International Journal of Mathematical and Computational Methods
http://www.iaras.org/iaras/journals/ijmcm

An Application of Intuitionistic Fuzzy Sets to Ethics

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Abstract: - A number of ethical philosophers pointed out the need of introducing a multiple-valued logic in ethics, but nobody has attempted to propose a concrete framework materializing this idea. In the present paper intuitionistic fuzzy sets are used as tools for mathematicizing the ethical rules and interpreting the ethical dilemmas. Our examples are focused on the trolley problem and the dilemma of the searching assassin and the outcomes are compared to the outcomes of our earlier attempt to use soft and neutrosophic sets as tools for the same reason.

Key-Words: - Ethical rules, ethical dilemmas, intuitionistic fuzzy sets, neutrosophic sets.

Received: July 16, 2023. Revised: February 22, 2024. Accepted: March 7, 2024. Published: April 26, 2024.

1. Introduction

The term *ethics* means a framework of principles, rules and laws for the right human behavior, which is acceptable by a certain community or social setting. *Morality* on the other hand is usually understood as something which is personal. Assume, for example, that your local community considers that sex before marriage is immoral, but you have no strong feelings about it. In this case your personal morality contradicts the ethics of your community. The two terms, however, are frequently used interchangeably, loosely meaning the same thing.

Humans are characterized by deep differences among each other, which makes the development of a universally acceptable theory for ethics difficult. This becomes even more difficult due to the fact that the various theories about an ideal system of ethics, developed during human history (e.g. see [1, 2] etc.), are based on principles of the *bivalent logic*. As a result, every human action or behavior is characterized only as “good” or “bad”, without taking into account how good or bad it is.

This frequently creates *moral dilemmas,* i.e. situations in which one has moral reasons to do a series of actions, he/she is able to do each of them, but not all of them. No matter what he/she does, he/she will do something that seems condemned to moral failure.

A characteristic example is the ethical dilemma of *the trolley problem* [3]: Suppose that you suddenly realize that there is a runaway trolley coming in your direction. It is clear that if the trolley continues on its present path, it will kill five workmen who are repairing the track. Also you realize that there is a switch in front of you, which would turn the trolley onto a side track. Unfortunately, there is one workman on the side track who would certainly be killed if you intervene by turning the switch on and diverting the trolley. Even more, imagine the mother of the workman on the side track in place of a neutral observer. What is the right thing to do in both of these cases?

Such kind of dilemmas made a number of philosophers start thinking about the need of introducing a multi-valued logic in ethics, where degrees of truth are used. Apart from some general suggestions (e.g. see [4, 5], etc.), however, nobody has attempted to present a concrete way for materializing this idea.

The next dilemma of the *searching assassin* [6] (pp. 187–195) in which the moral principle of not killing conflicts with the moral obligation of always telling the truth, marks out more emphatically the need of introducing degrees of truth characterizing the moral principles. This happens, because assassination is a much more serious action than telling a lie, since its consequences, i.e. the loss of human lives, cannot be corrected. According to the dilemma of the searching assassin, an individual who searches for his enemy on the purpose of killing him, comes to the house where his enemy is hidden, and asks the housekeeper where he can find him. Let us imagine some of the housekeeper's answers: A_1: "I don't know him", A_2: "I have not seen him during the last few days", A_3: "He passed from here for a few minutes and then he left", A_4: "He was here just before, but now he has left", A_5: "He is hiding in the basement", etc. Obviously only the last of the previous answers is true. Consequently, with respect to Kantian ethics [7] and the principles of BL, this is the unique moral answer. Common sense, however, suggests that it is better to tell a lie in order to save a life.

In an earlier work [8] we have proposed the use of *soft* or *neutrosophic sets* as tools for the introduction of a multiple-valued logic in ethics, which helps to interpret the ethical dilemmas and to suggest better solutions for them. In this paper we show that in case of complete information the same thing may be done equally well by using *intuitionistic fuzzy sets* as tools

The paper is formulated as follows: Section 2 contains the mathematical background needed for the understanding of the rest of the paper. The use of intuitionistic fuzzy sets in ethics is described in Section 3. For reasons of comparison the same examples with [8] are used here to illustrate our results, i.e. the Moses' ethical rule "Thou shalt not kill" and the dilemma of the *searching assassin*.The paper closes with Section 4 including a discussion about the outcomes of our research, the final conclusions and some hints for further research.

2. Mathematical Background

Zadeh, in 1965, extended the concept of a crisp set to that of a *fuzzy set* [9], on the purpose of tackling mathematically the existing in everyday life partial truths, as well as the definitions having no clear boundaries, like "high mountains", "clever people", "good players", etc. Zadeh's idea was to replace the objective function of a crisp set with the *membership function* in a fuzzy set, which takes values in the interval [0, 1]. In this way a *membership degree* between 0 and 1 is assigned to each element of the universal set U with respect to the corresponding fuzzy set. A crisp subset A of U is a fuzzy set in U with membership function defined by m(x) = 1, if $x \in A$ and $m(x) = 0$, if $x \notin A$, $\forall x \in U$.

Before the introduction of fuzzy sets, probability used to be the unique tool in hands of the experts for tackling mathematically the existing in real world uncertainty, which is created by the shortage of knowledge for an observed phenomenon.

Several types of uncertainty exist, including *randomness, imprecision, vagueness, ambiguity, inconsistency*, etc. [10]. The uncertainty due to randomness is related to well-defined events whose outcomes cannot be predicted in advance, like the turn of a coin, the throwing of a die, etc. Imprecision occurs when the corresponding events are well defined, but the possible outcomes cannot be expressed with an exact numerical value; e.g. "The temperature tomorrow will be over 20^o C". Vagueness is created when one is unable to clearly differentiate between two properties, like a good and a mediocre student. In case of ambiguity the existing information leads to several interpretations by different observers; for example the successful result of a game is usually a victory, but under certain conditions could be a draw too. Inconsistency appears when two or more pieces of information cannot be true at the same time. As a result the obtainable in this case information is conflicted or undetermined. For example, "There is an 80% chance for rain tomorrow, but this does not mean that the chance of not raining is 20%, because they might appear hidden for the moment weather factors".

Probability was proved to be effective only for tackling the uncertainty due to randomness, in contrast to fuzzy sets which were proved to be effective for tackling other forms of uncertainty as well, and in particular the uncertainty due to vagueness [11].

Following the introduction of fuzzy sets, several generalizations and other theories related to them have been proposed on the purpose of tackling more effectively the existing uncertainty, e.g. see [12]. None of these theories, however, was proved to be sufficient for tackling effectively all the forms of the existing uncertainty alone, but the synthesis of all of them forms an adequate framework towards this direction.

In 1986, Atanassov enriched Zadeh's degree of membership with the *degree of non-membership* and expanded fuzzy set to the concept of *intuitionistic fuzzy set* in the following way [13]:

Definition 1: An intuitionistic fuzzy set A in the universal set of the discourse U is of the form

$$A = \{(x, m(x), n(x)): x \in U, m(x), n(x) \in [0, 1], \ 0 \leq m(x) + n(x) \leq 1\} \quad (1)$$

In Eq. (1) m: U → [0, 1] is the *membership function* and n: U → [0, 1] is the *non-membership function* of A with m(x) and n(x) being the degrees of membership and non-membership respectively with respect to A, for each x in U. Further, h(x) = 1 - m(x) - n(x), is said to be the *degree of hesitation* of x with respect to A.

For example, if A is the intuitionistic fuzzy set of the good players of a team and (x, 0.6, 0.3) ∈ A, this means that there is a 60% belief that x is a good player, but simultaneously a 20% belief that he is not a good player, and a 10% hesitation to decide about it. The intuitionistic fuzzy sets are suitable for tackling the uncertainty due to imprecision, which appears frequently in human reasoning [14].

A fuzzy set with membership function y = m(x) is an intuitionistic fuzzy set with non-membership function n(x) = 1 – m[x) and hesitation degree h(x) = 0, for all x in U. The basic concepts and operations defined on crisp and fuzzy sets, like subset, complement, union, intersection, etc., are extended in a natural way to intuitionistic fuzzy sets [13, 14].

Here, for simplicity, we will denote an intuitionistic fuzzy set A by A = <m, n> and the elements of A in the form of *intuitionistic fuzzy pairs* (m, n), with m, n in [0, 1], 0 ≤ m + n ≤ 1.

3. Representation of the Ethical Rules Using Intuitionistic Fuzzy Sets

Here we show that the ethical rules of a community or a social setting can be represented using intuitionistic fuzzy sets as tools. As a result, a moral theory, being a collection of ethical rules, could be considered as a collection of intuitionistic fuzzy sets. This consideration, which gives satisfactory solutions to the ethical dilemmas, could be proved in future as the starting point of a modern approach of ethics on a mathematical basis.

Let us consider, for example, the Moses' rule "Thou shalt not kill", which is widely acceptable by people, regardless of their religion and origin. The following question is raised in this case: Have all the assassinations the same degree of immorality? Compare, for instance, an assassination designed with every detail and performed under complete soberness (case C_1), with an assassination performed under a condition of "boiling soul" (C_2), or with the assassin being in defense trying to protect his/her life (C_3), or even with the instinctive reaction of the mother in the trolley problem to save the life of her son (C_4). Consider also the case of a neutral observer in place of the mother, who decides to turn the switch on to save the lives of the five workmen, thus causing the death of the other one, who is working on the side track (C_5).

From the previous discussion it becomes evident that there is a need for evaluating the degree of immorality of each assassination. This need, however, cannot be materialized with the help of the bivalent logic, which simply considers all assassinations as being immoral actions.

To overcome this difficulty, let us consider the set U of all assassinations as the set of the discourse and the intuitionistic fuzzy set A in U of the assassinations with lightning. Then each assassination can be characterized by an intuitionistic fuzzy pair in A. We may have, for example, that C_1 (0, 1), C_2 (0.2, 0.7), C_3 (0.6, 0.2), C_4 (0.5, 0.2), C_5 (0.9, 0) and so on. This means that the assassination C_1 has no lightning at all, C_2 is with a 20% lightning and a 70% guilt, but there is also a 10% hesitation to decide about it, etc.

Similarly for the dilemma of the searching assassin, considering as set of the discourse U the set of all the housekeeper's possible answers and defining the intuitionistic fuzzy set A in U of the "right" answers, according to common sense, we may have that A_1(0.3, 0.6), A_2(0.4, 0.4), A_3(0.5, 0.3), A_4(0.8, 0.2), A_5(0.2, 0.8). This means that the answer $A_1$is characterized 30% as right and 60% as wrong (unnecessary lie) with a 10% hesitation to decide about it, and so on.

As said before, this way of thinking gives satisfactory solutions to the ethical dilemmas. In the trolley problem, for example, the action of a neutral observer, who decides to save the lives of the five workmen by causing the death of the other one who works in the side track, was characterized as having a 90% lightning and a 10% hesitation to decide about it. Also the mother's instinctive reaction to save the life of her son was characterized with a 50% lightning, a 20% guilt and a 30% hesitation to decide about it.

The previous characterizations depend of course on the objectivity of the criteria of the decision-maker (e.g. the judge, the jury, etc.), which means that could not be the ideal ones. These characterizations, however are much better than those based on the principles of bivalent logic (innocent – guilty).

Remark 1: In 1995 Smarandache added to the degrees of membership (m) and non-membership (n) the degree of *indeterminacy* or *neutrality* (i) and extended further the concept of intuitionistic fuzzy set to that of *neutrosophic set* [15]. The elements of a neutrosophic set A can be written in the form of *neutrosophic triplets* (m, i, n), with m, i, n ∈ [0, 1] and 0 ≤ m + i + + n ≤ 3. Let x = (m, i, n) ∈ A. Then, if m + i + n < 1, x is characterized by incomplete information with respect to A, if m + i + n = 1 by complete, and if m + i + n > 1 by inconsistent (i.e. contradiction tolerant) information.

A neutrosophic set may simultaneously contain elements corresponding to all types of information. An intuitionistic fuzzy set A is a neutrosophic set with indeterminacy (i) equal to hesitation (h) and m + n + + h =1, ∀ x ∈ A (complete information).

In an earlier work [8] we have used neutrosophic instead of intuitionistic fuzzy sets in ethics. The advantage of neutrosophic sets is that they can handle inconsistent or incomplete information, which is not possible when using intuitionistic fuzzy sets. Also in the same work [8] we have proposed a parametric approach for a mathematical representation of the ethical rules using soft sets as tools

5. Discussion and Conclusions

In this work intuitionistic fuzzy sets were used as tools for introducing a multiple-valued logic in ethics, which helps to interpret the ethical dilemmas and to suggest satisfactory solutions for them.

A "weak" point of the theory of fuzzy sets is that there is no general rule for defining the membership functions. The methods used for this are either statistical or intuitive depending on the subjective criteria of each observer and resulting in definitions which are not unique. In the fuzzy set of the expensive cars, for example, one may consider all the cars with price over 25000 euros as being expensive and another one all cars with price over 30000 euros, etc.

This "weakness" is transferred to the membership and non-membership functions of the intuitionistic fuzzy sets. As a result, the quality of the results obtained in this work depend on the objectivity of the decision-maker's criteria. The same holds when using neutrosophic instead of intuitionistic fuzzy sets. Neutrosophic sets, however, can tackle all the types of information, in contrast to intuitionistic fuzzy sets, which can tackle complete information only.

The combination of two or more theories related to fuzzy sets and their generalizations appears to be an effective way for tackling more effectively the uncertainty in various human or machine activities, like assessment, decision-making etc. (e.g. see [16-18]). Consequently this is a promising area for further research in the field of ethics too.

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Sources of funding: This research received no external funding.