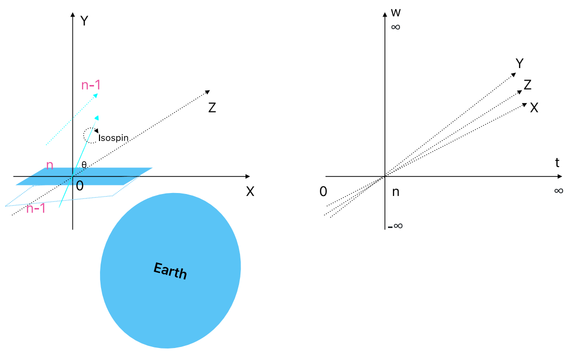

Figure 4 Illustration on the fifth dimension according to time and space relations.

The principle of least action doesn't necessarily apply in the causal inference when the fifth dimension is introduced. Phase transitions in quantum chromodynamics (QCD) crossing the chiral symmetry at high density and low temperature were expected to be involved in violent astrophysical phenomena with infrared fixed point in the renormalization group in the asymptotic freedom beyond the Fermi surface [15]. Consequentially, the normative causal descriptions of time and space, and the energy quantity derived from the former presumptions break apart, but not with the fifth dimension that is outside of the Big Bang framework of interpretations. Namely, the fifth dimension, empirically in the relevant researches, displays the explosive astrophysical morphologies seen in the video Pachankis [16].

3.2 Broken Timelines

Even with the SI definition of time and with atomic clocks, the reference of the observer's locality or the solar calendars is still normatively implied. The problem of broken timelines is an anthropological question, and a question on if the concept of time and space are adequate for contemporary [astro]physics, even when geological / archeological time is applied in cosmological calculations [3, 17]. The unification of fundamental interactions with relative space for reference, however, can take a preliminary solution to the problem [13].

It can be seen in Fig. 4 that the arrow of isospin can be regarded as the arrow of time, giving meaning to the fifth dimension. The concept is similar to gravitational wave, but in essence different. The conceptualization on the arrow of time is epistemologically subjective, on the basis of the unidirectional flow of life forms. Hereby, I intend to convert the fifth dimension to an objective epistemology, and isospin is the nonlinear concept that is well-established.

The set of the fifth dimension, the numerical form of the coordinate geometry, is the objective epistemology of the concept of time. Albeit the number line of this dimension is two-dimensional, its correlational angles with other dimensions are conformal to the theory of general relativity. Only that instead of the warp of the spacetime fabric as a whole, the fifth dimension's angle separates gravity from space, and time from gravity. Namely, the more condense the gravity, the more accelerated the passage of time; the more spacious the occupation of mass, the less accelerated the fixed direction of time [18]. Fig. 5 illustrates the concept.

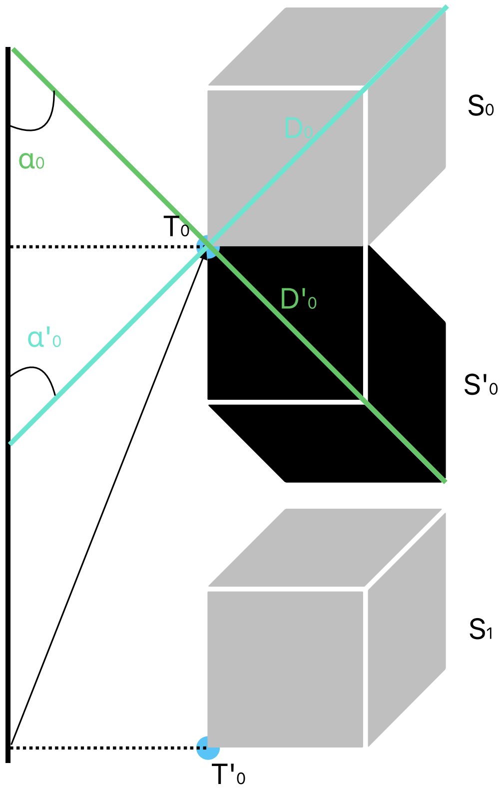

Figure 5 Conceptualization of spacetime curvature.

With the premise of definitive time point and spacetime curvature, the curvature angle on the fifth dimension determines the spatial morphology, and consequentially matter content component, of the spacetime. For example, in Fig. 5, the isospin of the matter content in time T_0 may project four or more morphologies with the same distance D_0 relative to the fifth dimension with the same angle α_0 in geometrically numerical terms. While at T'_0 on a direct line parallel to the fifth dimension, the spatially identical morphology S_1 is differentiated with S_0 by the dimension of time, the path time takes from T'_0 to T_0, however, can differ by the morphological paths of space projected on the fifth dimension from S_1 to S_0. In terms of concrete detections, the linearized presumption of light clock or atomic clocks rests on the measurable hypothesis that the duration measured from two points of time has the unchanging spatial dimension. The theoretical framework corroborates with the observations by Weinberg with nuclear time-reversal violation (T violation) in particle physics [19].

The broken timelines in terms of the fifth dimension render a theoretical framework for the inevitable formation of the time-like and space-like singularities. The projection of the four-dimension spacetime on the fifth dimension is dependent on the parametric isospin on the time-like or space-like point (the particle-filtering process in observations can be seen in Pachankis [20], and the isospin-violating activities were discussed in van Kolck [19]), and apart from the geographic meaning with seen in Fig. 5, its physical properties, with Heisenberg's indeterminacy principle, can be said to be the speed of time passage determined by the parametric isospin. The concept has been developed in the direction of oscillation from chips and circuit quantum electrodynamics and from the more detailed interpretations of studies on galactic disks [21-23].

3.3 Time Shift Regarding Space

The theory of general relativity covered the explanatory paradigms regarding possible space-time curvature from T'_0 to T_0 in Fig. 5, especially with black holes, however, the matter content capable of such warping was rarely questioned or explored (a preliminary discussion on the inverted Fermi surface can be seen in Pachankis [24], [25], and a recent discussion on nuclear densities regarding Fermi surface from the perspective of Quarkyonic matter in transmutation density can be seen in McLerran and Miller [26]). Therefore, the analytic process from redshifts to gravitational lensing in the determination of observational results is further influenced by another parameter derived from spacetime curvature [23].

The conceptualization of the parameter is differentiated from decadal surveys, but shares similar inferential characteristics [27]. While decadal surveys presume linear regression analyses from the questions of cosmic origins, the parameter of time shift proposed here does not, hence the fifth dimension. The Bayes' theorem can be one of the methodological approaches to the fifth dimension in description of time shift phenomena, which would be post-empiricist [28].

3.4 The Degree of Randomness and Discrete Mathematics

The study of entropy is predominantly considered discrete with the isomorphic premises on the large-scale structure of the universe. With our human perceptions and apparatus dependence on constructing the third dimension of space dependent upon the passage of time (e.g. with the arrival of light particles or transmission of sound waves etc.), the premises of isomorphism seem plausible within the physical limits, i.e., the maximum speed of information transmission by the speed of light, leaving the matter of discreetness to the different combinations to graph theory with a generating set of the group [29, 30]. In other words, we can know and obtain the evidence when our cosmological theories are incomplete, but we do not know by how much afterwards. So, empirical amendments to discrete mathematics are more often than not the cases how the "discrepancies" are dealt with.

The categorical terms of fixed effect and random effects meta-analysis are borrowed from the social and clinical sciences to resolve the indeterminacy and uncertainty problems in terms of incompleteness [31]. In the context of meta-analysis on randomness with the emerging mind-body duality debates in the physical sciences, Delanoy [32] provided two examples of "participants attempting to make a random system behave in a non-random manner" that "under certain circumstances, consciousness interacts with random physical systems". Therefore, the regions of minimum entropy (near the earth) differentiated from those of maximum entropy (around and inside of black holes) contain the premise of the set completeness from the Big Bang Theory with a linearized perspective in geometric sequences [33] .

What I propose, with the conceptualizations outlined, is that the observational results on the large-scale structure of the universe is a combination of both fixed (path-integral applicable [34, 35]) and random effects, and the degree of randomness is determined by the graphic representations of the sets between the third dimension (*d*) and time (*t*) and between time and the fifth dimension (*f*), i.e., the sets of $T = (f, t, \phi)$ and $D = (t, d, \phi)$. The set D constitutes red / blue shifts and parallax, and the connectivity in the set D can be determined by gravity to a threshold with a weighed graph. The weighed graph in the set T, then, constitutes the gravitational lensing with $T \cap D = (t, f, d, \phi_T, \phi_D)$. The degree of randomness between the observed universe and the actual universe is, therefore, denoted $T \cup D - T \cap D$.

3.5 Flavors of Deep Orbit

Whereas the part of $T \cap D$ can be summarized by linearized and hyperbolic and inverse functions, the part of $T \cup D$ can only be resolved by number theory and infinite graphs. The work of Imrich, Klavžar [36] proved that monochromatic 2^n nonisomorphic graph $G = Q_{\sum m<n^m} = Q_n$, and according to the Lagrange theorem, there must be negative space in order for G to be a finite connected graph [37]. Either charge-coupled devices or cryogenic plates, or the other emerging technologies [38], pixelated and qubit information collected and stored in the negative image as negative space by the depth of view can be either treated pixel by pixel as in quantum chromodynamics (QCD) with wavelength sensitivity from the qubits, or in quantum electrodynamics (QED) with the pixel depth of the qubits. In extreme and strong-coupling regimes, flavor-breaking is inevitable even when one tries to correct the QED results to conform to the QCD, and proportional matrices are often adopted in relation to Majorana fermion or mass to bridge the graph theoretical methods with particle physics [39, 40]. In the alternative cases, Scigliuzzo, Calajò [41] justified the atom-bound photonic representation from the QED basis to the QCD [42].

4 Conclusion

From the conceptualization and the empirical review for the solutions, the time-like and space-like singularities can be explained in terms of quantum indeterminacy. The photonic representation of masses with QED follows the isospin detections of space, when the orbital effects are taken into consideration with the qubit data; meanwhile, the time-like singularity is reserved for the particle physics and flavor explanations [43]. Gravitational lensing adopted the spatial fetching technique from the combination of Einstein-Maxwell theories, namely EDO [44, 45], and the strong-coupling regimes justify the theoretical notion of negative space to an extent that is tolerable by the current particle physics theories in the quantum realm.

There are no current solutions to the broken timelines with the conservation of angular momenta. The position element of the particles incident on the detection plates is essential to the concept of time that coordinate human actions and the coordination of instrumentations we depend upon. The time delay functions in gravitational lensing is essential to the limitations of the method itself [46, 47], and the search for the timelike particles that match the anchoring of instrumentation and spatial calculations have been the current approach, with Von Neumann–Bernays–Gödel sets to match the Zermelo–Fraenkel sets in the quantum realm [48]. With the negative space proposition, the transformation of the negative time to the concept of the continuum of space in the quantum realm may supplement the gaps between the observational gravitational lensing and the analytic particle physics possibilities between the sets of *T* and *D*.

In summary, two distinct but correlated topologies are proposed in the research, which are marginal to each other with the element of time that may be both proportional and disproportional to each other. The proportional element combination is the applied and empirical compared to the disproportional element combination, and statistical analyses can be performed to coordinate the potential biases between the observational and the analytic facets of astrophysics and cosmology. Time crystal and time particle anchoring is essential to the engineering and mechanical coordination in further scientific instrumentation and exploration approaches, especially with regard to the strong-coupling regimes and the EDO inferences with the gravitation data in the quantum realm.

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The AI tool ChatGTP is used during the revision of the article for sourcing citations and figure generation, including the model’s mass sourcing capacities that enabled the fine-tuned generation of

Figure 2. The chat history can be reviewed with the URL: https://chatgpt.com/share/686fe9f7-a4a8-800a-88ea-8b0f6c459a28.