Indicies of faintness for ferrotoroidic property of grey groups

G. SIREESHA

Department of Mathematics

VNR Vignana Jyothi Institute of Engineering and Technology,

Hyderabad, INDIA.

Abstract: The basic faintness index of ferrotoroidic polarizability is the smallest of the faintness indices with respect to the individual components of the ferrotoridic polarizability tensors. In this paper the ferroic species and their indices of faintness for ferrotoroidic polarizability are determined by considering grey group as prototypic point group. When the prototypic point group is a grey group, the ferroic species are associated to the corresponding irreducible representations of the grey groups and their corresponding components with faintness indices are tabulated.

Keywords: Ferrotoroidic property, Grey group, faintness indices, axialvector, polar vector

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1. Introduction:

The domain states in crystals can be differentiated by spontaneous polarization, magnetization (or) strain are named as primary ferroic crystals. A ferroic crystal contains two (or) more equally stable domains of the same structure but of different spatial orientation [5]. These domains can coexist in a crystal and may be distinguished by the values of components of certain macroscopic tensorial physical properties of the domains. A fourth type of primary ferroic crystals, a ferrotoroidic crystal has been recently observed (Vanken, 2007) where the domains are distinguished by a toroidial moment [2, 8]. Aizu has given all possible 773 species of the ferroic crystal in phase transitions. This method has been extended by D. B. Litvin, he has calculated twin laws of domain pairs for ferrotoroidic ferroic species of eleven types of physical property tensors. The concept of ferroic species, prototypic and ferroic point groups, evaluation of indices of faintness for all the zero wave number vibrational modes whose oversoftening causes ferroelectricity (or) ferroelasticity are given by Aizu by considering point group as prototypic point group. There is a difference between the ordinary point group and the grey point group i.e in the ordinary point group the antisymmetry operation R_2 is not present at all, where as in the later it is an operation of the group and has the effect of doubling the order of the point group. Since R_2 commutes with all the elements of the point group G. Hence the grey groups are therefore direct product groups of one of the 32 crystallographic point groups G and 1′ (1′ is a group consisting of identity and time inversion operation R_2) and it is denoted as G1′ [1]. Crystals in which the domain states are distinguished by the piezoelectric tensor is an example of secondary ferroic crystals named as Ferromagnetotoroidic (ev^2), Ferromagnetoelastic (aev [v^2]) crystals respectively. Here "V" denotes a polar vector and "e" and "a" denotes zero rank tensors that change under spatial inversion and time inversion respectively [6].

2. Results and Discussion:

The phenomenon of ferromagneto-electric polarizability is the production of a magnetic field I on the application of an electric field E in a direction normal to it. Landau and Lifshitz (1960) had shown that this effect is likely to appear in crystals possessing magnetic structures [3]. Its actual occurrence has been verified in the trioxides of chromium (Astrov, 1960) and Titanium (AL shin and Astrov 1963) in their anti – ferromagnetic state. I and E are connected by the relation

$$I_i = \sum_j \lambda_{ij} E_j \, (i, j, = 1, 2, 3)$$

where λ_{ij} represents a magneto-electric polarizability tensor. Since E is a polar vector and its components are (x, y, z) and I is an axial vector and its components are (x^1, y^1, z^1), λ is a second rank tensor which transforms according to the representation formed by the product of the representation of polar and axial vectors which can be expressed as a matrix in the form of either (xx^1, xy^1, xz^1, yx^1, yy^1, yz^1, zx^1, zy^1, zz^1) (or) (λ_{11}, λ_{12}, λ_{13}, λ_{21}, λ_{22}, λ_{23}, λ_{31}, λ_{32}, λ_{33}) [7]. The character $\chi'(R)$ corresponding to a symmetry element R in this representation is $\chi'(R) = (1 \pm 2\cos\phi)(2\cos\phi \pm 1)$ where the +ve and –ve signs are to be taken accordingly as the symmetry operation R is a pure rotation (or) a rotation – reflection. The faintness indices range from 1 to 6, whether it is = 1 (or) > 1 defines whether the property is normal (or) faint. The components and the indices of faintness for ferrotoroidic polarizability are evaluated for all 32 grey groups and the results are tabulated in the following table.

Table:

Grey Group	Irreducible representations	Ferroic species	Tensor components and indices of faintness for ferrotoroidic polarizability
$\bar{1}1'$	A'	$\bar{1}1'F\bar{1}$	$(xx', xy', xz', yx', yy', yz', zx', zy', zz')$ **1**
	B'	$\bar{1}1'F\bar{1}'$	….
$21'$	A'	$21'F2$	$(xx', xy', yx', yy', zz')$ **1**
	B'	$21'F2'$	(xz', yz', zx', zy') **1**
$m1'$	A_1'	$m1'Fm$	$(xx', xy', yx', yy', zz')$ **1**
	A_1''	$m1'Fm'$	(xz', yz', zx', zy') **1**
$2/m1'$	A_g'	$2/m1'F2/m$	$(xx', xy', yx', yy', zz')$ **1**
	B_g'	$2/m1'F2'/m'$	(xz', yz', zx', zy') **1**
	A_u'	$2/m1'F2/m'$	….
	B_u'	$2/m1'F2'/m$	….
$2221'$	A_1'	$2221'F222$	(xx', yy', zz') **1**
	$B_i'(i=1,2,3)$	$2221'F2'2'2$	(xy', yx') **1**
$mm21'$	A_1'	$mm21'Fmm2$	(xx', yy', zz') **1**
	A_2'	$mm21'Fm'm'2$	(xy', yx') **1**
	$B_i'(i=1,2)$	$mm21'Fmm'2'$	(xz', zx') **1**
$mmm1'$	A_g'	$mmm1'Fmmm$	(xx', yy', zz') **1**
	$B_{ig}'(i=1,2,3)$	$mmm1'Fm'm'm$	(xy', yx') **1**
	A_u'	$mmm1'Fm'm'm'$	….
	$B_{iu}'(i=1,2,3)$	$mmm1'Fmm'm$	….
$41'$	A'	$41'F4$	(xx', yy', zz') **1**

	B'	$41'F4'$	(xy', yx') **1**
	E'	$41'F2'$	(xz', yz', zx', zy') **1**
$\bar{4}1'$	A'	$\bar{4}1'F\bar{4}$	(xx', yy', zz') **1**
	B'	$\bar{4}1'F\bar{4}'$	(xy', yx') **1**
	E'	$\bar{4}1'F2'$	(xz', zx', yz', zy') **1**
$4/m1'$	Ag'	$4/m1'F4/m$	(xx', yy', zz') **1**
	Bg'	$4/m1'F4'/m$	(xy', yx') **1**
	Eg'	$4/m1'F2'/m'$	(xz', yz', zx', zy') **1**
	Au'	$4/m1'F4/m'$	....
	Bu'	$4/m1'F4'/m'$	....
	Eu'	$4/m1'F2'/m$	....
$4221'$	A_1'	$4221'F422$	$\left(\frac{xx'+yy'}{2}, zz'\right)$ **1**
	A_2'	$4221'F42'2'$	(xy') *where* $(\lambda_{21} = -\lambda_{12})$ **1**
	$B_i'(i=1,2)$	$4221'F4'2'2$	(xy', yx') **1**
	E_1'	$4221'F2'$	(xz', yz', zx', zy') **1**
		$4221'F1$	(xz', yz', zx', zy') **1** $(xx', xy', yx', yy', zz')$ **3**
$4mm1'$	A_1'	$4mm1'F4mm$	$\left(\frac{xx'+yy'}{2}, zz'\right)$ **1**
	A_2'	$4mm1'F4m'm'$	(xy') **1**
	$B_i'(i=1,2)$	$4mm1'F4'mm'$	(xy', yx') **1**
	E	$4mm1'F2'$	(xz', yz', zx', zy') **1**
		$4mm1'F1$	(xz', yz', zx', zy') **1** $(xx', xy', yx', yy', zz')$ **3**
$\bar{4}2m1'$	A_1'	$\bar{4}2m1'F\bar{4}2m$	$\left(\frac{xx'+yy'}{2}, zz'\right)$ **1**
	A_2'	$\bar{4}2m1'F\bar{4}2'm'$	(zy') **1**

	$B_1^{'}$	$\bar{4}2m1'F\bar{4}'2m'$	(xy') *where* $(\lambda_{21}=-\lambda_{12})$ **1**
	$B_2^{'}$	$\bar{4}2m1'F\bar{4}'2'm$	(xz') **1**
	E'	$\bar{4}2m1'F2'$	(xz',yz',zx',zy') **1**
		$\bar{4}2m1'F1$	(xz',yz',zx',zy') **1** (xx',xy',yx',yy',zz') **3**
$4/mmm1'$	$A_{1g}^{'}$	$4/mmm1'F4/mmm$	$\left(\frac{xx'+yy'}{2},zz'\right)$ **1**
	$A_{2g}^{'}$	$4/mmm1'F4/m'm'm$	(xy') *where* $(\lambda_{21}=-\lambda_{12})$ **1**
	$B_{ig}^{'}(i=1,2)$	$4/mmm1'F4'/m'mm$	(xy',yx') **1**
	$E_g^{'}$	$4/mmm1'F2'/m'$	(xz',yz',zx',zy') **1**
	$A_{1u}^{'}$	$4/mmm1'F4/m'm'm'$	….
	$A_{2u}^{'}$	$4/mmm1'F4/m'mm$	….
	$B_{iu}^{'}(i=1,2)$	$4/mmm1'F4'/m'm'm$	….
	$E_u^{'}$	$4/mmm1'F2'/m$	….
$31'$	A'	$31'F3$	$\left(\frac{xx'+yy'}{2},xy',zz'\right)$ **1**
	E'	$31'F1$	$(xz',yx',yy',yz',zx',zy')$ **1** (xx',xy',zz') **3**
$\bar{3}1'$	$A_g^{'}$	$\bar{3}1'F\bar{3}$	$\left(\frac{xx'+yy'}{2},xy',zz'\right)$ **1**
	$E_g^{'}$	$\bar{3}1'F\bar{1}$	$(xz',yx',yy',yz',zx',zy')$ **1** (xx',xy',zz') **3**
	$A_u^{'}$	$\bar{3}1'F\bar{3}'$	….
	$E_u^{'}$	$\bar{3}1'F\bar{1}'$	….
$321'$	$A_1^{'}$	$321'F32$	$\left(\frac{xx'+yy'}{2},zz'\right)$ **1**

	A_2'	$321'F32'$	(xy') where $(\lambda_{21}=-\lambda_{12})$ **1**
	E'	$321'F1$	$(xx',xz',yx',yz',zx',zy')$ **1** (yy',xy',zz') **3**
$3m1'$	A_1'	$3m1'F3m$	$\left(\frac{xx'+yy'}{2},zz'\right)$ **1**
	A_2'	$3m1'F3m'$	(xy') where $(\lambda_{21}=-\lambda_{12})$ **1**
	E'	$3m1'F1$	$(xz',yx',yy',yz',zx',zy')$ **1** (xx',xy',zz') **3**
$\bar{3}m1'$	A_{1g}'	$\bar{3}m1'F\bar{3}m$	$\left(\frac{xx'+yy'}{2},zz'\right)$ **1**
	A_{2g}'	$\bar{3}m1'F\bar{3}m'$	(xy') where $(\lambda_{21}=-\lambda_{12})$ **1**
	A_{1u}'	$\bar{3}m1'F\bar{3}'m'$	….
	A_{2u}'	$\bar{3}m1'F\bar{3}'m$	….
	E_g'	$\bar{3}m1'F\bar{1}$	$(xx',xz',yx',yz',zx',zy')$ **1** (xy',yy',zz') **3**
$61'$	A'	$61'F6$	$\left(\frac{xx'+yy'}{2},xy',zz'\right)$ **1**
	B'	$61'F6'$	….
	E_1'	$61'F2'$	(xz',yz',zx',zy') **1**
	E_2'	$61'F2$	(yx',yy') **1** (xx',xy',zz') **3**
$\bar{6}1'$	A_1'	$\bar{6}1'F\bar{6}$	$\left(\frac{xx'+yy'}{2},xy',zz'\right)$ **1**
	E_1'	$\bar{6}1'Fm$	(xz',yz',zx',zy') **1**
	A_1''	$\bar{6}1'F\bar{6}'$	….
	E_1''	$\bar{6}1'Fm'$	(xx',yx') **1** (xy',yy',zz') **3**

$6/m1'$	A_g'	$6/m1'F6/m$	$\left(\frac{xx'+yy'}{2}, xy', zz'\right)$ **1**
	B_g'	$6/m1'F6'/m'$	....
	E_{1g}'	$6/m1'F2'/m'$	(xz', yz', zx', zy') **1**
	E_{2g}'	$6/m1'F2/m$	(xx', yx') **1** (xy', yy', zz') **3**
	A_u'	$6/m1'F6/m'$	....
	B_u'	$6/m1'F6'/m$	....
	E_{1u}'	$6/m1'F2'/m$	....
	E_{2u}'	$6/m1'F2/m'$	....
$6221'$	A_1'	$6221'F622$	$\left(\frac{xx'+yy'}{2}, zz'\right)$ **1**
	A_2'	$6221'F62'2'$	(xy') *where* $(\lambda_{21} = -\lambda_{12})$ **1**
	$B_i'(i=1,2)$	$6221'F6'22'$	....
	E_1'	$6221'F2'$	(xz', yz', zx', zy') **1**
	E_2'	$6221'F2$	(yx', yy') **1** (xx', xy', zz') **3**
$6mm1'$	A_1'	$6mm1'F6mm$	$\left(\frac{xx'+yy'}{2}, zz'\right)$ **1**
	A_2'	$6mm1'F6m'm'$	(xy') *where* $(\lambda_{21} = -\lambda_{12})$ **1**
	$B_i'(i=1,2)$	$6mm1'F6'm'm$	....
	E_1'	$6mm1'F2'$	(xz', yz', zx', zy') **1**
	E_2'	$6mm1'F2$	(yx', yy') **1** (xx', xy', zz') **3**
$\bar{6}2m1'$	A_{1u}'	$\bar{6}2m1'F\bar{6}2m$	$\left(\frac{xx'+yy'}{2}, zz'\right)$ **1**

	$A_{2u}^{'}$	$\bar{6}2m1'F\bar{6}2'm'$	(xy') where $(\lambda_{21}=-\lambda_{12})$ **1**
	$E_{1}^{'}$	$\bar{6}2m1'Fm$	(xz',yz',zx',zy') **1**
	$A_{1u}^{''}$	$\bar{6}2m1'F\bar{6}'2m'$	….
	$A_{2u}^{''}$	$\bar{6}2m1'F\bar{6}'2'm$	….
	$E_{1}^{''}$	$\bar{6}2m1'Fm'$	(yx',yy') **1** (xx',xy',zz') **3**
$6/mmm1'$	$A_{1g}^{'}$	$6/mmm1'F6/mmm$	$\left(\frac{xx'+yy'}{2},zz'\right)$ **1**
	$A_{2g}^{'}$	$6/mmm1'F6/m'm'm$	(xy') where $(\lambda_{21}=-\lambda_{12})$ **1**
	$B_{ig}^{'}(i=1,2)$	$6/mmm1'F6'/m'm'm$	….
	$E_{1g}^{'}$	$6/mmm1'F2'/m'$	(xz',yz',zx',zy') **1**
	$E_{2g}^{'}$	$6/mmm1'F2/m$	(xx',yx') **1** (xy',yy',zz') **3**
	$A_{1u}^{'}$	$6/mmm1'F6/m'm'm'$	….
	$A_{2u}^{'}$	$6/mmm1'F6/mm'm$	….
	$B_{iu}^{'}(i=1,2)$	$6/mmm1'F6'/mm'm$	….
	$E_{1u}^{'}$	$6/mmm1'F2'/m$	….
	$E_{2u}^{'}$	$6/mmm1'F2/m'$	….
$231'$	A'	$231'F23$	$\left(\frac{xx'+yy'+zz'}{3}\right)$ **1**
	E'	$231'F2$	(xy',yx') **1** (xx',yy',zz') **3**
	T'	$231'F1$	$(xz',yy',yz',zx',zy'zz')$ **1** (xx',xy',yx') **3**
$m31'$	$A_{g}^{'}$	$m31'Fm3$	$\left(\frac{xx'+yy'+zz'}{3}\right)$ **1**

	$E_g^{'}$	$m31'F2/m$	(xy',yx') **1** (xx',yy',zz') **3**
	$T_g^{'}$	$m31'F\bar{1}$	$(xx',xz',yy',yz',zx'zy')$ **1** (xy',yx',zz') **3**
	$A_u^{'}$	$m31'Fm'3$	….
	$E_u^{'}$	$m31'F2/m'$	….
	$T_u^{'}$	$m31'F\bar{1}'$	….
$4321'$	$A_1^{'}$	$4321'F432$	$\left(\frac{xx'+yy'+zz'}{3}\right)$ **1**
	$A_2^{'}$	$4321'F4'32'$	….
	E'	$4321'F2$	(xy',yx') **1** (xx',yy',zz') **3**
	$T_i^{'}(i=1,2)$	$4321'F1$	$(xx',xz',yy',yz',zx'zy')$ **1** (xy',yx',zz') **3**
$\bar{4}3m1'$	$A_1^{'}$	$\bar{4}3m1'F\bar{4}3m$	$\left(\frac{xx'+yy'+zz'}{3}\right)$ **1**
	$A_2^{'}$	$\bar{4}3m1'F\bar{4}'3m'$	….
	E_1'	$\bar{4}3m1'F2$	(xy',yx') **1** (xx',yy',zz') **3**
	$T_i^{'}(i=1,2)$	$\bar{4}3m1'F1$	$(xz',yy',yz',zx'zy',zz')$ **1** (xx',xy',yx') **3**
$m3m1'$	$A_{1g}^{'}$	$m3m1'Fm3m$	$\left(\frac{xx'+yy'+zz'}{3}\right)$ **1**
	$A_{2g}^{'}$	$m3m1'Fm3m'$	….
	$E_g^{'}$	$m3m1'F2/m$	(xy',yx') **1** (xx',yy',zz') **3**
	$T_{ig}^{'}(i=1,2)$	$m3m1'F\bar{1}$	$(xx',xz',yz',zx'zy',zz')$ **1**

			(xy', yx', yy') **3**
	A'_{1u}	$m3m1'Fm'3m'$	….
	A'_{2u}	$m3m1'Fm'3m$	….
	E'_{u}	$m3m1'F2/m'$	….
	$T'_{iu}(i=1,2)$	$m3m1'F\bar{1}'$	….

3. Conclusions:

In this present paper ferrotoroidic polarizability property is considered and the ferroic species are associated to their corresponding irreducible representations for all 32 grey groups. The faintness indices for the ferrotoroidic polarizability components of corresponding ferroic species are obtained.

Data Availability

The data used to support the findings of this study are available from the corresponding author or within the article upon request.

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